Quadratics and Rocketry

Kevin Regardie

This is a multi-faceted lesson based on quadratic functions and their application to the study of rocketry. Quadratic functions have important applications in science and engineering. In this lesson, students will apply their knowledge of quadratic functions in three distinct modular themes. By considering real-world examples in the classroom, students have an opportunity to broaden their perspectives, make connections to future careers, and build excitement in the application of mathematical concepts. The focus on Science, Technology, Engineering, and Mathematics (STEM) concepts and skills provides cross-curricular opportunities and is vital to becoming a productive member of our workforce.

This themed unit studies rocket flight characteristics and their applications to quadratic equations. Using the HP 39GII calculator, students will complete an activity designed to deepen their understanding of quadratic concepts, including graphing, finding the vertex, uses of the discriminant, and quadratic inequalities. The lessons consist of an interactive demo, lesson plans and student activities. The demo, and calculator activities may be used separately or as described in the sample instructional plan. It may even be spread out into two days or extended into a homework assignment.

The avalanche component is designed as a review and extension of graphing quadratic functions as it requires some previous understanding of this process. Concepts such as extrema, roots, axis of symmetry, and sketching a graph are emphasized. The model rocket component is best applied after covering factoring, completing the square, and vertex form of a quadratic equation. Previous work with regression or lines of best fit is recommended as well. The fireworks component wraps up a chapter covering quadratic equations by covering the discriminant and transformations of quadratic graphs. It also touches on quadratic inequalities.

As a result of utilizing these lessons, students will be able to model real world problems using quadratic functions; develop depth of understanding of the interconnected nature of solutions, graphs, and representations of quadratic functions; analyze and interpret applications of quadratic functions.

It's been my experience that many existing text book examples and/or internet resources do not develop desired depth or critical thinking skills that students should be expected to demonstrate.
While you are not required to use the HP 39gII calculator, it is advantageous as it offers many features that don't exist in other graphing calculators on the market.

For more information on the HP 39gII Graphing Calculator, click here to download the data sheet.

To download the Interactive Rocketry Demo, please click on this link: Interactive Demo

All six PDF versions of the lesson plans can be seen and printed below.

About the Author

Kevin Regardie holds a B.S. in Aerospace Engineering from the University of Southern California and a MS in Education from National University. In his 10 years as a GATE certified teacher, Kevin has taught an array of high school mathematics courses including AP Calculus. As a the faculty advisor for his school's Astronomy Club, Team America Rocketry Challenge (TARC) team, and Science Fair Competition, Kevin strives to increase student engagement and facilitate hands-on experience in STEM learning. His work demonstrates that technology integration and hands on learning can revive and reinvigorate mathematics education.

Quadratics and Rocketry

Lesson Plan

Description:

This is a multi-faceted lesson based on quadratic functions and their application to the study of rocketry. Quadratic functions have important applications in science and engineering. In this lesson, students will apply their knowledge of quadratic functions in three distinct modular themes.

By considering real-world examples in the classroom, students have an opportunity to broaden their perspectives, make connections to future careers, and build excitement in the application of mathematical concepts. The focus on Science, Technology, Engineering, and Mathematics (STEM) concepts and skills provides cross-curricular opportunities and is vital to becoming a productive member of our workforce. This themed unit studies rocket flight characteristics and their applications to quadratic equations. Using the HP 39GII calculator, students will complete an activity designed to deepen their understanding of quadratic concepts, including graphing, finding the vertex, uses of the discriminant, and inequalities.

Grade: 10th, 11th, 12th
Subjects: Intermediate Algebra, Pre-Calculus, Mathematical Analysis
Engineering Concepts: Rocket design & modeling, aerodynamics, flight characteristics
Topics: Quadratic Equations, Graphing, Solving Equations
Time Needed: 3 50-minute class period

Objectives:

- Students will be able to model real world problems using quadratic functions
- Students will develop depth of understanding of the interconnected nature of solutions, graphs, and representations of quadratic functions
- Students will analyze and interpret applications of quadratic functions

Standards:

Common Core Standards	**Reasoning with Equations and Inequalities** A – REI	**Solve equations and inequalities in one variable** 4. Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the Quad-ratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. **Solve systems of equations** 7. Solve a simple system consisting of a linear equation and a Quadratic equation in two variables algebraically and graph-ically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.
		Analyze functions using different representations 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more comp-licated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. y and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.

