hp calculators

HP 9s Logarithmic Functions

Logarithms and Antilogarithms

Practice Solving Problems Involving Logarithms

Logarithms and antilogarithms

The logarithm of *x* to the base *a* (written as $\log_a x$) is defined as the inverse function of $x = a^y$. In other words, the logarithm of a given number is the exponent that a base number must have to equal the given number. The most usual values for *a* are 10 and e, which is the exponential constant and is defined by the infinite sum: 1+ 1/1! + 1/2! +1/3! + ...+1/n! +... Its value is approximately 2.718 and is a transcendental number, that is to say: it cannot be the solution of a polynomial equation with rational coefficients.

Logarithms to base 10 are called common logarithms and also Briggsian logarithms. They are usually symbolized as $\log_{10} x$ or simply log 10, and on the HP 9s, they correspond to the [log] key. These logarithms are used in calculations.

Logarithms to base e are called natural logarithms, Naperian logarithms and also hyperbolic logarithms. Their symbol is ln x or $\log_e x$. They are calculated with the [ln] key on the HP 9s. This kind of logarithms is most used in mathematical analysis. There is still another kind of logarithms, though somewhat unusual; they are the binary logarithms, which are logarithms with base 2 ($\log_2 x$).

The following formula is very useful to change logarithms from one base to another:

$$\log_n x = \frac{\log_m x}{\log_m n}$$

The denominator, $\log_m n$, is known as the *modulus*.

The inverse function of the logarithm is called the antilogarithm. If $y = \log_a x$, then $x = a^y$ is the antilogarithm of y. If the base is e then the inverse function is called the exponential function, e^x , which is also known as the compound interest function and the growth (if x > 0) or decay (if x < 0) function. Perhaps the most important property of the exponential function is that its derivative is also e^x , that is, it's the solution of the differential equation dy/dx = y for which y = 1 when x = 0.

On the HP 9s, the keys that carry out these calculations are [ln] , [2ndF] [e^x] , [log] and [2ndF] [10^x] . The function x^y (x [x^y] y [ENTER]) can be considered the generic antilogarithm function: if 10^x is the inverse of $\log_{10} x$ and e^x is the inverse of $\log_e x$, then x^y is the inverse of $\log_x y$. Refer to the HP 9s learning module *Solving Problems Involving Powers and Roots* for more information on the x^y function.

Practice solving problems involving logarithms

Example 1: Find the common logarithm of 2

Solution: On the HP 9s the logarithm is a postfix function, i.e. the argument is keyed in before pressing the function key. [ENTER] is not necessary, since the result is displayed as soon as the function key is pressed. In this example:

[2] [log]

<u>Answer:</u> 0.301029995. This is the result of *truncating*—because the number is less than 1—the internal 12-digit answer: 0.301029995664 to nine decimals. Press [2ndF] FIX [9] to *round* the number to nine decimal places: 0.301029996, and [F↔E] to view the tenth decimal digit. Please refer to the HP 9s learning *module operating Modes and Display Format* for more information on the available display settings.

<u>Example 2:</u> What is the numerical value of the base of the natural logarithms?

<u>Solution:</u> Simply press:

[1] [2ndF] e^x

<u>Answer:</u> 2.718281828. Note that the pattern 18-28-18-28 is really easy to remember!

<u>Example 3:</u> Calculate $\ln(8)+\ln(5)$

<u>Solution:</u> [8] [ln] [+] [5] [ln] [ENTER]

It is important to bear in mind that the [ln] key, being a postfix function, returns the logarithm of the number being displayed, no pending calculation is affected unless the displayed number is negative, which is an error condition. The [ENTER] key *is* necessary to perform the pending addition because the second [ln] returns the logarithm of the displayed number only, i.e. 5.

<u>Answer:</u> 3.688879454

<u>Example 4:</u> Verify that $\ln(8)+\ln(5)=\ln(8\times 5)$

<u>Solution:</u> Before electronic calculators replaced logarithmic tables, logarithms were used for multiplying and dividing large numbers quickly because the logarithm of the product is equal to the sum of logarithms of the multiplicand and the multiplier. We already calculated the left-hand side in the previous example, let's now calculate the right-hand side of the equation by pressing:

[8] [x] [5] [ENTER] [ln]

Note that the [ENTER] key (which performs the multiplication) must be pressed *before* [ln] .

