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Volume 10 December 2008

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## Patterns and sequences

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## Feature calculator of the month: 35 s Scientific Calculator

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Get professional performance from the Ultimate RPN Scientific Programmable Calculator. The 35s has both scientific and algebraic notation so you can switch between both. The 35s features a two line display with adjustable contrast. There are 42 built in physical constants and 30 kb of memory with $800+$ independent storage registers.

Get more information with the overview of the HP 35s Scientific Calculator.

## Patterns and Sequences

Patterns occur all around us. Sometimes these patterns are geometric like floor tiles or sidewalks. The bar code on items you buy consists of a pattern. In mathematics, we look for patterns in pictures as well as in numbers. This activity will explore both picture and number patterns.

## Exercise 1

Consider the picture pattern below.


Figure 1


Figure 2


Figure 3

1. Look for a pattern, create Figures 4 and 5, and then draw Figures 4 and 5. Describe the pattern you detected.
2. Now look for a pattern in how many squares are used in each figure. Make a list of these numbers.

What you have just created is called a sequence. A sequence is just a listing of numbers. Many times when we create a sequence, we look for a pattern in the numbers.
3. Is there a relationship between the numbers in your sequence? Describe it in words.

To avoid confusion with sequences, we often name our sequences. The name $u(n)$ means that we have named our sequence $u$ and we will use the letter $n$ when we talk about the figure number. In other words, $u(1)$ means the $1^{\text {st }}$ term in the sequence named $u$. We could say that $u(1)=3$.

Many times when working with sequences, we describe the pattern we see in terms of the previous term. The way we represent the previous term in the sequence named $u$ is to use the notation $u(n-1)$. With the sequence we are looking at here, we are adding two to the previous term. We can represent that at $u(n)=u(n-1)+2$.

In addition to describing patterns as sequences and in words, we also use tables.
4. Create a table to represent your sequence.

| Figure Number | Number of Squares |
| :---: | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
|  |  |
|  |  |

5. Do you see the same pattern in the table that you saw when you listed the sequence? Use this pattern to predict the number of squares for Figures 6 and 7 .

Let's create this table on the 39gs. Start the Sequence aplet. You will be prompted to input the first and second terms. You will then need to enter an expression for the sequence. You can use the $u(n)$ we defined earlier in this exercise. Once you input this information in the SYMB menu, you can press NUM and see the table of values.

Verify your values for the $6^{\text {th }}$ and $7^{\text {th }}$ figures in your table are the same as what you see on the 39gs.

The sequence you have created here is an arithmetic sequence. A sequence is arithmetic if it has a common difference. You can find a common difference by subtracting two consecutive terms. It is called a common difference because no matter which two consecutive terms you subtract, you will always get the same answer. The common difference in our sequence is two.

## Exercise 2

In Exercise 1, we looked at a picture pattern and then used that to create a sequence of numbers. We also looked at our sequence in a table format, both on paper and on the calculator. Another way to look at sequences is with a graph.

1. Use your table from Exercise 1 to create a graph of your data.


Make sure the values on your graph match the values in the table as well as your predicted values for Figures 6 and 7.

You already have all of your sequence information entered in your 39gs. By going to PLOT SETUP and inputting the appropriate settings, you can press PLOT and see your graph on the calculator.
2. You now have a graph of an arithmetic sequence. If you were to connect the points, what kind of graph would you have?
3. How could you use this connected graph to continue make predictions?
4. How does the calculator help you make predictions?

## Exercise 3

Below is another picture pattern. Look for a pattern and draw or create Figure 4.


Figure 1


Figure 2


Figure 3

1. Count the number of triangles in each figure and list them in a sequence.
2. Create a table using the figure number and the number of triangles in the figure.

| Figure Number | Number of Triangles |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
|  |  |

3. What pattern do you see in the sequence and table? Describe what you see.
4. Use your information from the table to create a graph of your sequence.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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5. Is this graph linear like the sequence from Exercises 1 and 2?

What you have created is a geometric sequence. Rather than having a common difference as the arithmetic sequence did, geometric sequences have a common ratio. Based on how you computed a common difference, how do you think you will compute a common ratio? What is the common ratio for the sequence in this exercise?

## Extension 1

Create a sequence, table, and graph for the number of segments needed to create each figure. Do the same for the perimeter of each figure.