	Linear, Quadratic, and Exponential Models F - LE	**Interpret expressions for functions in terms of the situation they model** 6. Apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. (CA Standard Algebra I – 23.0)
NCTM Standards		• analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions • interpret representations of functions of two variables • draw reasonable conclusions about a situation being modeled

Instructional Plan: 3 50-minute periods

Day 1:
Intro: 5-10 minutes
Show the 'Avalanche flipchart'
Review concepts of quadratic graphs
Model: 10-15 minutes
Show the Avalanche portion of the interactive demo
Practice: 25-30 minutes
Have students complete the Avalanche calculator activity
Close: 5 minutes
Answer questions, check for understanding

Day 2:
Intro: 5-10 minutes
Show the 'Model Rocket' flipchart
Review concepts of solving quadratic equations
Model: 10-15 minutes
Show the Model Rocket portion of the interactive demo
Practice: 25-30 minutes
Have students complete the Model Rocket calculator activity
Close: 5 minutes
Answer questions, check for understanding

Day 3:
Intro: 5-10 minutes
Show the 'Fireworks' flipchart
Review concepts of transformations of quadratic functions and inequalities

Model: 10-15 minutes
Show the Fireworks portion of the interactive demo
Practice: 25-30 minutes

Have students complete the Fireworks calculator activity
Close: 5 minutes
Answer questions, check for understanding

Teacher Notes:
The flipchart, demo, and calculator activity may be used separately or as described in the sample instructional plan. It may even be spread out into two days or extended into a homework assignment.

Avalanche:

This component is designed as a review and extension of graphing quadratic functions as it requires some previous understanding of this process. Concepts such as extrema, roots, axis of symmetry, and sketching a graph are emphasized.

Model Rocket:

This component is best applied after covering factoring, completing the square, and vertex form of a quadratic equation. Previous work with regression or lines of best fit is recommended as well. The solution to Part 3 follows:

$\mathbf{y = ax^2 + bx + c}$	standard form
$\mathbf{y = a(x^2 + \frac{b}{a}x + (\ \ )^2) + c - a(\ \ )^2}$	factor a, set up to complete the square
$\mathbf{y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)}$	complete the square
$\mathbf{y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)}$	complete the square
$\mathbf{y = a(x - h)^2 + k}$	vertex form
$\mathbf{h = -\frac{b}{2a} \qquad k = c - \frac{b^2}{4a}}$	**h,k in terms of b,a**
$\mathbf{0 = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)}$	solve by setting y equal to zero
$\frac{b^2}{4a} - c = a\left(x + \frac{b}{2a}\right)^2$	subtract
$\frac{b^2}{4a^2} - \frac{c}{a} = \left(x + \frac{b}{2a}\right)^2$	divide by a
$\frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$	common denominator
$\pm\sqrt{\frac{b^2-4ac}{4a^2}} = \left(x + \frac{b}{2a}\right)$	square root
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$	quadratic formula

Fireworks:

This component wraps up a chapter covering quadratic equations by covering the discriminant and transformations of quadratic graphs. It also touches on quadratic inequalities.

Resources:
Interactive Demo
'Avalanche' flipchart, calculator activity, solution guide
'Model Rocket' flipchart, calculator activity, solution guide
'Fireworks' flipchart, calculator activity, solution guide
HP 39gII calculator or other graphing calculator

Avalanche Calculator Activity

INTRO

Avalanches are dangerous events that can cause loss of life and can destroy settlements, roads, railways and forests. Avalanche control uses guns to fire explosive projectiles onto slopes too distant or dangerous for patrollers to approach on skis. As a gunner for the Mile High Avalanche Mitigation Company, your job is to calculate the proper velocity to fire the rocket from an initial height of 140 meters that will hit the designated target at a height of 1380 meters. You know from experience that the rocket should be in flight for 20 seconds.

$$y = -\frac{1}{2}gt^2 + vt + h$$

where :

{ y is the height of the target (1380 m), g is the gravitational constant (9.8 m/s^2), v is the initial velocity (m/s), t is the time (20 seconds), h is the initial height (140

Part I

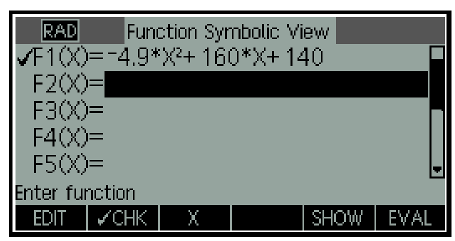
1. Let's begin by solving for the initial velocity of the avalanche gun using

the Solve App. Press 

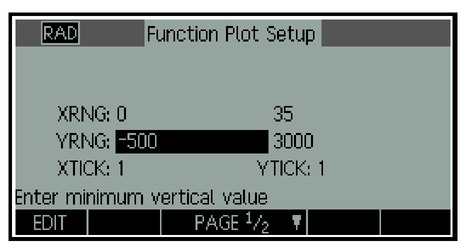
, select **Solve,** and press the **Start**

menu key(F6). Enter the equation above as well as the values given. What did you find? How do the values change if the gun is fired from ground level?

