



## hp calculators

### HP 9s Solving Problems Involving Fractions

Basic Concepts

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### Basic concepts

Those numbers that can be written as one integer over another, i.e.  $\frac{a}{b}$ , ( $b$  can't be zero) are called *rational numbers*.

When written as the quotient of two integers, rational numbers are called fractions. In arithmetic there are three basic rules for fractions:

- ◆  $\frac{a}{b} > \frac{c}{d}$  if  $ad - bc > 0$  (same for  $<$  and  $=$ )
- ◆  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
- ◆  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b}$  is also referred to as a *vulgar* fraction when  $a$  and  $b$  are positive integers (the sign is considered apart).  $a$  is called the numerator (corresponds to the dividend in a division) and  $b$  is the denominator (which corresponds to the divisor). When the numerator is 1 (or  $-1$ ), it is a *unit* fraction. A proper fraction is a fraction in which the numerator (apart from the sign, remember) is *smaller* than the denominator. Therefore, proper fractions always lie between  $-1$  and  $1$ . If the numerator is *greater* than the denominator, the fraction is called improper.

Vulgar fractions that have the same value are called equivalent fractions: for example  $\frac{3}{4}$  and  $\frac{6}{8}$ . Reducing a vulgar fraction to its lowest terms means to find the simplest equivalent fraction, which can be done by dividing the numerator and the denominator by the same number. This process is also called *cancellation*.

Mixed numbers are those improper fractions written as an integer followed by a proper fraction. For example:  $4\frac{3}{4}$  or  $2\frac{1}{2}$ . It is important to understand that there's no implicit multiplication in  $a\frac{b}{c}$  (also written as  $a\frac{b}{c}$ ). In fact, it is an *addition* that is implicit:

$$a\frac{b}{c} = a + \frac{b}{c} = \frac{ac + b}{c}$$

When the numerator and the denominator of a fraction are not both integers then the fraction is called *complex*, for example:  $\frac{13}{3\frac{3}{4}}$ . Finally, a number where the part which is a proper fraction is expressed as a set of digits placed after a

decimal point, is called a decimal (also known as decimal fraction) e.g.  $3\frac{7}{50} = 3.14$ .

### Fractions on the HP 9s

The HP 9s has two keys to handle fractions, namely  $\frac{a}{b/c}$  and  $\frac{2ndF}{-d/c}$ . The symbol used by the HP 9s to display a fraction (i.e. the equivalent to the "/" symbol, which is called *solidus*) is "⌵" and is entered into the display by pressing  $\frac{a}{b/c}$ . Thus,  $7\lrcorner 8$  means  $7/8$  and is entered by pressing  $\frac{7}{\frac{a}{b/c}} \frac{8}{\frac{a}{b/c}}$ . Mixed numbers are also keyed in using the  $\frac{a}{b/c}$  key, but *twice*, for example:  $7\frac{8}{9}$  is input by pressing  $\frac{7}{\frac{a}{b/c}} \frac{8}{\frac{a}{b/c}} \frac{9}{\frac{a}{b/c}}$ . As soon as the second  $\frac{a}{b/c}$  is pressed, the displayed ⌵ is replaced by the following symbol: ⌵. So,  $7\frac{8}{9}$  is finally displayed as  $7\lrcorner 8\lrcorner 9$ .

$\frac{2ndF}{-d/c}$  is used for converting a mixed number to an improper fraction and vice versa. Let's illustrate all this with various examples.

### Practice working problems involving fractions

Example 1: Enter the proper fractions  $\frac{3}{9}$  and  $\frac{21}{124}$

Solution: Note that pressing  $\text{3} \text{ } \frac{\square}{\square} \text{ } \text{9} \text{ } \text{ENTER}$  does not return a fraction but a decimal number, which is the result—within the accuracy of the calculator—of dividing 3 by 9. As stated above, fractions are entered with the  $\frac{\square}{\square}$  key, which separates the numerator from the denominator. Press:

$\frac{\square}{\square}$  (so that  $\text{ENTER}$  does not repeat the calculation “/ 9”) and  $\text{3} \frac{\square}{\square} \text{9}$

The display now reads  $3 \square 9$ . Press  $\text{ENTER}$ . The fraction now displayed is  $1 \square 3$ , which is equivalent to the entered fraction but reduced to its simple form. The HP 9s always tries to find the simplest equivalent fraction. Let's enter the second fraction:

$\text{2} \text{1} \frac{\square}{\square} \text{1} \text{2} \text{4} \text{ENTER}$

No reduction is possible this time, the fraction displayed is therefore  $21 \square 124$ .

