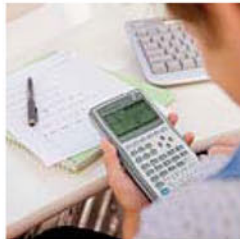




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**Volume 4
May 2008**

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Event	Dates - Location
ASEE	June 22-25 - Pittsburgh, PA
NECC	June 29-July 2 - San Antonio, TX
AP Annual	July 16-20 - Seattle, WA
WIPTe	October 15-16 - Westville, IN
Educause	October 28-31 - Orlando, FL

RPN Tip #4

Learn how the calculator thinks and the difference between RPN and ALG problems and solutions, and learn about the RPN Stack. Get this months RPN tip.

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New! HP 39gs Graphing Calculator Classroom Kit

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Announcing the HP 39gs Graphing Calculator Classroom Kit for Middle Grades.

Easy to use and powerful, HP Calculators for grades 6-8 are designed to perform above expectations for math and science students on all levels. These reliable calculators are equipped with easy-to-use problem solving tools, flexible connectivity & customizing options, plus award-winning HP support.

Our HP 39gs Graphing Calculator Classroom Kit is a unique solution that provides teachers with the tools and training to engage and motivate middle school students in mathematics. We believe that successful integration of technology into the mathematics classroom is enhanced when it is undertaken as a group or team effort, a belief that is reinforced by independent studies, so we designed the Classroom Kit to help middle school teachers establish a mathematics leadership team on campus. The Classroom Kit features a three-day professional development workshop and supporting curricula to help ensure the successful integration of the HP 39gs Graphing Calculator into math classes.

The HP 39gs Graphing Calculator is the heart of the HP Graphing Calculator Classroom Kit, and this all-in-one kit includes calculators, carrying case, classroom-ready teaching materials, and a 3-day teacher workshop. The HP 39gs Graphing Calculator was designed by teachers for classroom use and to align with NCTM (National Council of Teachers of Mathematics) Standards. Importantly, and uniquely, the HP 39gs has dedicated keys for symbolic, graphic and numeric views.

The Classroom Kit allows teachers to save time in their digital classroom. Using our kit and included calculator emulator software, the teacher can project a virtual calculator during the lesson so students can follow along easily. Plus, teachers can use HP Aplets, either that they've written or downloaded from numerous free resources online. Teachers can set up powerful examples beforehand, saved with common names to ease retention, and then send them to students wirelessly! By utilizing this powerful feature, and given the ability to collect or create these aplets beforehand, teachers save precious class time, and are secure in the knowledge that students will quickly get on the same page, ready to learn.

See the [Graphing Calculator Classroom Kit](#) for more information.

Feature Calculator of the Month: HP 10bII Financial Calculator

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The HP 10bII Financial Calculator is the smart choice for business, finance and accounting needs, for professionals and students alike. With over 100 time-saving business functions you can calculate loan payments, interest rates and conversions, standard deviation, TVM, cash flows and more. Algebraic notation makes it easy to learn and use. With so many features, at a great price, the HP 10bII adds up to a wise investment. The HP 10bII is permitted for use on the CFP® Certification Exam [1].

Fun facts about the HP 10bII Financial Calculator:

- Born: Introduced December 1st, 2001
- Replaced the HP 10b, which was introduced January 3rd, 1989. HP also introduced the HP 20s and HP 21s on the same date.
- The HP 10BII has a memory called the M register that is usable directly from the basic keyboard. There are three keys associated with the M register. The $\rightarrow M$ key will store the number presently displayed into the M register. Note that this will overwrite any value previously stored in the M register. The RM key will recall the number presently in the M register to the display where it can be used. The M+ key will add the number presently in the display to the number already in the M register. If pressed repeatedly, it will add the number in the display to the number in the M register each time. This is often used to keep up with a running total. For example, to solve $(2 + 3) \div (5 - 2)$ using the M register, press $5 - 2 = \rightarrow M$ $2 + 3 \div RM =$ To quickly find the total of 12, 44, 17 and 36, press $12 \rightarrow M$ $44 M+$ $17 M+$ $36 M+$ RM
- It is HP's lowest cost financial model, well suited for finance students who do not need the advanced features of other HP models, but who want HP reliability. It provides a simple interface, all the important operations needed in common financial calculations but avoids the complications of programming, scientific functions and RPN.

Click here for more information about the HP 10bII Financial Calculator.

[1] CFP® is a registered trademark of the Certified Financial Planner Board of Standards, Inc.

RPN Tip #4

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Introduction (by Wlodek Mier-Jedrzejowicz, Richard J. Nelson, & Jake Schwartz)

Hewlett-Packard has always made calculators with a user interface, the method used to solve problems, most suitable for the general class of problems to be solved. The four basic types of calculator user interface are: Arithmetic, ATH; Algebraic, ALG; Reverse Polish Notation, RPN; and Command Line Interface, CLI. RPN is, over all, the most effective and efficient way to solve a very large class of problems, and HP is well known for its RPN machines.