## Extension 2

The area (number of triangles) is geometric. Create a sequence, table, and graph for the number of segments. Describe what you see in words. What type of sequence do you think you have?

## Extension 3

In the activity "Perimeter and Area of Similar Shapes", you looked a similar squares, rectangles, and circles, created tables with dimensions and perimeter and circumference, and the differences in perimeter and circumference. How are the differences for similar figures related to the sequences created in this activity?

Go back and look at similar figures and tables. Look at differences column from previous activity and make connection.

## Teacher Notes

The exercises in this activity address the teaching standards for middle grades mathematics listed below.

- Use tables and symbols to represent and describe proportional and other relationships such as those involving arithmetic sequences
- Use letters to represent an unknown in an equation
- Generate formulas from situations
- Graph data to demonstrate relationships in familiar concepts
- Use words and symbols to describe the relationship between the terms in an arithmetic sequence and their positions in the sequence
- Compare and contrast proportional and non-proportional linear relationships
- Make connections among various representations of a numerical relationship

Many teachers have tried toothpicks as a hands-on approach to building the figures and been very successful. However, others have found that Q-tips work better since they do not have sharp ends.

It is important to have students verbalize the patterns they see in the sequences. This is a direct link to the symbolic representation used in a recursive definition of a sequence.

The notation for sequences is very difficult for many students. Emphasize that $u$ is simply the name of the sequence. The letter n is just a way to represent any term in the sequence. When we talk about the first term, $\mathrm{n}=1$. When we talk about the $12^{\text {th }}$ term, $\mathrm{n}=12$. The biggest difficulty is often in talking about "the previous term". The $u(n-1)$ notation is cumbersome to quite a few students. However, since describing patterns in terms of the previous terms is the easiest way for students to begin their approach to sequences, the notation, though difficult, is necessary.

The common difference concept in arithmetic sequences will develop the idea of slope in linear functions. Students can notice the table shows common differences in $x$ and $y$ and in the graph, the height of the steps are the same size.

The exercises have been set up so that students experience as many representations as possible: concrete, numeric, table, graph, verbal and written.

## Answer Key

## Exercise 1

1. 



Figure 4


Figure 5

Students can use manipulatives to create the $4^{\text {th }}$ and $5^{\text {th }}$ figures. Their descriptions will probably be varied. Some may talk about the number of squares needed, others may describe their pattern based on how many "sticks" or manipulatives they needed, while others may talk about adding squares to the top and the side. As long as the description is correct, students have a lot of latitude with their description in this part. However, once the question has been asked about the number of squares in the figure, there is a precise answer students should provide.
2. The list of numbers is $3,5,7,9,11$.
3. The pattern is that you are adding two each time. Try to get students working with the terminology that they are adding two to the previous term.
4.

| Figure Number | Number of Squares |
| :---: | :---: |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |
| 6 | 13 |
| 7 | 15 |

5. The same pattern holds by adding two to the previous term.

To see the table in the 39gs, use the following input screen.


By pressing NUM, you should see the table of values for the sequence.


To see additional values in the table, you can scroll down.

| H | U1 |  |  |
| :---: | :---: | :---: | :---: |
| 5 | 10 |  |  |
| 5 | $1{ }^{12}$ |  |  |
| 寿 | 16 |  |  |
| 10 | $1{ }^{1}$ |  |  |
| 26 |  |  |  |
| EIn $x^{1}$ |  | EII | [吅: |

## Exercise 2

1. 



2. If the points are connected, you would use a line.
3. If you extend the line, you can use it to make predictions.
4. The calculator helps make predictions by showing values on the graph and the table.

## Exercise 3

1. The sequence for the number of triangles is $2,4,8$.
2. 

| Figure Number | Number of Triangles |
| :---: | :---: |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

3. You are multiplying by 2 each time. Another way to phrase it is to say that you are multiplying the previous term by 2.
4. SETUP screens to see the graph on the calculator are shown below.

|  |
| :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |



5. This graph is not linear. The sequence in this exercise is geometric. Geometric sequences do not produce linear graphs, but rather exponential graphs.

