Interest rate conversions

Practice solving interest rate conversion problems
Interest rate conversions

The interest rate applied to a value will compute an amount of accrued interest. When this calculation is related to a single period, it is called a simple interest calculation. The interest rate may also be applied to a value and the amount of accrued interest added to it in a finite number of compounding periods. This is called a compound interest calculation. Interest may be compounded once a year (annually), twice a year (semiannually), four times a year (quarterly), twelve times a year (monthly), etc. Eventually, if the compounding frequency continues to be increased, it will reach an upper limit. This situation is referred to as continuous compounding. Converting interest rates is primarily used when finding solutions for TVM problems where interest and payment periods are not the same and when investments with different compounding periods are compared with each other.

When calculating compound interest with a finite number of periods, the effective interest rate is annual interest rate which would have provided the same results as the interest rate compounded more frequently than once a year. The following relationships are valid:

$$E_{FF} = \left(1 + \frac{N_{OM}}{C}\right)^{C} - 1$$

where:
- $E_{FF}$ is the effective rate (in decimals);
- $N_{OM}$ is the nominal rate (in decimals);
- $C$ is the number of compounding periods.

Effective and nominal rates can also be calculated when the interest is based on continuous compounding. In this case, the following relationship is valid:

$$E_{FF} = \left(e^{N_{OM}} - 1\right)$$

Practice solving interest rate conversion problems

Example 1: What is the effective annual interest rate if the nominal rate is 8.4%, compounded monthly? Use the TVM application to calculate the effective rate.

Solution: To clear out any values left from previous computations, clear the TVM registers contents to zero and set END mode:

```
f g FIN 9 END
```

First, $n$ is set as the number of compounding periods ($C$) and $i$ is set as the nominal rate applied to each composition period:

```
8 × 4 ENTER 1 2 n ÷ i
```

0.70

Figure 3
The following keystroke sequence calculates the effective percent rate.

\[ 100 \rightarrow M + \]

![Figure 4](image)

If \( PMT = 0 \), the compound interest rate expression is reduced to:

\[ PV = -FV(1+i)^n \Rightarrow FV = -PV(1+i)^n \]  \hspace{1cm} \text{Figure 5}

Given that \( i = \left( \frac{\text{NOM}}{n} \right) \) and \( n = C \), and provided that \( PV = -100 \), then \( FV = \left( \frac{EFF}{100} + 1 \right) \) or \( FV\% = \left( \frac{EFF}{100} + 1 \right) \).

**Answer:** The effective interest rate equivalent to a nominal rate of 8.4%, compounded monthly is 8.73%.

**Example 2:** Find the nominal interest rate, compounded quarterly, that is equivalent to an effective annual rate of 6.2%. Use the TVM application to calculate the nominal rate.

**Solution:** To make sure there is no residual value from previous calculations, clear the TVM registers contents to zero and set END mode prior to start the calculation:

\[ \boxed{f \ M I N \ 9 \ \text{END}} \]

First, \( n \) is set as the number of composition periods (\( C \)) and \( PV \) is set as 100.

\[ 4 \ n \ 1 \ 0 \ 0 \ \text{ENTER} \ PV \]

![Figure 6](image)

The following keystroke sequence calculates the nominal rate

\[ 6 \ \ast \ 2 \ \text{CHS} \ \text{FV} \ 1 \ \text{RCL} \ n \ \text{X} \]

![Figure 7](image)

**Answer:** The nominal rate of 6.06%, compounded quarterly, is equivalent to an effective annual rate of 6.2%.
Example 3: Find the effective rate resulting from a 7.5% rate compounded continuously.

Solution: The following keystroke sequence calculates effective rate:

\[ 1 \text{ ENTER } 7.5 \text{ b g> à } \]

Figure 8

Answer: The effective rate is 7.79% per year.

Example 4: An annual interest rate of 8.9% compounded continuously applies to a savings account that was started with an initial $2,500 balance. At the end of each month, a $100.00 deposit is made in this account. What will be the balance for this account after 10 years?

Solution: The first step is to convert the continuous nominal rate to an effective rate:

\[ 1 \text{ ENTER } 8.9 \text{ b g> à } \]

Figure 9

This annual effective rate must now be converted to an interest rate compounded monthly in order to evaluate the deposits and solve the problem.

\[ \text{f g à } \text{FIN } \text{END} \text{ } 1 \text{ } 0 \text{ } 0 \text{PV} \text{+ CHS} \text{FV} \text{12} \text{ n } i \]

Figure 10

A new TVM calculation must take place, and the value in the display is the correct interest rate to use.

\[ \text{f g à } \text{FIN } \text{i} \text{2} \text{5} \text{0} \text{0 CHS} \text{PV} \text{1} \text{0} \text{0 CHS PMT} \text{1} \text{0} \text{9} \text{12X} \text{FV} \]

Figure 11

Answer: The balance for this account after ten years will be $25,366.22.