RPN is a Really Productive Notation

RPN is different, and to most people it is unfamiliar. It is easy to learn and it is the most efficient way to solve a large class of problems because you treat each part of the problem in exactly the same way. The rules, compared to Algebraic, ALG, are fewer and simpler. The purpose of the Newsletter series of RPN Tips is to describe RPN techniques so new and experienced users may improve their calculator skill.

The User Interface

The two most popular general purpose user interfaces for midrange calculators are RPN and ALG. The user interface is the means by which the calculator user presses keys to solve problems. The user interface is the "rules" that the user (and the machine) must follow in order to give the machine the numbers (data) and especially the operators +, -, ×, & ÷ so the machine solves the problem correctly.

Suppose you had the problem of adding one and two and multiplying the result by three. The problem would be mathematically written as $(1 + 2) \times 3 = 9$. In order for the machine to know that the problem is not $1 + (2 \times 3) = 7$, both the user and the calculator must know the rules of what the problem is, and how it is to be solved. Using parentheses and inserting the operators between the data is the problem form taught

in school and most people are familiar with it. The use of parentheses (and brackets and curly braces) is usually taught in a beginning algebra class.

The problem, $1+2\times3=$, and pressing the keys as shown, is often used to identify the user interface. If the answer is 7 it is ALG. If the answer is 9 it is ATH. It is interesting to note that without parentheses you can't solve the problem using RPN. This is another example of "the rules."

The calculator, however, can't "think" like we can so it must be presented with the data and the operators in a very precise order. This precise order is where the "rules" are very important. The simple example problem above requires the machine to understand the proper sequence of performing the operators. The parentheses group the various portions of the problem and they show how the portions are related.

In addition to knowing the correct order of the data and the operators, the machine must also be able to store intermediate results so that they may be correctly integrated with the current calculation. In this case the addition of one and two is performed first and the result is then multiplied by three. This method of solving the problem, e.g. $(1 + 2) \times 3$, and representing and solving problems of this type is called Algebraic, ALG, notation. From a mathematical logic perspective this is called infix notation.

In order to present the problem to the calculator the three numbers must be entered, the two operators (+ and \times) must be entered, and the order in which calculation is to be made must be conveyed. In addition to this, the machine must know that a previously calculated result is to be used with another number and operator.

The parentheses group the various operations and the general rule is to solve the problem from the "inside out" by performing the operations in the inner most parentheses first. This is true no matter what calculator you are using. There is another notation called postfix notation. Using postfix notation the problem above may be represented as $1\ 2\ +\ 3\ \times$. Solving the problem in postfix notation is what the machine does internally regardless if the user interface is ALG or RPN. Infix is $1 + 2 \times 3$; Postfix is $1\ 2\ +\ 3\ \times$.

RPN uses postfix notation as the logical interface. RPN does not require an equal key. RPN does not use parentheses. The machine must, however, know when the user presses the 1 and the 2 key that there are two numbers of interest, 1 and 2, and not the single number 12. In other words any number keyed into the calculator must be terminated (completed). RPN machines use the ENTER key to terminate numeric inputs. In terms of the keys pressed to solve our example problem the notation would be: 1 ENTER 2 + 3 \times . The desired answer, 9, appears in the display immediately when the multiply key, \times , is pressed. The intermediate result, of 1 + 2, also appears when the add operator is pressed and 3 appears in the display. Equal (or a key) is not used for the RPN user interface.

The ability to integrate parts of a calculator problem by storing intermediate results is a very important aspect of calculator problem solving rules that must be learned by the user. For the ALG interface the use of parentheses and the hierarchy of operations are the most important rules. The use of parentheses is the primary means of storing intermediate results. RPN does not have a list of hierarchy rules. ALG may six or more, in some cases as many as 11, depending on how many operators the machine has. RPN uses an automatic memory stack, see below, to store intermediate results. Neither ALG nor RPN is able to solve all problems exactly as they are written on paper. A more accurate form of using mixed data and operators utilizes a user interface that is predominantly found on the higher end calculators, the Graphing Calculators, and it is called Command Line Interface, CLI. All graphics calculators (and computers) use CLI.

Arithmetic, ATH, is the user interface found on adding machines. It is the simplest and it doesn't have any "rules" except that it does the operators such as +, -, \times , & \div when the keys are pressed.

The RPN Stack

The ENTER key, \uparrow , and a group of four data storage registers, called an automatic RPN stack, is what makes RPN very powerful and simple to use. The stack may be visualized as a stack of “shoe boxes” as shown in figure 1. Each stack shoe box, register, is vertically related to each other. The contents of the X register are shown in the display. The original RPN machine, the HP-35A, utilized a single line display. More recent machines that have a multiline display may also show additional stack registers in the order of Y to T.