2. Next, go to the Function App and enter the equation, including the initial velocity, into F1(X). Will the parabola open up or down? Explain?



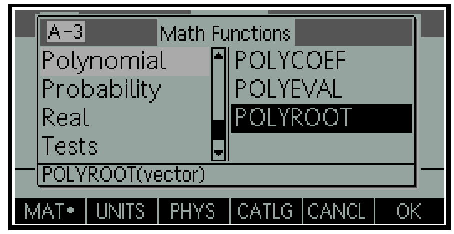
3. Use the viewing window to the right.



4. Find the y-coordinate of the vertex. Is this the maximum or minimum of the function? What do the coordinates of the vertex tell you about the flight of the projectile? Explain why negative values for y and t do not make sense for this problem. Repeat this procedure for the other velocities by graphing the next two equations (into F2(X) and F3(X)) and completing the table to the right.

Velocity	Maximum Altitude
130	
160	
190	

5. When will the projectile hit the ground? How far away does it hit the ground? What method did you use to find the roots of the equation? An alternative method exists using the Math Menu in the 39gII graphing calculator. Go to Home Modes, then, Math Cmds B scroll to polynomial and select polyroot. This calculator function returns the roots of a polynomial with specified coefficients. What values does polyroot([-4.9,160,140]) return? Complete the table for the remaining velocities. Which roots are valid for this problem? Why? Are the results from #4 and #5 consistent with your expectations? Explain.



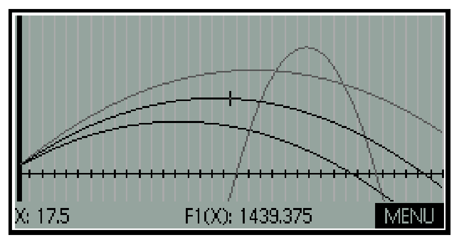
Velocity	Roots
130	
160	
190	

Part II

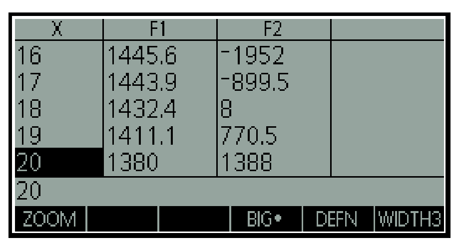
6. Let's assume we have a snow covered mountain that is shaped like a parabola and is defined by the equation:

$$y = -72.5x^2 + 3445x - 38512$$

Enter this equation into F4(X).



7. Find the vertex of the mountain equation. Estimate the roots by examining the graph as well as the table of values (an example is provided to the right). Then find the exact roots using one of the methods described previously.



8. Find the points of intersection between the gun equations and the left side of the mountain equation. Complete the table.

Velocity	**Points of Intersection**
130	
160	
190	

Part III

9. A sky diver is planning on jumping from a plane and landing on the mountain. His descent is modeled by the equation:

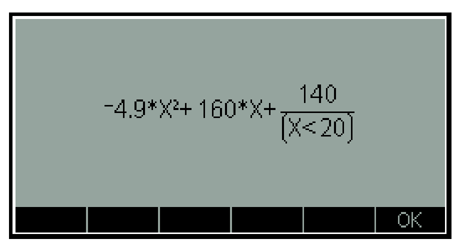
$$y = -4.9x^2 + 49x + 2777.5$$

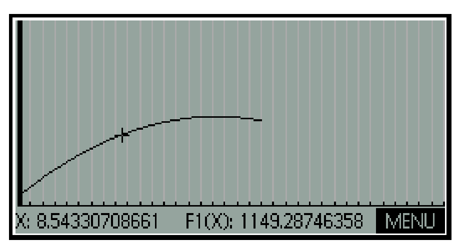
Enter this equation in F5(X). Find the vertex. Write an equation for the plane (assume horizontal motion). Enter this equation in F6(X).

10. Find the point of intersection between the skydiver equation and the left side of the mountain equation. How long was the skydiver in the air? Is the skydiver in danger of being hit by the avalanche gun? Explain.

Part IV

11. Finally, we want to 'clean up' the graph by graphing only the segments that are applicable to this activity. You can do this by dividing the equation by the domain, in inequality form, that you wish to display. An example is shown to the right. Complete this procedure for the remaining functions to create a clean pictograph of the mountain, avalanche gun, skydiver, and airplane.