## Extension 1

Number of segments
10, 16, 22


Perimeter


## Extension 2

The sequence for the number of segments is $5,9,17$. Students may need to draw an additional figure to come up with the next term. The sequence is neither arithmetic nor geometric, however there is a pattern. Each time you are adding increasing powers of 2 (first you add 4, then 8, then 16). Although you are adding each time, you are not adding the same thing each time. Therefore the sequence is not arithmetic. Also, since you are not multiplying by the same number each time, the sequence is not geometric. The goal is to get students to realize that there are many sequences that will not fall into either category. Even an advanced student will have difficulty coming up with $u(n)=3 n^{2}-5.8 n+8$.

## Extension 3

In the table for perimeter for similar squares, the common difference is 8 . Students can write a recursive formula $u(n)=u(n-1)+8$. The symbolic input screen and graph are shown below.


In the table for perimeter for similar rectangles, the common difference is also 8. These similar rectangles have the same recursive formula and graph as similar squares. Make sure to point out
that this relationship and common difference of 8 is only for the rectangles with the dimensions in the problem. If the dimensions are different, there will be a different common difference in the perimeters.

In the table for circumference for similar circles, the common difference is $4 \pi$. Students can write a recursive formula $u(n)=u(n-1)+4 \pi$. The symbolic input screen and graph are shown below.


## HHC 2008

Richard J. Nelson
The annual Hewlett-Packard Handheld Conference for 2008 was recently held at HP in Corvallis Oregon on September $27^{\text {th }}$ and $28^{\text {th }}$. This is the $35^{\text {th }} \mathrm{HHC}$ since the first one was held September 22, 1979 at the HP Sales office in Santa Clara California. Fifty three HP calculator users attended this year with 13\% from other countries. Conference details, past and present, may be found at the HHC 2008 website. The hot topic of HHC 1979 was the new HP solver of the HP-34C. This year the hot topic was the $\$ 40$ HP20b and its ability to be user re-flashed to make it any machine you desire.


Group photo taken of attendees on Saturday morning. Enthusiasts from Canada, the UK, \& Germany attended. JH
Sam Kim, Cyrille de Brebisson, and G.T. Springer of the HP Calculator Division, along with 19 others made presentations during the two very full Conference days.


Charlie Patton, ex HP calculator designer and HP calculator patent holder describes Cognitive Science and Calculator design.


Pavneet Arora from Canada discusses HP50g Construction Applications. Powerful programs save time for professionnals. Pavneet was voted the Conference Best Speaker. Js

Registration started at 7:30 AM Saturday and presentations concluded at 10 PM . Sunday started at 10 AM and concluded at 6 PM followed by a Halo tour and then caravanning 10 miles north to Albany for a tour of Jim Donnelly's model shop which finished at 11 PM. Attendees received an HP 10*, an HP20b*, at least three door prizes, and bound \& printed Conference proceedings in addition to a 1 GB Thumb drive* electronic form of all the Conference materials. * - Thank you HP!


L to R-G.T. Springer, Cyrille de Brebisson, and Sam Kim answer general questions from the audience. This is an unusual opportunity "To Ask HP."


Diana Byrne studies the HHC 2007 Calendar of HP Personal Calculators. Diana worked on the HP48G series machines.

In addition to enjoying a programming contest (Allen Thomson won), the presentation of topics ranging from the Evolution of Dynamic Geometry software, an HP Calculator Ten Commandments, and New Root-seeking Algorithms, HP and the HP Calculator User Community is generous in donating a wide range of prizes to be randomly drawn by each attendee. This year was an exceptional year in that there were three times as many door prizes as attendees.

HHC 2008, like previous HHC's, was an intense opportunity to share and document the previous year of the activities of the HP User Community. Our proceedings, electronic records, and video tapes have documented our work for 29 years.

Sharon Butterfield, Order processing Administrator, describes HP-35A orders of \$1million in cash kept in two filing cabinets "in the early days."


Left to right: Wlodek Mier-Jedrzejowiez, UK, Gene Wright, HHC Committee, Detlef Mueller, and Pavneet ss Arora, Canada.


One of two door prize tables; calculators, books, etc.

How does my calculator find the Sinus, Cosine or Tangent of a number?
HP calculators store numbers as a set of 3 items: a sign, + or -, a 12 digit number called mantissa always greater or equal to 100000000000 and an exponent.
For example, 34.432 is stored as ' + ', 344320000000,1 .
To calculate trigonometric values such as Tangent, sinus and cosines, your calculator uses an algorithm called CoRDiC or Coronate Rotation Digital Calculations.