Fig. 1 – Automatic four high stack.

Basic RPN

Aside from the general rule we learned in school that is used for all moderately complex problem solving – perform the operations by solving the parentheses from the inside out – the basic RPN rule is: if an operation is indicated, do it. The example given above: $(1+2) \times 3 = ?$ is 1 ENTER 2 + 3 \times . Pressing the ENTER key performs three operations.

- (1) Pressing ENTER terminates the number so the calculator knows that the complete number has been keyed into the calculator.
- (2) Pressing ENTER also makes a copy of the X register data by also storing it on the stack in the Y register. The contents of the registers above are pushed up i.e. the contents of the Y register are copied into the Z register. The contents of the Z register are copied into the T register and the contents of the top register, T, are lost.
- (3) Pressing ENTER also prepares the machine for accepting additional data or a keyboard operation.

RPN Stack Operators (see Table 1)

Using and controlling the RPN stack utilizes the five RPN operators. RPN utilizes postfix notation and just important is that RPN implies the automatic stack as shown in figure one above. Postfix notation, the use of a stack, and five stack operators is what defines RPN as a user interface. The five basic RPN Stack operators are:

1. ENTER, \uparrow , is the most important RPN operator. See the description under Basic RPN above. ENTER is never a shifted operator on the RPN keyboard.
2. $X \rightleftarrows Y$ is the second most important RPN operator. $X \rightleftarrows Y$ exchanges the contents of the X and Y registers. $X \rightleftarrows Y$ is never a shifted operator on the RPN keyboard.
3. Roll down, $R\downarrow$: The third most important RPN operator. Roll down rotates the (shoe box) stack downward. The contents of X are copied into T, T into Z, X into Y, and Y into X. Roll down is a primary operator on all RPN models except the HP-34C, 37E, and 38E/C.
4. LAST X is the very important error correction operator. LAST X recalls the value of the X Register prior to the most recent operation performed. LAST X is a shifted operator on all RPN models except the six early models that do not have it – the HP-35A, 21, 22, 37E, 70A, and 80A.
5. Roll up, $R\uparrow$, rotates the stack upward. The contents of X are copied into Y, Y into Z, Z into T, and T into X. $R\uparrow$ is a shifted operator and it is not found on 24 of the 43 RPN models (58%).

The roll down, $R\downarrow$, operator allows a quick verification of the stack contents by pressing the primary stack roll key four times in succession.

RPN Stack Diagrams

In the days before RPN calculators were programmable and the basic RPN calculator was the optimum choice for complex problem solving, it was useful to make a stack diagram of the keystrokes used to repeatedly solve very complex problems. This was especially useful for iterative solutions and engineers carried 3 x 5 cards in their shirt pockets with these "keystroke solutions" written on them. To illustrate a stack diagram let's show the stack to make a calculation involving three values, each used twice. The best approach is one that utilizes the stack in such a way that the values are keyed only once to avoid errors.

An example problem is shown below.

$$\frac{(A + B)(A - C)}{B - C} = 8.4$$

Where:

$$A = 4$$

$$B = 3$$

$$C = -2$$

The stack registers are identified at the left. The tilde symbol indicates any (don't care) value. The press row indicates the inputs that are keyed to solve the problem. These are the keystrokes that are recorded to solve the problem. The stack diagram is primarily used for illustration and analysis. The values involved in the example are shown in both symbolic and numerical form (where practical) so the user may solve the problem by pressing the inputs as indicated and see the stack values. The solution steps are numbered above the stack diagram.

Step	1	2	3	4	5	6	7	8
9	10	11						
T	~	~	~	B=3	B=3	B=3	B=3	B=3
Z	~	~	B=3	B=3	B=3	B=3	B=3	B=3
Y	~	B=3	B=3	B=3	B=3	B=3	A+B=7	A=4
X	B=3	B=3	B=3	B=3	A=4	A+B=7	A=4	C=-2
press	B=3	↑	↑	↑	A=4	+	LASTX	C=-2
								-
								LASTX
								R↓
Step		12		13	14	15	16	
17		18						
T		C=-2		42	42	42	42	C=-2
Z		C=-2		C=-2	C=-2	42	42	42
Y		B=3		C=-2	B=3	C=-2	B-C=5	42
X		(A+B)(A-C)=42		B=3	C=-2	B-C=5	C=-2	B-C=5
press		×		R↓	X↔Y	-	X↔Y	R↓
								÷
								$\frac{(A+B)(A-C)}{B-C} = 8.4$

The reader is encouraged to solve the problem for practice.

The stack diagram is useful for the following reasons.

- (1) A stack diagram illustrates how the stack works.
- (2) Solving complex problems repeatedly is faster if an optimum keystroke solution is known. What is the value of the above expression if:

$$A = 1.41421357348,$$

$$B = -1.73205091868, \text{ and}$$

C = 2.23606808860?