Avalanche Calculator Solutions

INTRO

Avalanches are dangerous events that can cause loss of life and can destroy settlements, roads, railways and forests. Avalanche control uses guns to fire explosive projectiles onto slopes too distant or dangerous for patrollers to approach on skis. As a gunner for the Mile High Avalanche Mitigation Company, your job is to calculate the proper velocity to fire the rocket from an initial height of 140 meters that will hit the designated target at a height of 1380 meters. You know from experience that the rocket should be in flight for 20 seconds.

$$y = -\frac{1}{2}gt^2 + vt + h$$

where :

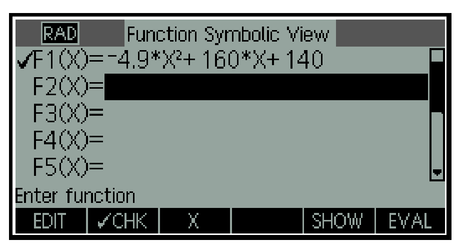
{ y is the height of the target (1380 m), g is the gravitational constant (9.8 m/s^2), v is the initial velocity (m/s), t is the time (20 seconds), h is the initial height (140

Part I

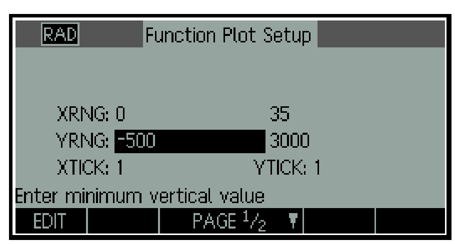
1. Let’s begin by solving for the initial velocity of the avalanche gun using the Solve App. Press Apps Info, select **Solve,** and press the **Start** menu key(F6). Enter the equation above as well as the values given. What did you find? V=160 m/s How do the values change if the gun is fired from ground level? V=167 m/s

2. Next, go to the Function App and enter the equation, including the initial velocity, into F1(X). Will the parabola open up or down? Explain?

Down. The leading coefficient is negative.

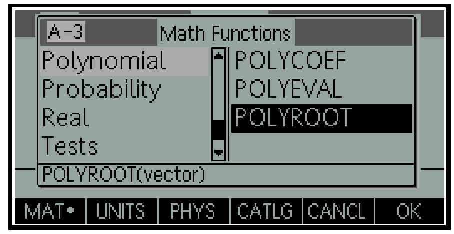


3. Use the viewing window to the right.



4. Find the y-coordinate of the vertex. 1446 m. Is this the maximum or minimum of the function? Max. What do the coordinates of the vertex tell you about the flight of the projectile? The x value of the vertex represents the time when the projectile reaches the maximum height, the y value represents the maximum height. Explain why negative values for y and t do not make sense for this problem. Time is defined for values greater than or equal to zero. Negative values for y would indicate the projectile is below the ground. Repeat this procedure for the other velocities by graphing the next two equations (into F2(X) and F3(X)) and completing the table to the right.

Velocity	Maximum Altitude
130	1002 m
160	1446 m
190	1982 m



5. When will the projectile hit the ground? 33.51 seconds. What method did you use to find the roots of the equation? MENU, FNC, Root. An alternative method exists using the Math Menu in the 39gII graphing calculator. Go to Home Modes, then Math Cmds B, scroll to polynomial and select polyroot. This calculator function returns the roots of a polynomial with specified coefficients. What values does polyroot([-4.9,160,140]) return? (33.51, - .85). Complete the table for the remaining velocities. Which roots are valid for this problem? Positive. Why? Negative roots are outside the domain for the time. Are the results from #4 and #5 consistent with your expectations? Explain. Answers vary.

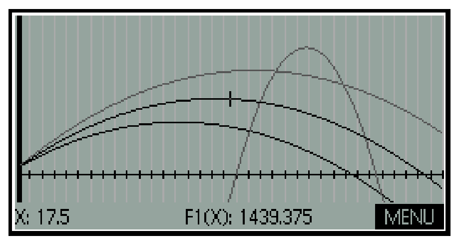
Velocity	Roots
130	-1.04, 27.57
160	- .85, 33.51
190	- .72, 39.5

Part II

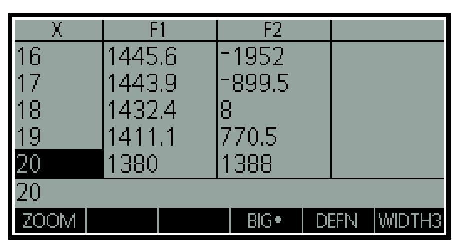
6. Let's assume we have a snow covered mountain that is shaped like a parabola and is defined by the equation:

$$y = -72.5x^2 + 3445x - 38512$$

Enter this equation into F4(X).