But before we go into detail as to the inner workings of the algorithm, let us make a note about the appropriate input ranges. The calculator trigonometric function can, of course, take any number as an input, and work with any coordinate system (Radian, degree or gradient).
Before starting the algorithm, the calculator will do a modulo 2*PI, 360 or 400 as appropriate and then further reduce the angle to a value between 0 and PI/4, 45 or 50 as appropriate. The appropriate result will be returned using basic trigonometric transformations such as $\sin (180+x)=-$ $\sin (\mathrm{x}), \sin (180-\mathrm{x})=\sin (\mathrm{x}), \sin (90-\mathrm{x})=\cos (\mathrm{x}) \ldots$
Since the core part of the algorithm assumes radians, any angle that is not in radians will be further converted to radians so that the input for the algorithm is always between 0 and PI/4 $(\sim 0.785)$.

The core of the algorithm is as follows: given an angle Alpha and a point at coordinates $\mathrm{X}_{1}, \mathrm{Y}_{1}$, you can find the coordinates $\mathrm{X}_{2}, \mathrm{Y}_{2}$, of the point which is the result of the rotation of the first point by the angle Alpha around the center of the Cartesian plan, by the following calculation:
$\mathrm{X}_{2}=\mathrm{X}_{1}{ }^{*} \operatorname{Cos}\left(\right.$ Alpha) $-\mathrm{Y}_{1} * \operatorname{Sin}$ (Alpha)
$\mathrm{Y}_{2}=\mathrm{X}_{1} * \operatorname{Sin}\left(\right.$ Alpha) $+\mathrm{Y}_{1} * \operatorname{Cos}($ Alpha)
Using the following identities $\operatorname{Cos}(\alpha)=\frac{1}{\sqrt{1+\operatorname{Tan}^{2}(\alpha)}}$ and $\operatorname{Sin}(\alpha)=\frac{\operatorname{Tan}(\alpha)}{\sqrt{1+\operatorname{Tan}^{2}(\alpha)}}$
We can transform the 2 equations above into
$\mathrm{X}_{2}=\mathrm{Cte} *\left(\mathrm{X}_{1}-\mathrm{Y}_{1} * \operatorname{Tan}(A l p h a)\right)$
$\mathrm{Y}_{2}=\mathrm{Cte} *\left(\mathrm{X}_{1} * \operatorname{Tan}(A l p h a)+\mathrm{Y}_{1}\right)$
With Cte $=\frac{1}{\sqrt{1+\operatorname{Tan}^{2}(\text { Alpha) }}}$
Note that when Alpha is small (and our steps will be small), Cte is close to 1 , so even after lots of rotations, the total cumulative Constant will be close to 1 and will not cause a major shift in the magnitudes of the values for X and Y . So, the algorithm will, for the moment forget about it!

But, how do you use this to calculate Sinus and Cosines? Well, the gist of it is that you start with the point $\mathrm{X}=1, \mathrm{Y}=0$ and you rotate it slowly, one step at a time until you get to the angle that you want. At that point, you have the coordinates of the point $\mathrm{X}_{\mathrm{n}}, \mathrm{Y}_{\mathrm{n}}$, which correspond to the Cosine and Sinus of the angle you were looking for...

But in fact, as always, things are a little bit more complicated. First, what will your step be? And will all your steps be of the same size? We obviously need to do multiplication by Tangent of the 'step'. In a computer, multiplications are 'expensive', ie: they take a long time. And Trigonometric
values are exactly what we are trying to calculate, so it would not be good to need to calculate Tangents to calculate... Tangents...
So, what we do is look for angle steps that yield easy numbers for the Tan... for example, powers of 10 ! As multiplying or dividing by a power of ten is just a shift of the decimal point.