Solve the problem without looking at the stack diagram. See the correct answer at the end. Did you get this answer on your first try without using the stack diagram?

- (3) A stack diagram shows the values needed for future use in order to determine the best solution sequence – especially in preparation for reason 4.
- (4) The optimum solution for a program that may solve the problem multiple times and program solution speed is an issue. The stack diagram is a powerful optimization tool.

RPN Stack Operator Table

Table 1 below shows all RPN models to date (April 2008) grouped into their series in approximate introduction order. It also shows the classic RPN stack operators with a weight value assigned to each of them. The weight value is used to compare the various models to each other for their “RPNness” in terms of the RPN keyboard and the included operators. The colors provide a quick overview of the model and the operator use on that model, i.e. if it is used, and if it is a shifted operator. The ideal RPN machine would have an index of 9. Obviously there are many keyboard tradeoffs that must be made for the intended class of problem the model has to solve.

Table 1 – HP RPN Calculator Stack Operators And Keystroke Index

		Weight	3	2	2	1	1	Ideal Index = 9
			ENTER	R↓	X<>Y	R↑	LASTx	Ks Index
Classic Series	35A	y	y	y	n	n		7
	45A	y	y	y	n	s		7.5
	55A	y	y	y	n	s		7.5
	65A	y	s*	s*	s	s		6 undefined
	67A	y	s*	s*	s	s		6 undefined
	70A	y	y	y	n	n		7
	80A	y*	y	y	n	n		7 * = key marked SAVE
Woodstock	21	y	y	y	n	n		7
	25	y	y	y	n	s		7.5
	29C	y	y	y	n	s		7.5
	22	y	y	y	n	n		7
	27	y	y	y	n	s		7.5
Spice/Spike Series	31E	y	y	y	n	s		7.5
	32E	y	y	y	n	s		7.5
	33E/C	y	y	y	n	s		7.5
	34C	y	s	y	s	s		7
	37E	y	s	y	n	n		6
	38E/C	y	s	y	n	s		7.5
41C/CV/CX		y	y	y	ALPHA	s		8
Voyager Series	10C	y	y	y	n	s		7.5
	11C	y	y	y	s	s		8
	12C	y	y	y	n	s		7.5
	15C	y	y	y	s	s		8
	16C	y	y	y	s	s		8
	12Cp	y	y	y	n	s		7.5

Topcat Series	91	y	y	y	y	s	8.5	
	92	y	y	y	y	s	8.5	
	95C	y	y	y	y	s	8.5	
	97	y	y	y	s	s	8	
<hr/>								
Pioneer Series	19C	y	y	y	n	s	7.5	
	32S	y	y	y	n	s	7.5	
	32SII	y	y	y	s	s	8	
	42S	y	y	y	ALPHA	s	8	
	17BII	y*	y	y	n	s	7.5 INPUT	
	19BII	y*	y	y	n	s	7.5 * = key marked INPUT	
	33S	y*	y	y	s	s	8 * = SMALL ENTER key	
	35S	y	y	y	s	s	8	
<hr/>								
	17BII+ Gold '03	y*	y	y	n	s	7.5 * = key marked INPUT	
	17BII+ Silver '07	y*	y	y	n	s	7.5 * = key marked INPUT	
<hr/>								
Total:	43 machines							Table by Jake Schwartz 1/08

*Color legend: Green indicates primary operator.
Red indicates stack operator not on the machine.
Yellow indicates shifted operator.*

RPN And ALG

All of the four basic user interfaces mentioned above (ATH, ALG, RPN, and CLI) have been used in HP calculators e.g. the 10 model number has been used five times since 1972. One was ATH, two were ALG, and two were RPN. All graphing machines are CLI. Each interface has its advantages and disadvantages. Over all, RPN is the most effective (requires less thinking, which is hard work, is faster, and usually requires fewer keystrokes) for a very large class of problems. ALG often seems "easier" because your problem is often expressed in an "algebraic" form. If you understand RPN and you don't use your calculator frequently you will always be able to remember the rules of how to solve problems because the rules are very few and they are obvious.

There are two primary reasons that RPN is more effective compared to ALG. The first is the basic structure of RPN. The second reason is that you are able to see the problem "unfold" as work your way through it. All too often ALG keeps parts of the problem "hidden" until the final EQUAL.

RPN users do not need to clear anything (the stack, or the X register) before starting a problem. In fact, the vast majority of RPN machines do not have a clear stack operator. You just start keying in the data.

ALG users, however, know that if they don't start with a "clean slate" so to speak, they will get wrong answers and may have to do the problem all over again – which usually means doing it over twice for insurance that they have the correct answer. ALG users soon learn to clear their machines before starting and if you watch an ALG user solve a problem you will see them pressing clear multiple times, "just to be sure." In a similar way you will notice an inexperienced RPN user press zero and ENTER three times to clear the stack, "just to be sure."

