**hp calculators**

**HP 9s** Powers and Roots

Powers and Roots

Practice Solving Problems Involving Powers and Roots

## Powers and roots

The number $a$ in $a^m$ is said to be *raised to the <u>power</u> m*, which is also called index or exponent. It obeys the so-called index laws, namely:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

where $a$ is any number and $m$ and $n$ are rational numbers. The process of raising to a power is often referred to as *involution*. The opposite of a power is called a <u>root</u>. For example, if $5^3 = 125$ then 5 is the third root of 125, and it is often written thus:

$$5 = \sqrt[3]{125}$$

The process of finding a root is known as *extraction* and also as *evolution*. Evolution is the inverse of the involution. Notice the following important relationships:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

The HP 9s has numerous functions to calculate powers and roots. These are: $\boxed{x^2}$ , $\boxed{2ndF}\,\overset{10^x}{\boxed{F}}$ , the exponential function $\boxed{2ndF}\,\overset{e^x}{\boxed{E}}$ , $\boxed{2ndF}\,\overset{x^{-1}}{\phantom{x}}$ (same as 1/x), the power function $\boxed{x^y}$ , $\boxed{\sqrt{\phantom{x}}}$ which calculates square roots (i.e. second roots), $\boxed{2ndF}\,\overset{\sqrt[3]{\phantom{x}}}{\boxed{C}}$ , that calculates cube roots (the third root), and the x*th*-root function $\boxed{2ndF}\,\overset{\sqrt[x]{\phantom{x}}}{\boxed{B}}$ .

### Practice solving problems involving powers and roots

<u>Example 1:</u>   Is $e^\pi$ greater than $\pi^e$ ?

<u>Solution:</u>       We'll calculate the difference $\pi^e - e^\pi$

$\boxed{2ndF}\,\overset{\pi}{\boxed{A}}\,\boxed{x^y}\,\boxed{1}\,\boxed{2ndF}\,\overset{e^x}{\boxed{E}}\,\boxed{-}\,\boxed{1}\,\boxed{2ndF}\,\overset{e^x}{\boxed{E}}\,\boxed{x^y}\,\boxed{2ndF}\,\overset{\pi}{\boxed{A}}\,\boxed{ENTER}$

The $\boxed{x^y}$ and $\boxed{2ndF}\,\overset{e^x}{\boxed{E}}$ keys are postfix functions, i.e. their argument is the number being displayed. They do not cancel any pending operation (unless they give an error, e.g. $\boxed{2}\,\boxed{3}\,\boxed{1}\,\boxed{2ndF}\,\overset{e^x}{\boxed{E}}$ , which is an overflow condition). Because of the lower priority of the $\boxed{-}$ key, when it is pressed the partial result $\pi^e$ is displayed (22.45915772). The $\boxed{ENTER}$ key at the end carries out all pending operations and returns the final result.

We can save one keystroke by storing (i.e. pressing $\boxed{x-M}$ ) the value of $e$ in the M register as soon as it's displayed. $e$ can now be retrieved by pressing one key: $\boxed{MR}$ (instead of $\boxed{1}\,\boxed{2ndF}\,\overset{e^x}{\boxed{E}}$ ):

$\boxed{2ndF}\,\overset{\pi}{\boxed{A}}\,\boxed{x^y}\,\boxed{1}\,\boxed{2ndF}\,\overset{e^x}{\boxed{E}}\,\boxed{x-M}\,\boxed{-}\,\boxed{MR}\,\boxed{x^y}\,\boxed{2ndF}\,\overset{\pi}{\boxed{A}}\,\boxed{ENTER}$

Press $\boxed{0}\,\boxed{x-M}$ or $\boxed{ON/C}\,\boxed{x-M}$ to clear the M register. (The sequence given in the manual, $\boxed{CE}\,\boxed{x-M}$ , won't work here because there's no *entry* to be cleared by $\boxed{CE}$ once $\boxed{ENTER}$ has been pressed!).

Answer:   The difference is –0.681534914 = $\pi^e - e^\pi < 0$, therefore $e^\pi > \pi^e$.

Example 2:   Find the hypotenuse of a triangle the catheti of which are 8 and 15 (see figure 1).

Solution:   The hypotenuse of a right triangle is given by Pythagoras' theorem (even though the Babylonians already knew this relationship!):

$$\text{Hypotenuse} = \sqrt{a^2 + b^2}$$

where *a* and *b* are the two catheti. In this example:

(8) ($x^2$) (+) (1) (5) ($x^2$) (ENTER) ($\sqrt{}$)

The (ENTER) key is necessary to perform the addition. If the (ENTER) key had been pressed at the end instead, the calculation performed would have been $8^2 + \sqrt{15^2}$ . In fact, we can avoid pressing the (ENTER) key:

( ( ) (8) ($x^2$) (+) (1) (5) ($x^2$) ( ) ) ($\sqrt{}$)

Even though the sequence with the (ENTER) key is one keystroke shorter, you might prefer using parentheses for being clearer.

Answer:   17 . (The set of numbers 8, 15 and 17 is an example of a Pythagorean triple, i.e. integers that can be the sides of the same right triangle. Some of the simpler sets were already known by the ancient Egyptians.) Refer to the HP 9s learning module *Polar/Rectangular Coordinate Conversions* to learn a faster way of calculating the hypotenuse.

Example 3:   Calculate $0^0$

Solution:   Although some calculators return 1, on the HP 9s (0) ($x^y$) (0) (ENTER) is an error condition because $0^0$ is mathematically an indeterminate (or undetermined) form, much like 0 / 0 or log 0. Press (ON/C) to remove the error indicator.