7. Find the vertex of the mountain equation. 2412 m. Estimate the roots by examining the graph as well as the table of values (an example is provided to the right). Roots are between 17 and 18 as well as between 29 and 30. Then find the exact roots using one of the methods described previously. 17.99 and 29.53



8. Find the points of intersection between the gun equations and the left side of the mountain equation. Complete the table.

Velocity	Points of Intersection
130	(19.1, 835.7)
160	(20, 1380.5)
190	(21.3, 1964.4)

Part III

9. A sky diver is planning on jumping from a plane and landing on the mountain. His descent is modeled by the equation:

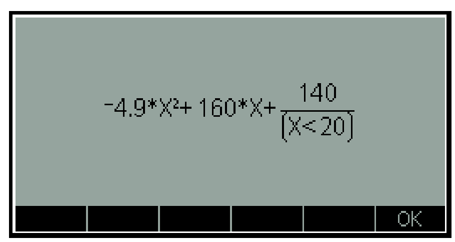
$$y = -4.9x^2 + 49x + 2777.5$$

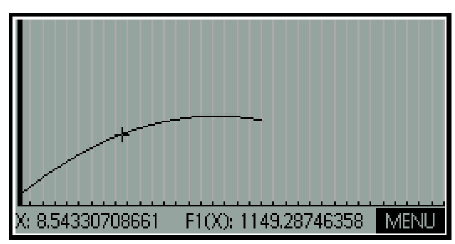
Enter this equation in F5(X). Find the vertex. (5, 2900) Write an equation for the plane (assume horizontal motion). Y=2900 Enter this equation in F6(X).

10. Find the point of intersection between the skydiver equation and the left side of the mountain equation. (20.6, 1702.9) How long was the skydiver in the air? 14.6 seconds (20.6 sec — 5 sec) Is the skydiver in danger of being hit by the avalanche gun? Explain. Only at the velocity set to 190 m/s.

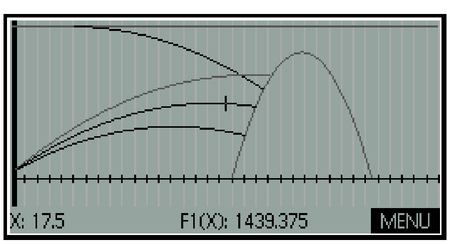
Part IV

11. Finally, we want to 'clean up' the graph by graphing only the segments that are applicable to this activity. You can do this by dividing the equation by the domain, in inequality form, that you wish to display. An example is shown to the right. Complete this procedure for the remaining functions to create a clean pictograph of the mountain, avalanche gun, skydiver, and airplane.





Final Pictograph:



Model Rocket Calculator Activity

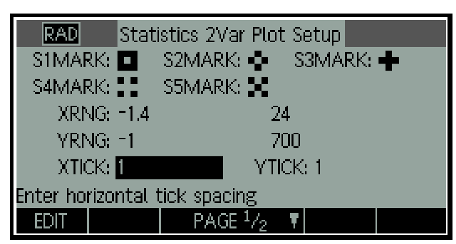
Model rockets have evolved after generations of research and experimentation from weapons of war to the modern version of this safe and widespread hobby. Model rockets utilize various recovery systems and motor performance in applications such as aerial photography, experimentation, and high powered rocketry.

Part I

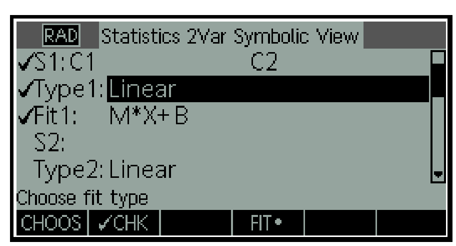
As a student at White Sands High School, you are building a model rocket for the science fair competition. The table shows the height of the rocket y measured in meters after x seconds. Find and graph a linear regression equation as well as a quadratic regression equation. Determine which is a better fit for the data.

Time (sec.)	1	2	3	4	5	6	7	8
Height (m)	105.1	200.4	286.9	361.6	427.5	483.6	529.9	566.4

1. To find the linear regression equation, start the Statistics 2Var App and enter the times into C1 and the heights into C2. Use the Plot setup shown below for a proper viewing window. Press Symb Setup , choose linear, and press Plot Setup . How many points does the line cross through? Is this a good fit for the data? Why or why not?



2. Find the quadratic regression equation. Press Symb Setup and choose quadratic. How many points does the line cross through? Is this a good fit for the data? Why or why not?