The user interface of choice is often the one we first learned. Taking the time to master RPN, however, will save you endless key pressing hours. If, however, you are an HP calculator user it you may select the either ALG or RPN on many models.

The HP-42S Owner's Manual provides a very nice list of the advantages of RPN.

“Advantages of RPN”

Remember: This method of entering numbers, called Reverse Polish Notation (RPN), is unambiguous and therefore does not need parentheses. It has the following advantages:

- You never work with more than two numbers at a time.
- Pressing a function key immediately executes that function so there is no need for an $\boxed{=}$ key.
- Intermediate results appear as they are calculated, so you can check each step as you go.
- Intermediate results are automatically stored. They reappear as they are needed for the calculation—the last result stored is the first to come back out.
- You can calculate in the same order as you would with pencil and paper.
- If you make a mistake during a complicated calculation, you don't have to start over (Correcting mistakes is covered in chapter 2,)
- Calculations with other types of data (such as complex numbers and matrices) follow the same rules.
- Calculations in programs follow the same steps as when you execute them manually.

Keyboard or Program Solution

Many RPN calculators are programmable. A program is simply a “memorized” list of keys pressed to solve a problem. In addition to the normal operators provided on the keyboard a programmable model provides a means of naming, starting, and stopping, a program. Another feature of a program is a format for inputting the data and running the program to get an answer. Once a program is proved to solve a complex problem (debugged) it may be run whenever the problem needs to be solved. This may be long after you have forgotten how to solve the problem.

Another very important aspect of solving your problem using a program is that the program may test values and make decisions based on these tests. This feature is what makes running a program so powerful.

Still another advantage of a program is executing the same series of keystrokes multiple times. This is called looping. Looping is an important feature of a programmable calculator. The straightforward logic and structure of RPN makes keystroke programming easy to learn and use. In some instances the program may make many calculations in a loop and speed becomes an important consideration. RPN saves keystrokes and speeds up programs.

Conclusion

No matter what midrange calculator you use, all problems are not solvable “as you write them on paper.” It takes a high end graphing calculator with its Command Line Interface to be able to do this. You may find examples where one of the simpler machines will solve the problem correctly, but as you increase the complexity of the problem you will have to “know more of the user interface rules.” The Arithmetic, ATH, user interface has no built in logic. This leaves ALG and RPN. Of these two, RPN has fewer rules and is always consistent in the process in which you solve problems. Like any well thought out system there are many ways in which a problem may be solved. The HP Solve newsletter series of RPN Tips provides techniques to better understand and improve your RPN skills.

Answer to three variable, A,B,C problem: Δ NSV-0:5265482868520

Click here to learn more about [RPN](#).

Let's make music with math!

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It may come as a surprise to some that music and mathematics are intimately connected. Both are consequences of human creativity, both involve intricate patterns, both can be at once beautiful and daunting, and both can be appreciated more deeply through conscientious study. Pleasant harmonious patterns of music reveal themselves in exquisite numerical patterns of mathematics. This activity focuses on some of the connections between mathematics and music.

Exercise 1:

Start the MUSICMATH applet on the HP39gs. You are presented with two columns of numbers in a table. The numbers in column C1 are used to designate different musical notes. The numbers in column C2 are the frequencies. The chart below is provided for those with some musical background, and relates the numbers in C1 to the names of different musical notes. The frequencies are repeated here, rounded to the nearest hundredth, for your convenience.

Number	Note	Frequency (hz)
1	A4	440
2	A#	466.16
3	B4	493.88
4	C5	523.25
5	C#	554.37
6	D5	587.33
7	D#	622.25
8	E5	659.26
9	F5	698.46
10	F#	739.99
11	G5	783.99
12	G#	830.61
13	A5	880

The 39gs has a built-in speaker, and a command to play any note for any length of time. To play the first note in the table (A4) for 2 seconds, go to the HOME screen and enter the command BEEP 440;2 as shown in the screen below.



Find another student with a 39gs. Have your partner play the last note (A5 or 880 hz) at the same time you play the first note (A4 or 440 hz). Hold the calculators close together, so the notes blend.

Try playing two E's together by executing BEEP 660;2 on one calculator, and BEEP 1320;2 on another at the same time. (Notice that 660 is pretty close to the frequency for E5).

The harmonies you just heard results from playing the same note in two different octaves (an octave consists of one complete set of twelve notes). These are the simplest type of harmonies.

1. What relationship do you notice between the frequencies of these harmonious notes (440 with 880; 660 with 1320)?

Create a second list of frequencies one octave above those in column C2 by executing the command shown in the screen shot below.

```

RAD MUSICMATH
BEEP 440;2
1
2*C2>C3
(880,932.32,987.76,10...
STOP
  
```

You can play the entire scale by using the column name with the BEEP command. Try it!