PI/4 is $\sim=$ to 0.785 so, we will decide to do steps of $\operatorname{Tan}^{-1} 0.1, \operatorname{Tan}^{-1} 0.01 \ldots$. Which correspond to steps of $0.09968 \ldots, 0.009999666686 \ldots$ radians. I will use step ${ }_{n}=\mathrm{TAN}^{-1} 1 / 10^{\wedge} \mathrm{n}$ in the rest of this document.
So, a rotation by step $\mathrm{p}_{\mathrm{n}}$ can be performed (ignoring the constant) using the following equations: $\mathrm{X}_{2}=\mathrm{X}_{1}-\mathrm{Y}_{1}{ }^{*} / 10^{\wedge} \mathrm{n}$ and $\mathrm{Y}_{2}=\mathrm{X}_{1} / 10^{\wedge} \mathrm{n}+\mathrm{Y}_{1}$ since the division is just a shift of the decimal point, it takes no time to perform!

Now, the next question is: how many steps do we need to do for each step of size step ${ }_{\mathrm{n}}$ ?
Well, we take the angle, and start by seeing how many time we can remove Step from it, then how $^{\text {for }}$ many times we can remove step ${ }_{2}$, and continue until step ${ }_{7}$.
Why stop at 7 you might ask? Well, step 7 is equal to roughly $1 \mathrm{E}-7$ ???????
Note that since the difference between step ${ }_{x}$ and step $p_{x+1}$ is less than $1 / 10^{x}$, this means that there is a maximum of 10 steps for every step value.

Ok, we now have our angle A decomposed into a series of number of rotation steps for various step sizes. Now, we are ready to do the rotation... or are we really?

The next question is: what will we use as a starting point? Earlier, we talked about using the $\mathrm{X}=1$, $\mathrm{Y}=0$ point, but we might be able to do better...
$\sin (x)$ when $x$ is small is roughly equal to $x$ (as a matter of fact, $\sin (1 E-7)-1 e-7$ is smaller than $2 e-$ 22 !). Since the calculations of $\mathrm{X}_{2}$ and $\mathrm{Y}_{2}$ will involve adding X (a number close to 1 ) to $\mathrm{Y}_{1}$, a number close to $1 \mathrm{e}-7$, an error at the 22th decimal would not change anything when 15 digits are used to do the calculations. This means that we can safely use the $\sin (x)=x$ approximation and use $\mathrm{Y}=$ remaining of the angle after we remove all the steps. This allows to slightly improve the precision of the calculation.

Ok, now we have a point $\mathrm{X}_{1}=1, \mathrm{Y}_{1}=$ leftAngle and an array (let us call it NumberOfSteps) of number specifying the number of time we need to rotate by step $\mathrm{p}_{\mathrm{n}}$.
The rotation phase will be as follow:
For stepindex:=1 to 7 do
For counter: $=1$ to NumberOfSteps[stepIndex] do
Begin
temp: $=\mathrm{X}-\mathrm{Y} / 10^{\wedge} \mathrm{n}$;
$\mathrm{Y}:=\mathrm{X} / 10^{\wedge} \mathrm{n}+\mathrm{Y}$;
$\mathrm{X}:=$ temp ;
End ;
So, are we done yet? Not really, do you remember that constant that we 'forgot' ? Well, we now need to deal with it! We were trying to get a X and Y that corresponded to $\sin (\mathrm{a})$ and $\cos (\mathrm{a})$, but in
fact, we have a X and Y that are cte* $\sin (\mathrm{a})$ and cte* $\cos (\mathrm{a})$, or, to put it more graphically, X and Y indicate a point which is on the line from 0 to the point $\sin (a) \cos (a)$, but not at $\sin (a), \cos (a)$. Here is a graphical representation:


This means that $\mathrm{X} / \mathrm{Y}$ is equal to tangent of angle! And that is exactly how the constant gets handled. The calculator divides X by Y . if the trigonometric operation is a tangent, it is the end of the algorithm. If the operation is a sinus, it uses one of these identity
$\operatorname{Cos}(\alpha)=\frac{1}{\sqrt{1+\operatorname{Tan}^{2}(\alpha)}}$ and $\operatorname{Sin}(\alpha)=\frac{\operatorname{Tan}(\alpha)}{\sqrt{1+\operatorname{Tan}^{2}(\alpha)}}$ to calculate the result!

## RPN Tip \#10

RPN Tips \#8 and \#9 illustrated the advantages of sketching a stack diagram for frequently solved problems. Spending a few minutes to better understand your RPN calculator builds confidence and skill. This understanding will also especially prepare you for a possible next step - programming your RPN calculator.