Example 2: Enter the improper fraction  $\frac{1000}{101}$  and the complex fraction  $\frac{1.8}{9}$

Solution: These fractions are entered exactly as in the previous example, but the results displayed after pressing the  $\text{ENTER}$  key are different. Let's enter the first fraction by pressing:

$\text{1} \text{0} \text{0} \text{0} \frac{\square}{\square} \text{1} \text{0} \text{1} \text{ENTER}$  <sup>1</sup>

The fraction displayed ( $1000 \square 101$ ) changes to  $9 \square 91 \square 101$  which means  $9 \frac{91}{101}$ . Improper fractions are always converted into mixed numbers (i.e. an integer plus a proper fraction) after pressing  $\text{ENTER}$ . As with proper fractions, the HP 9s tries to give the simplest form.

As to the complex fraction, try pressing  $\text{1} \text{.} \text{8} \frac{\square}{\square}$ . The decimal point disables the  $\frac{\square}{\square}$  key because on the HP 9s the numerator and the denominator must be integers. We can always use the  $\frac{\square}{\square}$  key, though:

$\text{1} \text{.} \text{8} \frac{\square}{\square} \text{9} \text{ENTER}$

which returns 0.2. In the following example we'll learn a way of converting that decimal into the fraction  $\frac{1}{5}$ .

<sup>1</sup> 1000 cannot be entered as 1E3 (i.e.  $\text{1} \text{EXP} \text{3}$ ) because the  $\frac{\square}{\square}$  key is disabled when an exponent is displayed. Likewise, the  $\text{EXP}$  key is disabled when a fraction is being entered.

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**Example 3:** Convert the decimal 1.23456 into a fraction.

**Solution:** Unlike the HP 9g, the HP 9s provides no function to convert decimal to fractions quickly. But, there is a simple procedure anyway. For example, consider:

$$0.2 = \frac{2}{10}$$

Then, simply press:  $\text{ON/C}$   $\text{2}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{ENTER}$  which results in the fraction  $\frac{1}{5}$ . Similarly, any number that has a finite number of decimal digits can be written as:

$$a_0.a_1a_2\dots a_n = a_0 + \frac{a_1}{10} + \frac{a_2}{100} + \dots + \frac{a_n}{10^n}$$

Let's now convert  $1.23456 = 1 + \frac{2}{10} + \frac{3}{100} + \frac{4}{1000} + \frac{5}{10000} + \frac{6}{100000}$ :

$\text{1}$   $\text{+}$   $\text{2}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{+}$   $\text{3}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{+}$   $\text{4}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{+}$   $\text{5}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   
 $\text{0}$   $\text{0}$   $\text{0}$   $\text{+}$   $\text{6}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{ENTER}$

which results in  $1\text{□}733\text{□}3125$ .

This method won't convert long decimals because—as stated in the instruction sheet included with your calculator—mixed numbers must be no more than ten characters in total (including  $\text{□}$  and  $\text{□}$ ). In general, it is better to add  $a_0$  last, so that there is enough room in the display for one more decimal digit.

**Answer:**  $1.23456 = 1\frac{733}{3125}$

**Example 4:** Enter the mixed numbers  $7\frac{2}{18}$ ,  $-1\frac{57}{125}$  and  $3\frac{19}{5}$ .

**Solution:** When entering mixed numbers, remember that the  $\text{a/b/c}$  key is used for separating both the integer from the proper fraction and the numerator from the denominator. To enter the first mixed number press:

$\text{ON/C}$   $\text{7}$   $\text{a/b/c}$   $\text{2}$   $\text{a/b/c}$   $\text{1}$   $\text{8}$   $\text{ENTER}$

The number returned is  $7\text{□}1\text{□}9$ , i.e.  $7\frac{1}{9}$  which is the same as  $7\frac{2}{18}$  after doing a cancellation.

Let's now enter the second fraction  $-1\frac{57}{125}$ . On the HP 9s negative fractions and negative mixed numbers are keyed in by pressing the  $\text{+/-}$  key after entering either part (integer, numerator or denominator). In this example press:

$\text{1}$   $\text{+/-}$   $\text{a/b/c}$   $\text{5}$   $\text{7}$   $\text{a/b/c}$   $\text{1}$   $\text{2}$   $\text{5}$   $\text{ENTER}$

No cancellation is possible this time, so the result is  $-1\text{□}57\text{□}125$ .

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Note that the third number is not actually a mixed number strictly speaking because its fraction part is not proper. Nevertheless, your HP 9s can handle it as well and will return the reduced, proper form. Press:

$\boxed{3} \boxed{\text{a/b/c}} \boxed{1} \boxed{9} \boxed{\text{a/b/c}} \boxed{5} \boxed{\text{ENTER}}$

which returns the mixed number  $6\frac{4}{5}$ .