Part II

As a student at White Sands High School, you are building a model rocket for the science fair competition. Your data recorder gives you the altitude for the first three seconds: (1, 75.2), (2,140.5), (3,196). Your goal is to find the maximum height to deploy the parachute and allow for a safe recovery. You will need to find the maximum height by using a quadratic regression model to fit a curve to the data points.

3. Find the quadratic regression equation. Copy the equation, open the function app, and paste into F1(X).

4. Adjust the view and find the vertex by pressing **MENU**(F6), **FCN** (F4), and then **Extremum**. What do the coordinates of the vertex tell you about the flight of the rocket? Use the graph to verify the original data points. If the parachute fails to deploy, when will the rocket hit the ground?

$$y = -\frac{1}{2}gx^2 + vx + h$$

where :

{y is the height of the rocket (in meters), g is the gravitational constant (9.8 m/s^2), v is the initial velocity (in m/s), x is the time (in seconds), h is the initial height (in meters).}

Part III

5. Given the quadratic equation in standard form to the right, solve for x by completing the square. What equation did you derive?

<u>Standard Form:</u>

$$y = ax^2 + bx + c$$

Part III

6. You have been tasked to build another rocket that satisfies the following information. Write an equation for the parabola that has a vertex of (5.1, 127.5) and crosses through the point (1, 45). Use vertex form of a quadratic equation.

Vertex form:

$$y = a(x - h)^2 + k$$

What options exist to verify that the graph passes through the points? Find the initial velocity.

Part IV

7. Use the discriminant to show that there are always two elapsed times at which the altitude is zero, assuming the initial velocity is positive. Then, find the maximum altitude of the rocket.

$$h(t) = -4.9t^2 + v_0 t$$

Model Rocket Calculator Solutions

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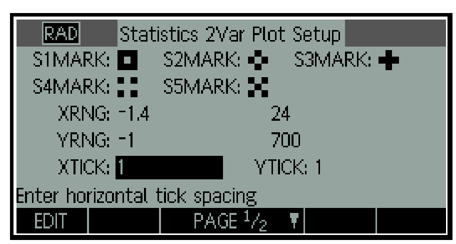
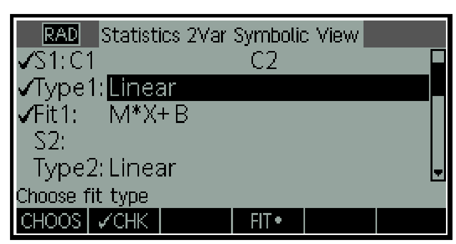


2. Find the quadratic regression equation. Press Symb Setup and choose quad- ratic. How many points does the line cross through? All. Is this a good



Part II

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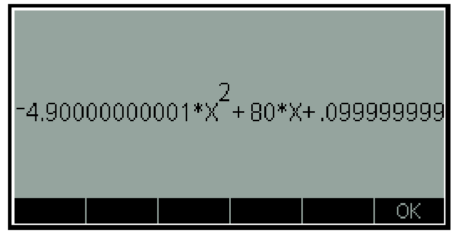


3. Find the quadratic regression equation. See screen shot to the right. Copy the equation, open the function app, and paste into F1(X).

4. Adjust the view and find the vertex by pressing **MENU**(F6), **FCN** (F4), and then **Extremum**. (8.16, 326.63) What do the coordinates of the vertex tell you about the flight of the rocket? It reaches a maximum height of 326.63 m after 8.16 seconds. Use the graph to verify the original data points. Go to M If the parachute fails

Part III

Standard Form:

$$y = ax^2 + bx + c$$

5. Given the quadratic equation in standard form to the right, solve for x by completing the square. See teacher notes. What equation did you derive? The quadratic formula.

Part III

Vertex form:

$$y = a(x-h)^2 + k$$

6. You have been tasked to build another rocket that satisfies the following information. Write an equation for the parabola that has a vertex of (5.1, 127.5) and crosses through the point (1, 45). Use vertex form of a quadratic equation. See below (plug vertex into h,k and point into x,y and solve for a)

$$y = -4.908(x-5.1)^2 + 127.5$$

What options exist to verify that the graph passes through the points? Graph or plug in points. Find the initial velocity. V=50 m/s (convert vertex form into standard form. Initial velocity is b.

Part IV

Let:

$$h(t) = -4.9t^2 + v_0 t$$

7. Use the discriminant to show that there are always two elapsed times at which the altitude is zero, assuming the initial velocity is positive. See solution to the right. Then, find the maximum altitude of the rocket.

