



hp calculators

HP 9s Solving Trigonometry Problems

The Trigonometric Functions

The Angular Unit

Practice Solving Problems Involving Trigonometric Functions



The trigonometric functions

The trigonometric functions are sine, cosine and their reciprocals (cosecant and secant, respectively) which repeat their values every 360° , and tangent and its reciprocal (cotangent), whose period is 180° . All these functions have their inverse functions (e.g. $\sin^{-1} x$ or $\arcsin x$) which are defined for specific ranges. The trigonometric functions are also known as circular functions because they are defined in geometric terms. These functions are extensively used in geometry, surveying, astronomy, building, design, etc. They play a lead role in electromagnetism. In fact, they describe the alternating current as well as the movement of a pendulum!

The HP 9s provides the three basic functions \sin , \cos and \tan , and their inverse functions: the “arc” functions \sin^{-1} , \cos^{-1} and \tan^{-1} . All these functions work in degrees, radians and grads. Additionally, π is provided as a shortcut on the shifted EXP key ($\text{2ndF} \pi \text{A}$).

The R-P and P-R functions carry out the related rectangular/polar conversions, which are discussed in the HP 9s learning module entitled *Polar/Rectangular Coordinate Conversions*.

The angular unit

In a complete circle there are 360 degrees, 2π radians (used in mathematical analysis) or 400 grads (which are common in surveying). Before doing any calculation involving trigonometric functions, you should always make sure that the appropriate angle unit is set. Just look at the top of the display, where the angle unit annunciator is located. Either DEG, RAD or GRAD is always lit. To change the mode, press the DRG key, which acts as a toggle switch.

Refer to the HP 9s learning module *Converting Angles and Times* for additional information on angular modes.

Practice solving problems involving trigonometric functions

Example 1: What is the sine of π ?

Solution: In Radian mode, press

$\text{2ndF} \pi \text{A} \sin$

Answer: 0. The value returned is **exact!** Indeed, the sine of the *irrational* number π (which has an infinite number of significant digits) *is* zero, but $\text{2ndF} \pi \text{A}$ actually returns an approximation to twelve digits: 3.14159265359.¹ Is the sine of 3.14159265359 smaller than 10^{-99} ? If so, the HP 9s would automatically substitute the number zero. But, that's not the case, the sine of 3.14159265359 is $-2.06761537357 \cdot 10^{-13}$. So, what's happening? The HP 9s evaluates to 0 the sine of any number x such that:

$$3.1415926535 \leq x \leq 3.1415926536 \text{ (} x \text{ is expressed in radians).}$$

That's not exactly cheating, but a way of producing *exact* answers by implementing a very important property of π , which is that its sine *is* zero. The question was: what is the sine of π ? Will you ever interested in the sine of 3.14159265359?

¹ Numbers with more than ten significant digits (up to fourteen) can be entered by splitting them: e.g. 3.141592653 + 5.9E-10.

Example 2: Find the height of the flag pole shown in Figure 1.

Solution: From the figure we know that:

$$\tan 75^\circ = \frac{\text{height}}{20} \Rightarrow \text{height} = 20 \cdot \tan 75^\circ$$

Press DRG twice (if RAD was lit) to set DEG mode. On the HP 9s the trigonometric functions are all postfix functions, i.e. their argument is the number in the display:

$\text{2} \text{0} \text{X} \text{7} \text{5} \text{tan} \text{ENTER}$

Answer: 74.64 feet, rounded to two decimal digits.

Example 3: Find the angle whose haversine is 0.4546

Solution: The versine, coversine, haversine and exsecant functions are frequently used in spherical trigonometry and, for example, in finding the courses of ships. They are defined as:

$$\text{vers } x = 1 - \cos x, \quad \text{covers } x = 1 - \sin x, \quad \text{hav } x = \frac{1 - \cos x}{2}, \quad \text{ex sec } x = \sec x - 1 = \frac{1}{\cos x} - 1$$

If the known value is the haversine, solving for x results in:

$$x = \arccos(1 - 2 \text{hav } x)$$

Press DRG to set RAD mode. To find the angle press:

$\text{1} \text{-} \text{2} \text{X} \text{.} \text{4} \text{5} \text{4} \text{6} \text{ENTER} \text{2ndF} \text{cos}^{-1}$

Notice the ENTER key before $\text{2ndF} \text{cos}^{-1}$, in order to display the partial result $1 - 2 \text{hav } x$, which is the argument of the COS^{-1} function.

Answer: Rounded to four decimal digits, 1.4799 radians.

Example 4: Show that the double angle formula for the tangent holds for α is 30°

Solution: The double angle identity is as follows:

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

Make sure that Degree mode is active and then store $\tan 2\alpha$ in the running memory:

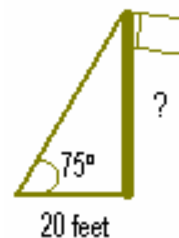


Figure 1.

$2 \times 3 \div \tan^{-1}$

And now calculate the right-hand expression:

$2 \times 3 \div \tan^{-1} (1 - 3 \div \tan^{-1})$

Answer: Pressing \ominus MR ENTER should return 0,² implying that both quantities are the same. Press ON/C X-M to clear the memory.

Example 5: A designer wants to use triangular tiles with sides 3 inches, 5 inches and 7 inches long, to put a mosaic on a floor. What is the angle opposite the 7 inch side? Will it be possible to lay three tiles next to each other with this angle pointing inwards?

Solution: The law of cosines states that for a triangle with sides a, b, and c and C being the angle opposite side c:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

C can be calculated as:

$$C = \arccos \frac{a^2 + b^2 - c^2}{2ab}$$

which can be calculated on the HP 9s by pressing:

$3 \text{ x}^2 + 5 \text{ x}^2 - 7 \text{ x}^2 \text{ ENTER } \div (2 \times 3 \times 5 \text{ ENTER } \text{2ndF} \text{cos}^{-1}$

Answer: The angle opposite the 7 inch side is 120 degrees. This means that three tiles will fit together exactly with this angle pointing inwards, as they would make up 360 degrees.

² Or, in many cases, slightly different than zero because of roundoff.