<u>Answer:</u> Both expressions evaluate to 3.688879454

<u>Example 5:</u> Calculate $3\ln(28.34\times 3.75)-\ln(6)$

<u>Solution:</u> The parentheses keys enable us to key in the problem as written, i.e. as it is mathematically stated from left to right:

[3] [x] [(] [2] [8] [•] [3] [4] [x] [3] [•] [7] [5] [)] [ln] [–] [6] [ln] [ENTER]

We need not press the [ENTER] key after the first [ln] because the [–] key already performs the pending multiplication ([x] takes priority over [+] and [–]). We can save one keystroke if we calculate the multiplication that is in parentheses *first*:

2 8 • 3 4 x 3 • 7 5 ENTER ln x 3 – 6 ln ENTER

Answer: 12.20633075

Example 6: Find the log to base 3 of 5. Confirm the result using the x^y function.

Solution: Using the formula given above, the log to base 3 of 5 can be calculated as $\frac{\log_{10} 5}{\log_{10} 3}$:

5 log ÷ 3 log ENTER

Let's confirm this result by pressing:

x^y 3 2ndF X–Y ENTER

We obtain 5, which means that the logarithm was correctly calculated.

Answer: 1.464973521

Example 7: What is the value of x in the equation $18^x = 324$?

Solution: To solve this equation, we will use an important property of logarithms which states that the logarithm of a number raised to a power is equal to the power multiplied by the logarithm of the number. This involves taking the logarithm of both sides of the equation. The original equation would then look like this:

$$\log 18^x = \log 324 \Rightarrow x \log 18 = \log 324$$

and *x* is therefore equal to:

$$x = \frac{\log 324}{\log 18}$$

3 2 4 log ÷ 1 8 log ENTER

Answer: 2. Note that the same answer will be found using natural logarithms instead.

Example 8: A rare species of tree has a trunk whose cross-section changes as 1/x with the height x. (Obviously this breaks down at ground level and at the tree top.) The cross section for any such tree is given by A/x, where A is the cross-section calculated at 1 meter above the ground. What is the volume of the trunk between 1 meter and 2 meters above ground?

Solution: The volume is obtained by integrating the cross-section along the length, so it is given by the integral:

$$\int_1^2 \frac{A}{x} dx$$

Although the HP 9s has no built-in integration function, this integral can be easily evaluated if we remember that the indefinite integral of 1/x is ln(x). The result is therefore:

$$V = A \times \ln(2) - \ln(1)$$

And since ln(1) = 0:

$$V = A \times \ln(2)$$

As no one is likely to measure tree heights to an accuracy of more than three significant digits, let's set our HP 9s to display the answer with just 3 digits after the decimal point, by pressing 2ndF FIX 3 .

Now let's find ln(2):

2 ln

Answer: The log to base *e* of 2 is close to 0.693, so the volume is 0.693A cubic meters. Remember to press 2ndF FIX • to restore the default display format, once you have finished this example.

Example 9: An activity of 200 is measured for a standard of Cr^{51} (with a half-life of 667.20 hours). How much time will have passed when the activity measured in the sample is 170?

Solution: This is the formula for half-life computations:

$$A = A_0 \times \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$$

Let's rearrange the equation to solve for t:

$$t = \tau \frac{\ln \frac{A}{A_0}}{\ln \frac{1}{2}} = 667.20 \frac{\ln \frac{170}{200}}{\ln 0.5}$$

Now it's up to you: use either the straightforward:

6 6 7 • 2 x (1 7 0 ÷ 2 0 0) ln ÷ • 5 ln ENTER

or the shorter:

1 7 0 ÷ 2 0 0 ENTER ln x 6 6 7 • 2 ÷ • 5 ln ENTER

Answer: 156.4352172 hours. Approximately, 156 hours and 26 minutes (2ndF).