Maximum altitude: $\frac{(v_0)^2}{19.6}$

{a=—4.9 $b = v_0$ c=0}. The discriminant is:

$$(v_0)^2 - 4(-4.9)(0) = (v_0)^2 > 0$$

Thus, there are two values of t for which h(t)=0.

Fireworks Calculator Activity

Fireworks date back to 7th century China. Today, fireworks displays are a common focal point of many celebrations around the world. City officials of San Diego want to arrange a fireworks show as part of the city's anniversary festival. Due to safety and legal constraints, and to allow for optimal viewing, they set the minimum height for each fireworks device to be set off at 300 meters. As an employee of The Garden State Fireworks Company, you have been tasked to set the fuse times of the fireworks to fit within these parameters.

We will be analyzing families of parabolas in vertex form. What conjecture can you make about the effect of changing the value of each of the constants a, h, and v on the graph of the parabola? You can use the HP39gII calculator to analyze these relationships.

Vertex form:

$$Y = a(X + h)^2 + v$$

Standard Form:

$$Y = aX^2 + bX + c$$

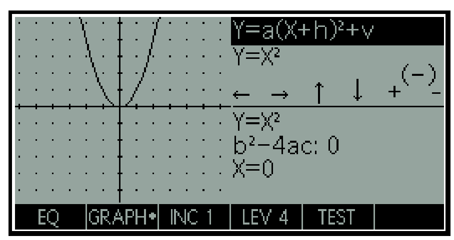
Part I

Let's begin by exploring the capabilities of the Quadratic Explorer App.

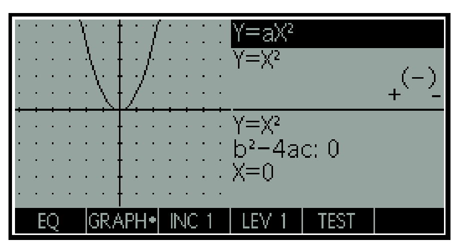
Press, 

select **Quadratic Explorer,** and press the **Start** menu key(F6).

1. Notice the information that is available. Listed first is the vertex form of a quadratic equation with variables and the next line with numerical values. Next are the available active calculator keys to adjust the graph. Then are standard form of a quadratic equation the discriminant, and roots X1 and X2, respectfully.

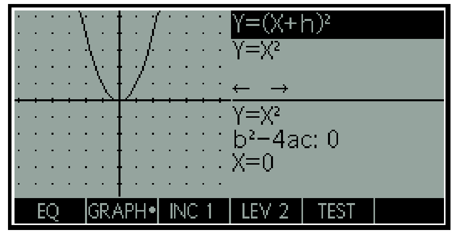


2. Press menu key F4 to access level 1. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? Are the roots and/or the discriminant affected by these keys? Why or why not?

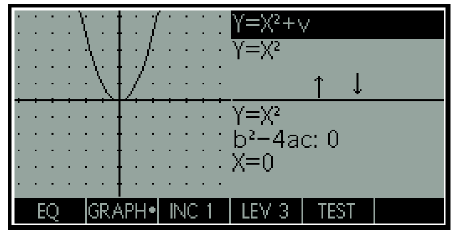


Fireworks Calculator Activity

3. Press menu key F4 to access level 2. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? How is h related to b and c? Write an expression for b and c in terms of h.

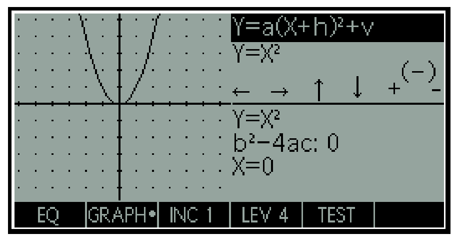


4. Press menu key F4 to access level 3. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? To the equation? What is a and h equal to? Why is the discriminant changing now? How is the discriminant related to the roots?

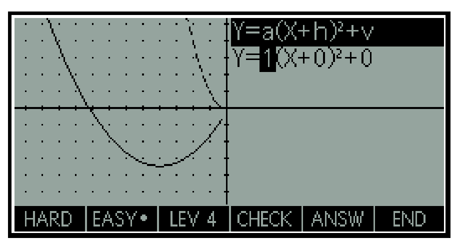


5. Press menu key F4 to access level 4.

- Click the negative sign (—). What changes?
- Click the plus and minus signs. What changes?
- Click the left and right arrows. What changes?
- Click the up and down arrows. What changes?