Fractions can be entered and displayed wherever ordinary decimal numbers can be used. For example, fractions can be used in ordinary arithmetic, stored in the M register (as displayed), used in calculations involving trigonometric and logarithmic functions, in CPLX and STAT modes, etc. But, fractions cannot be used in HEX, OCT and BIN modes: operations in these modes work with integers only.

Example 5: Calculate  $\sin(30^\circ) + \frac{3}{4}$ .

Solution: Calculations that involve both decimal numbers and fractions are expressed in decimal notation:

$\boxed{3} \boxed{0}$  (press  $\boxed{\text{DRG}}$  as needed)  $\boxed{\text{sin}}$   $\boxed{+}$   $\boxed{3} \boxed{\text{a/b/c}} \boxed{4} \boxed{\text{ENTER}}$

Answer: 1.25

Example 6: Add  $1\frac{3}{4}$  to  $2\frac{5}{8}$

Solution: We will add these two mixed numbers as we would add two integers:

$\boxed{\text{ON/C}}$   $\boxed{1} \boxed{\text{a/b/c}} \boxed{3} \boxed{\text{a/b/c}} \boxed{4} \boxed{+}$   $\boxed{2} \boxed{\text{a/b/c}} \boxed{5} \boxed{\text{a/b/c}} \boxed{8} \boxed{\text{ENTER}}$

Note that no parenthesis is necessary here.

Answer: The display shows  $4\frac{3}{8}$ , which means  $4\frac{3}{8}$ . As this calculation only contained fractions, the result is expressed as a fraction too.

Example 7: Express the previous result in decimal notation

Solution: Simply press  $\boxed{\text{a/b/c}}$ . Pressing  $\boxed{\text{a/b/c}}$  after  $\boxed{\text{ENTER}}$  converts the displayed proper fraction or mixed number into a decimal, which can be converted back into fraction by pressing  $\boxed{\text{a/b/c}}$  again.

Answer: 4.375.

Example 8: Are the fractions  $\frac{147}{489}$  and  $\frac{539}{1793}$  equivalent to each other?

Solution: There are several ways of testing whether two fractions are equivalent. We can subtract one from the other and see if the result is zero, or we can calculate the cross multiplication since if  $a/b = c/d$  then  $ad = bc$ . Doing the division is another way. But let your HP 9s do the hard part, and just enter both fractions. Since

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the Hp 9s carries out an automatic simplification, if the fractions displayed are the same then they are equivalent. Press:

$\text{ON/C}$   $\text{1}$   $\text{4}$   $\text{7}$   $\text{a/b/c}$   $\text{4}$   $\text{8}$   $\text{9}$   $\text{ENTER}$ , which returns  $49\text{.}163$

and  $\text{5}$   $\text{3}$   $\text{9}$   $\text{a/b/c}$   $\text{1}$   $\text{7}$   $\text{9}$   $\text{3}$   $\text{ENTER}$

Answer: Since the latter also returns  $49\text{.}163$ , they are equivalent fractions.

Example 9: Convert the mixed number  $2\frac{89}{133}$  to an improper fraction.

Solution: We have seen how improper fractions are automatically converted to mixed numbers when evaluated. But, the opposite is also possible. The  $\rightarrow \frac{d}{c}$  function ( $\text{2ndF}$   $\text{-d/c}$ ) carries out conversions between mixed numbers and improper fractions. Press:

$\text{2}$   $\text{a/b/c}$   $\text{8}$   $\text{9}$   $\text{a/b/c}$   $\text{1}$   $\text{3}$   $\text{3}$   $\text{2ndF}$   $\text{-d/c}$

which results in  $355\text{.}133$ . To convert it back to a mixed number, simply press  $\text{2ndF}$   $\text{-d/c}$  once again.

Answer:  $355\text{.}133$ , i.e.  $\frac{355}{133}$ . Incidentally, this is a good approximation to  $\pi$  (within 8.47 millionths of one percent) and very easy to remember because it's made by duplicating the first three odd numbers and inserting a division sign in the middle  $133\overline{)355}$ . And it was also the very first example in the HP-35 operating manual!

Example 10: Find a fraction which approximates  $\pi$  to four decimal places.

Solution: Rounding to four decimal digits  $\pi = 3.1416$ . Let's convert it to a fraction using the method described above:

$$3.1416 = 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \frac{6}{10000}$$

$\text{ON/C}$   $\text{3}$   $\text{+}$   $\text{1}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{+}$   $\text{4}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{+}$   $\text{1}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{+}$   $\text{6}$   $\text{a/b/c}$   $\text{1}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{0}$   $\text{ENTER}$

The mixed number  $3\text{.}177\text{.}1250$  is returned.

If a fraction is preferred, convert this number to an improper fraction by pressing  $\text{2ndF}$   $\text{-d/c}$

Answer:  $3\frac{177}{1250} = \frac{3927}{1250} = 3.1416$