<pre> RAD MUSICMATH BEEP 440;2 2*C2>C3 (880,932.32,987.76,10... BEEP C2;1 STOP </pre>	<pre> RAD MUSICMATH 2*C2>C3 1(880,932.32,987.76,10... BEEP C2;1 (880,932.32,987.76,10... BEEP C3;1 STOP COPY SHOW </pre>
--	---

Playing octaves on two calculators

Then try playing the scale in C2 while your partner plays the scale one octave higher, in C3, as shown above on the right. Try to start the scales simultaneously.

Now try playing 440 (A4) and 660 (E5) together.

2. Do they sound harmonious to you?

(If the speakers on the 39gs are not adequate to hear the harmonies, try the first web resource, or a piano!).

3. Can you see any connection between the frequencies 440 hz and 660 hz?

Legend has it that Pythagoras (the same guy from the Pythagorean Theorem) was one of the first to recognize the harmonious tone that results from playing notes such as these together.

4. Try playing an entire scale where the frequencies are in the ratio 3:2 by multiplying C2 by 1.5 and storing the result in C3. Use two calculators to play the resulting frequencies along with the originals. Then repeat after multiplying C2 by a nastier ratio like 17/13. What do you find?

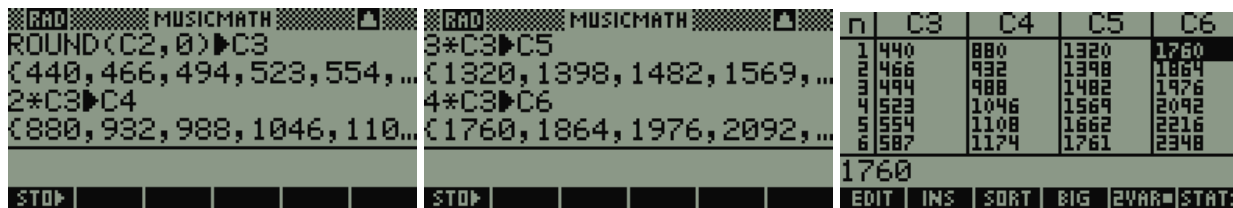
Exercise 2: What's So Special About Harmonious Notes

In Exercise 1, you saw (or rather heard!) that certain musical notes sound nice when played together. What attributes do these notes have that produces the pleasing sound?

Let's look at ratios.

1. What is the ratio of the frequency of A5 to the frequency of A4?
2. What is the ratio of the frequency of E5 to A4?

Let's look for notes whose frequencies are in small integer ratios. The 39gs can help in the hunt! First, we'll round the frequencies we have to integers. Then we'll multiply the rounded frequencies by small integers. Finally, we'll search the table of frequencies for matches. Execute the commands shown in the screen shots:



- Look around in the table for frequencies that are approximately the same. Then try playing them together and listen for the harmony. Record your results, and compare with other groups.

Harmonies occur when the notes are in "nice" integer ratios, such as 2:1 or 3:2. Notes in the ratio 2:1 are called an octave. Those in the ratio 3:2 are called a fifth. Ratios of 4:3 are called a fourth. C and F produce a fourth, since $4 \cdot 523.25 = 2093$, while $3 \cdot 698.46 = 2095.38$. Pretty close.

Try playing notes in the ratio 1:3 (such as A3, 220, and E5, 660) at the same time. Notes in the ratios 1:2, 1:3, 1:4, 1:5, etc are called *harmonics*. You may encounter the *harmonic series* in Precalculus or

Calculus. It is the sum $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$, where the ellipsis ... means the sum goes on forever.

Sometimes, three notes played together produce a pleasant sound. Find another partner and play A, C#, and E together. This chord is called a major triad.

- Can you spot the ratio of the frequencies? Hint: Round the frequency of C# to 550.

Exercise 3: A Musical Model

Take another look at the scale on the first page. Notice as we go from 440 hz (A4) to the next higher A, 880 hz (A5), we've doubled the frequency. We saw in Exercise 1 that pairs of notes that are in the ratio 1:2 produce a certain pleasant harmony. But how do the notes progress *within* one scale? What choices could there be?

One approach would be to divide the scale so that the arithmetic difference between adjacent notes was a constant. For example, $880 \text{ hz} - 440 \text{ hz} = 440 \text{ hz}$. Divide this into 12 equal steps.

- If the scale were made this way, what would the "step" between each note be?
- Fill in the table for the notes from 440 hz to 880 hz according to this "arithmetic" scale. Does this arithmetic scale match the scale from the table given in Exercise 1.