Once you understand the four high RPN stack you will begin to think in terms of its powerful capability (and limits) for problems of medium complexity. This RPN Tip will provide a comparison of the different RPN ways problems may be solved for a few common simple situations. Keystroke counts are not the only consideration for a particular method. Another consideration is how the method affects the stack. Pressing an extra key to preserve a stack value is usually well worth the effort. the tables below use the HP 35s for specific keystroke examples.

See Table 1 for operations examples. A and B are used for the data and are assumed terminated on the stack. $\uparrow$ is ENTER. ks is keystroke count ${ }^{1}$. Shaded solutions depend on special conditions. Table 2 shows numeric entry examples.

Table 1 - RPN Solutions For Common Calculation Situations

| Operation | Solution |  | Considerations and Notes - Operations | ks ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}^{2}$ | $\mathrm{X}^{2}$ | normal | Does not disturb the stack. | 2 |
|  | $\uparrow, \times$ |  | Pushes the T value off the stack ${ }^{2}$. | 2 |
|  | $2, \mathrm{Y}^{\mathrm{X}}$ |  | Pushes the T value off the stack ${ }^{2}$. | 2 |
| $\mathrm{A}^{3}$ | $3, \mathrm{Y}^{\mathrm{X}}$ | normal | Pushes the T value off the stack ${ }^{2}$. | 2 |
|  | $\uparrow, \uparrow, \times, \times$ |  | Pushes the Z \& T values off the stack. | 4 |
| 2 A | $2, \times$ | normal | Pushes the T value off the stack ${ }^{2}$. | 2 |
|  | $\uparrow,+$ |  | Pushes the T value off the stack ${ }^{2}$. | 2 |
|  | LASTX, + |  | Assumes a previous operation stored A into LASTX. | 3 |
| $\begin{gathered} \mathrm{A}-\mathrm{B} \\ \text { But } \mathrm{Y}=\mathrm{B} \end{gathered}$ | $\mathrm{X} \rightleftarrows \mathrm{Y},-$ | normal | Often done for clarity. Does not disturb the stack. | 2 |
|  | -, +/- (CHS) |  | Best for speed. Does not disturb the stack. | 2 |
| 1/A | 1/X | normal | Does not disturb the stack. | 1 |
|  | $1, \mathrm{X} \rightleftarrows \mathrm{Y}, \div$ |  | Pushes the T value off the stack ${ }^{2}$. | 3 |

Table 2 - RPN Solutions For Common Numeric Entry Situations

| Entry | Solution | Considerations and Notes - Numeric (and terminated ${ }^{3}$ ) | ks ${ }^{1}$ |
| :---: | :---: | :---: | :---: |
| 0.001 | Digit entry ${ }^{3,4}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 5 |
|  | $3,+/-$ (CHS), $10{ }^{\mathrm{X}}$ | Uses antilog (shifted) function ${ }^{2}$. | 4 |
| 0.01 | Digit entry ${ }^{3}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 4 |
|  | 1, $\uparrow$, \% | Pushes Z \& T values off the stack and leaves 1 in Y. | 4 |
| 0.1 | Digit entry ${ }^{3,4}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 3 |
|  | 1, $0,1 / \mathrm{X}$ | Pushes the T value off the stack ${ }^{2}$ (preserves Z ). | 3 |
| 0.25 | Digit entry ${ }^{3}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 4 |
|  | 4, 1/X | Pushes the T value off the stack. | 2 |
| 0 | $\begin{aligned} & \text { "C" }(\text { CLEAR }, \leftarrow) \\ & \text { normal } \end{aligned}$ | Does not disturb the stack. | 1 |
|  | $\uparrow$, - | Pushes the T value off the stack ${ }^{2}$. | 2 |
|  | LASTX, - | 2, assumes a previous operation stored A into LASTX. | 3 |
| 1 | Digit entry $^{3}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 2 |