Example 4:   Calculate $9^{-0.27}$ Use the exponential function to confirm the result.

Solution:   A convenient way of computing $x^y$ is as $e^{y \ln x}$ since $x^y = e^{\ln x^y} = e^{y \ln x}$ The HP-35, the world's first scientific electronic pocket calculator, used this method to save valuable space in ROM, which could be noticed by the fact that some results were not accurate to the last decimal place (e.g. $2^9$). Therefore, $9^{-0.27}$ can be calculated using the power function (($x^y$)) as follows:

(9) ($x^y$) (.) (2) (7) (+/−) (ENTER) (The "Change Sign"" key, (+/−) , must be pressed *after* or *while* keying in . 27)

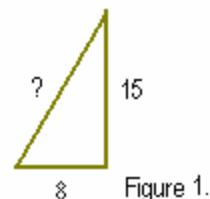and also as $e^{-0.27 \times \ln 9}$. To evaluate the latter expression press:

$\cdot$ 2 7 +/− × 9 ln ENTER 2ndF e$^x$ E

(or ON/C [1] ( $\cdot$ 2 7 +/− × 9 ln ) 2ndF e$^x$ E )

Answer:      Both methods return 0.552528294

Example 6:    Find the probability that at least one of 253 people has the same birthday as you.

Solution:     The probability that the birthday of at least one of $n$ persons is a particular day is given by:

$$1 - \left( \frac{354}{365} \right)^n$$ . In this example n = 253:

1 − ( 3 6 4 ÷ 3 6 5 ) $x^y$ 2 5 3 ENTER

Answer:      0.500477154. The probability is greater than 50%. You can easily check that for n ≤ 252, the probability is smaller than 50%, so 253 is the smallest number of persons such that the probability of at least one of them having the same birthday as you is greater than 50%!

Example 7:    Find the maximum shear stress on an element if the stress in the x –direction ($S_x$) is 25,000 psi, the stress in the y-direction ($S_y$) is –5,000 psi and the shear stress on the element for the Mohr circle input ($\tau_{xy}$) is 4,000 psi.

Solution:     The maximum shear stress is given by the following formula:

$$\tau_{max} = \sqrt{ \left( \frac{S_x - S_y}{2} \right)^2 + \tau_{xy}{}^2 }$$

2 5 EXP 3 − 5 +/− EXP 3 ENTER ÷ 2 ENTER $x^2$ + 4 EXP $x^2$ ENTER √

Answer:      15,524.1747 psi.

Example 8:    Find the geometric mean of the set of numbers { 2, 3.4, 3.41, 7, 11, 23 }

Solution:     For a set of n positive numbers { $a_1$, $a_2$, ..., $a_n$ } the geometric mean is defined as

$$G = (a_1 \cdot a_2 \cdot ... \cdot a_n)^{\frac{1}{n}}$$

To find G press:

2 × 3 $\cdot$ 4 × 3 $\cdot$ 4 1 × 7 × 1 1 × 2 3 ENTER $x^y$ ( 1 ÷ 6 ENTER

Remember that we can write the above equation this other way:

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[1] Remember that the ON/C key starts a new calculation. The ( key is only active after pressing ON/C , CE , ENTER , or a two-argument function (e.g. + , ÷ , $x^y$ , 2ndF nPr , etc.)

$$G = \sqrt[n]{(a_1 \cdot a_2 \cdot ... \cdot a_n)}$$

which can be calculated in fewer keystrokes:

2 × 3 · 4 × 3 · 4 1 × 7 × 1 1 × 2 3 ENTER 2ndF ⌐ B 6 ENTER

Answer:    G =5.873725441

There cannot be more than six pending operations on the HP 9s (e.g. 1 +  ( 1 + ( 1 + ( 1 + ( 1 + ( 1 + generates an error condition). Is that much limitation? Well, let's see if the Hp 9s can solve the following formula, which has been taken from the cover of the HP-67 Owner's Handbook and Programming Guide, and was used as an example of the value of HP's RPN logic system.

Example 9:  In a rather overoptimistic effort to break the speed of sound, high-flying pilot Ike Daedalus cranks open the throttle on his surplus Hawker Siddeley Harrier aircraft. From his instruments he reads a pressure altitude (PALT) of 25,500 feet with a calibration airspeed (CAS) of 350 knots. What is the flight mach number

$$M = \frac{\text{speed of aircraft}}{\text{speed of sound}}$$

if the following formula is applicable?

$$M = \sqrt{5\left[\left(\left\{\left[\left(1+0.2\left[\frac{350}{661.5}\right]^2\right)^{3.5}-1\right]\left[1-(6.875\times10^{-6})25500\right]^{-5.2656}\right\}+1\right)^{0.286}-1\right]}$$

Solution:   If the problem seems very complex and you are not sure that your HP 9s can handle it, evaluate the problem in parts, displaying intermediate results by pressing ENTER . In this particular case, though, the HP 9s can evaluate this formula as it is stated from left to right, without having to press the ENTER key at all:

( 5 × ( ( ( ( 1 + · 2 × ( 3 5 0 ÷ 6 6 1 · 5 )
x² ) x^y 3 · 5 − 1 ) × ( 1 − 6 · 8 7 5 EXP +/− 6 × 2
5 5 0 0 ) x^y 5 · 2 6 5 6 +/− + 1 ) x^y · 2 8 6 − 1 )
) √

Answer:    M = 0.835724535