Number	Note	Frequency (hz)
1	A4	440
2	A#	
3	B4	
4	C5	
5	C#	
6	D5	
7	D#	
8	E5	
9	F5	
10	F#	

11	G5	
12	G#	
13	A5	880

Since harmonies occur when the *ratio* of notes are small whole numbers, it makes more sense to arrange the scale so that the *ratios* of adjacent notes is the same. Let's calculate these ratios using the values in the MUSICMATH applet. In the 39gs, the first value in column C2 is referred to as C2(1). Calculate the ratio $C2(2)/C2(1)$, as shown.

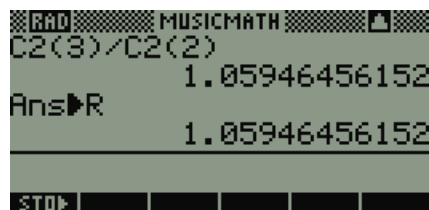


3. What is the ratio?

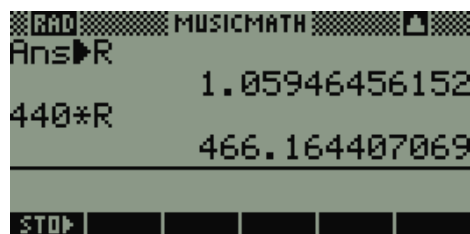
After you've calculated this ratio, press the up arrow key, highlight the expression $C2(2)/C2(1)$, and press the soft COPY key. Then edit (using the DEL key and inserting replacement digits) to produce the ratio $C2(3)/C2(2)$. Repeat for $C2(4)/C2(3)$.

4. What are $C2(3)/C2(2)$ and $C2(4)/C2(3)$?

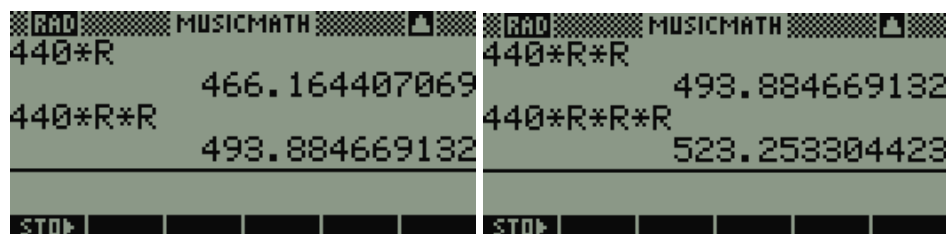
Clearly, the ratios of adjacent notes are the same through the scale. If we call this ratio r , then all we do to get the next note in the scale is to multiply the previous note by r . That is, with $A4 = 440$ hz, $A\#$ is $440r$. Store the ratio you calculated into the calculator variable R , as shown. Just press the STO> soft key on the HOME screen right after you calculated one of the ratios, then press ALPHA R.



Now to get the next note above $A4$, calculate $440 \cdot R$ as shown. Again, use ALPHA R to type the letter R:



The next note is always the previous note multiplied by R . To get $B4$, we could enter $440 \cdot R \cdot R$. (The COPY feature will help you out!).



This quickly becomes tedious. Mathematicians use exponents as a shorthand for repeated multiplications like this. For example $440 \cdot R \cdot R = 440R^2$, $440 \cdot R \cdot R \cdot R = 440R^3$, and so on. On the 39gs, exponents are entered using the key labeled X^y . Use this key to evaluate $440 \cdot R^{11}$ and $440 \cdot R^{12}$.

5. $440 \cdot R^{11} =$ _____ and $440 \cdot R^{12} =$ _____.

6. So, when R is multiplied by itself 12 times, what do you get? That is, what is R^{12} ?

We call the number R the twelfth root of 2 and write it like this: $R = \sqrt[12]{2}$. This number, like $\sqrt{2}$ and π , is irrational.

Overview

This exploration into the connection between music and mathematics begins with listening to octaves, which are notes with frequencies in the ratio of 2:1. Most, but not all, westerners sense the harmonies from these octaves. Also in exercise 1 is a 3:2 harmony. This exercise is largely qualitative.

Exercise 2 extends and quantifies the observations from Exercise 1. By making a large table of frequencies that are small integer multiples of each other, and searching for values that are close, students find more chords that produce harmonious tones. Exercise 2 also includes a triad (cord with 3 notes).

In Exercise 3, students explore the relationship between frequencies within one scale. In modern western music, the scale is based on "equal temperament", where the ratio between adjacent notes is exactly the same through the scale. Since there are 12 divisions within the scale, and the next octave has frequencies twice those in the previous, each adjacent frequency pair have a ratio of $\sqrt[12]{2}$.

The last observations in Exercise 3 lead to a mathematical model for an equal temperament scale, and this is explored in Extension 1. The extension should be used with students who have some experience graphing equations. Students graph the frequencies of the notes, enter the model, and see that the model fits the frequencies. They also create a scale based on equal *differences* between adjacent notes, and see that the graph of these leads to a linear model (though no attempt is made to create the symbolic form of the model).