| Entry | Solution | Considerations and Notes - Numeric (and terminated ${ }^{3}$ ) | ks ${ }^{1}$ |
| :---: | :---: | :---: | :---: |
|  | COS | Assumes zero in the X register in any angular mode. | 1 |
|  | $\uparrow, \div$ | Assumes X register $\neq 0$. | 2 |
| e | 1, $\mathrm{e}^{\mathrm{X}} \quad$ normal | Pushes the T value off the stack. | 3 |
|  | 2.71828182846 | Keying in the value, Pushes the Z \& T values off the stack. | 14 |
| $\pi$ | Function ${ }^{6}$ normal | Pushes the T value off the stack. | 1 |
|  | $355, \uparrow, 113, \div$ | Many business RPN machines do not have a $\pi$ function ${ }^{6}$. | 8 |
|  | 3.14159265359 | Keying in the value, Pushes the Z \& T values off the stack. | 14 |
| 10 | Digit entry ${ }^{3}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 3 |
|  | COS, $10^{X}$ | Assumes zero in X, Uses antilog (shifted) function. | 3 |
| 32 | Digit entry ${ }^{3}$ normal | Pushes the Z \& T values off the stack ${ }^{3}$. | 3 |
|  | "C" (CLEAR, $\leftarrow), \rightarrow^{\circ} \mathrm{F}$ | Uses shifted temperature conversion, Doesn't disturb the stack. | 3 |
|  | $\rightarrow{ }^{\circ} \mathrm{F}$ | Assumes zero in X, uses shifted temperature conversion. | 2 |
|  | $2, \uparrow, 5, \mathrm{y}^{\mathrm{X}}$ | Pushes the Z \& T values off the stack ${ }^{3}$. | 4 |
| 100 | Digit entry ${ }^{3,5}$ ormal | Pushes the Z \& T values off the stack $^{3}$. | 4 |
|  | 2, $10^{X}$ | Uses antilog (shifted) function ${ }^{5}$. | 3 |
|  | 1, E, (EEX), $2, \uparrow$ | One fewer keystroke on earlier RPN machines ${ }^{7}$. | 4 |
|  | $1, \uparrow, \%$, | Pushes the Z \& T values off the stack. | 4 |
| 1,000 | Digit entry ${ }^{3,5}$ normal | Keystroke intensive | 5 |
|  | 1, $\mathrm{E}^{7}$, (EEX), $3, \uparrow$ | One fewer keystroke on earlier RPN machines ${ }^{7}$. | 4 |
|  | 2, $10^{\text {x }}$ | Uses antilog (shifted) function ${ }^{5}$. | 3 |

## Notes

1. $k s$ - keystroke counts are for the HP35s and includes any data that is part of the solution for comparison purposes. The keystroke count may change on another RPN machine because some HP35s shifted functions may be primary functions and vice versa. Example: To enter 0.1 requires three keystrokes for the value to be terminated (., 1, 1) for three keystrokes. See Table 1 in Volume 4 RPN Tips for the primary or shifted keystroke counts of all HP RPN models and their stack functions.
2. Pushes the $T$ value off the stack. This is an important consideration when the full capacity of the stack is being used.
3. The $X$ value is assumed terminated. Digit entry will always include an ENTER, $\uparrow$, to terminate the value which will push the $T$ and $Z$ values off the stack. If loosing the $Z$ value is not acceptable press the digit entry keys followed by $\mathrm{X} \rightleftarrows \mathrm{Y}$ twice instead of pressing ENTER. This will add a keystroke, but could save you many more keystrokes if the $Z$ value has to be re-entered. Alternately, any function used on the digit entry will also avoid loosing the $Z$ value because the value is terminated after a function is executed and the stack is not disturbed.
4. Numbers from 0.0000000001 to 0.001 are more effectively/efficiently entered using +/- (CHS) and the shifted antilog function.
5. Numbers from 100 to $1,000,000,000$ are more effectively/efficiently entered using the shifted antilog function following the digit entry of the number of zeros.
6. Pi is not found on most early finance machines. Remembering two each of the first three odd integers, 113355, and dividing the $2^{\text {nd }}$ half by the $1^{\text {st }}$ half may be easier, and retained longer, than remembering seven significant digits of $\pi$. To 12 digits this divided value is 0.00000026676 high $\left(2.6676 \times 10^{-7}\right)$.
7. Most RPN (and RPL) machines prior to the HP35s assume a " 1 "mantissa (significand or coefficient) for scientifically entered numbers if the $E$ (EEX) key is pressed. The HP 35 s uses just an $E$ for the traditional EEX key because of this difference.