6. Press menu key F5 to access the **TEST** mode. You must manipulate the equation's parameters to make the equation match the target graph. When you feel that you have correctly chosen the parameters a **CHECK** menu key evaluates the answer and provide feedback. An **ANSW** menu key is provided for those who give up! Write down each of your guesses. Try again with the **HARD** option. Can the parabola given in your test be used as a model for the trajectory of a fireworks rocket launch? Why or why not? If not, what can be changed?



Fireworks Calculator Solutions

Fireworks date back to 7th century China. Today, fireworks displays are a common focal point of many celebrations around the world. City officials of San Diego want to arrange a fireworks show as part of the city's anniversary festival. Due to safety and legal constraints, and to allow for optimal viewing, they set the minimum height for each fireworks device to be set off at 300 meters. As an employee of The Garden State Fireworks Company, you have been tasked to set the fuse times of the fireworks to fit within these parameters.

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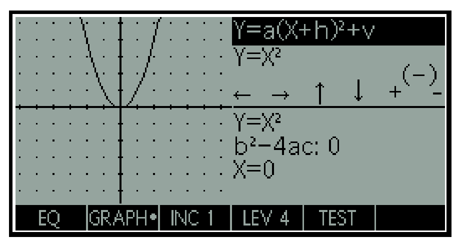
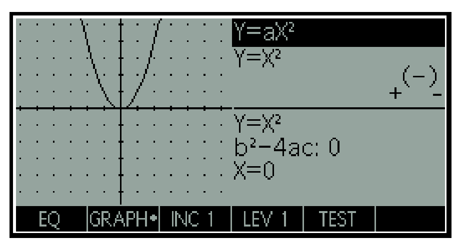
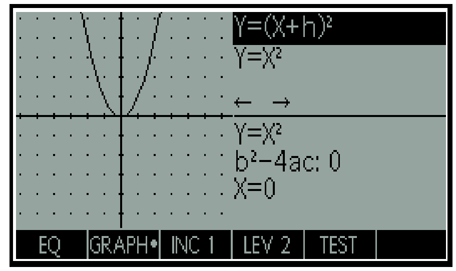


2. Press menu key F4 to access level 1. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? + and — changes the shape while (—) reflects the graph about the x-axis To the equation? + increases a, — decreases a, (—) changes the sign. Are the roots and/or the discriminant affected by these keys? No. Why or why not? Roots: double root at zero unaffected by a. Discriminant: b and c are zero.

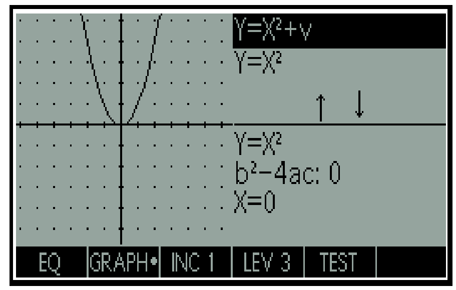


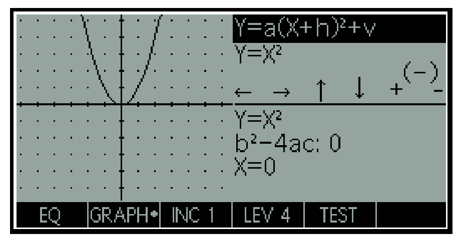
Fireworks Calculator Solutions

3. Press menu key F4 to access level 2. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? Arrows move graph left and right respectfully. To the equation? Arrows change the value of h. How is h related to a and b? Found by completing the square Write an expression for h in terms of a and b. h=—b/(2a)



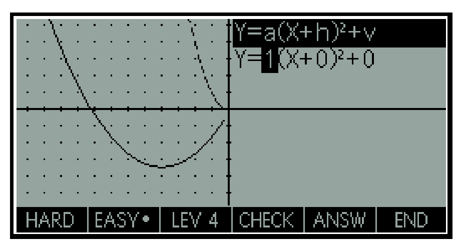
4. Press menu key F4 to access level 3. Notice the active calculator keys that are available. Try experimenting with each key. What does each key do to the graph? Arrows move graph up and down respectfully. To the equation? Arrows change the value of v. What is a and h equal to? a=1, h=0. Why is the discriminant changing now? Discriminant is related to the number of roots, so as the graph moves up and down the discriminant and the number and type of roots is affected. How is the discriminant related to the roots? Discriminant negative: no real roots, discriminant zero: 1 root, discriminant positive: 2 roots.





5. Press menu key F4 to access level 4.

- Click the negative sign (—). What changes? a, discriminant, roots
- Click the plus and minus signs. What changes? a, b, c, discriminant, roots
- Click the left and right arrows. What changes? h, b, c, roots
- Click the up and down arrows. What changes? v, c, discriminant, roots



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