Note that Exercise 3 stands by itself, and could be presented without doing Exercise 1 or 2 at all. Exercises 1 and 2 should be done together. Extension 1 fits with Exercise 3. Extension 2 fits with Exercises 1 and 2.

Extension 1:

Enter the notes created by the arithmetic scale in Exercise 3 into column C4 of the MUSICMATH applet. Then press SYMB. Define two scatter plots as shown below.

n	C1	C2	C3	C4
1		440	880	440
2		466.16	932.32	476.667
3		493.88	987.76	513.333
4		523.25	1046.5	550
5		554.37	1108.74	586.667
6		587.33	1174.66	623.333

NEW MUSICMATH SYMBOLIC SETUP		NEW MUSICMATH SYMBOLIC VIEW	
ANGLE MEASURE: Radians		✓S1: C1	C2
S1FIT: User	S2FIT: Linear	✓Fit1: b*EXP(m*X)	
S3FIT: Linear	S4FIT: Linear	✓S2: C1	C4
S5FIT: Linear		✓Fit2: m*X+b	
CHOOSE ANGLE MEASURE		ENTER USER DEFINED FIT	
CHOOSE		EDIT ✓CHK X	SHOW EVAL

Press VIEWS and select Autoscale.

1. Describe the difference in the shapes of the two graphs.

2. Evaluate the expression $440 \cdot 2^{\frac{X-1}{12}}$ when $X = 1$ and when $X = 2$. Recall that the first value (i.e. when $X = 1$) in our column C2 of notes is 440, and that the second is about 466.16.

Back in the SYMBOLIC view, enter $440 \cdot 2^{\frac{X-1}{12}}$ for S1FIT. Go to the PLOT, press the soft MENU key, then press the soft FIT key. You should see that S1FIT hits all the points on the S1 scatter plot!

Extension 2:

Research the musical term *dissonance*, and find out what it has to do with this activity.

Extension 3:

Find music for a song and distribute a measure to each student. Have them put the notes into a list, and organize themselves to play back the song.

Teacher Notes

The following TEKS (and others) can be found in this activity set:

- 6.1C, 6.1F, 6.2C, 6.3A, 6.5, 6.11A, 6.11D, 6.13, 7.2B, 7.2D, 7.14A, 7.15, 8.1C, 8.2D, 8.4, 8.14A, 8.15A, 8.16A

Here are the TEKS organized by Exercise and Extension:

Table 5: TEKS Covered, by Activity	
Activity	TEKS Covered
Exercise 1	6.1C, 6.1F, 6.2C, 6.11A, 6.11D, 7.2B, 8.2D, 8.14A
Exercise 2	6.1C, 6.1F, 6.2C, 6.11A, 6.11D, 7.2B, 8.14A, 8.16A
Exercise 3	6.3A, 6.13, 7.2B, 7.2D, 8.1C, 8.2D, 8.14A
Extension 1	6.3A, 6.5, 7.14A, 7.15A, 7.15B, 8.4, 8.14A, 8.15A
Extension 2	6.11A, 8.14A

Web Resources

Select and play combinations of notes and see their wave forms superimposed on this [musical scale](#).

Read the Wikipedia entry on [Consonance and Dissonance](#).

See the selection of questions and answers about music and math from [Dr Math](#).

Answers

Exercise 1

1. The frequencies are in the ratio 1:2
2. Most students should answer yes. To some, the harmony is not apparent.
3. There's no guarantee students will see what we want here. The relevant connection is the 2:3 ratio.
4. The scales in the ratio 3:2 sound harmonious to most people; the scales in the ratio 17:13 sound less pleasant.

Exercise 2

1. 2:1
2. 3:2
3. One possibility is C and F, in the ratio 4:3. A and D are also in the ratio 4:3. In general, notes that are seven rows apart in the table are in the ratio 3:2. Notes five rows apart are in the ratio 4:3.
4. 4:5:6

Exercise 3

1. $440/12 = 36.67$
- 2.

Number	Note	Frequency (hz)
1	A4	440
2	A#	476.67
3	B4	513.33
4	C5	550
5	C#	586.67
6	D5	623.33
7	D#	660
8	E5	696.67
9	F5	733.33
10	F#	770
11	G5	806.67
12	G#	843.33
13	A5	880

3. 1.059
4. Both are 1.059
5. 830.61 and 880, which are the frequencies of G# and A5
6. 2

Extension 1

1. S1 is curved; S2 is linear
2. 440 and 466.15, the first two notes in our scale.

Extension 2

According to Wikipedia, dissonance refers to harmonies which create more complex acoustical interactions (called 'beats'). Consonant harmonies are made up of tones that complement and increase each other's resonance.

Generally speaking, dissonant intervals involve frequencies with ratios that are larger integers. (Or, at least, that's what this non-music theorist believes!) However, this notion seems to be culturally dependent.

HP's most powerful financial calculator is now sleeker than ever

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