hp calculators

HP 9s Polar/Rectangular Coordinate Conversions

Rectangular and Polar Coordinates

Practice Solving Problems Involving Coordinate Conversions

Rectangular and polar coordinates

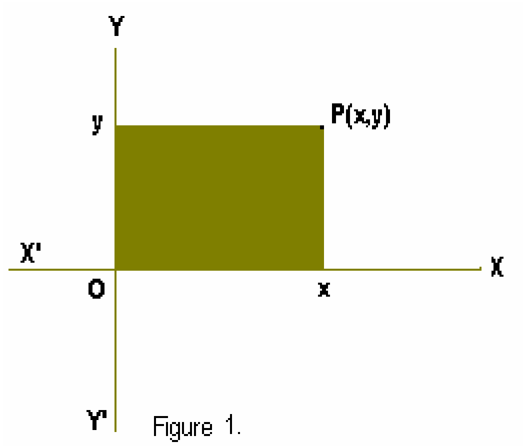

Figure 1.

Coordinates define the position of a point in space. In two dimensions, they are an ordered pair (a, b). There are two primary coordinate systems: rectangular and polar. The **rectangular** system is the commonest coordinate system and uses perpendicular lines in order to measure distances to the base lines or axes, which are also perpendicular to each other. It is often referred to as rectangular cartesian system, and even as cartesian system, even though cartesian coordinates may not be rectangular (when axes are not perpendicular to each other, coordinates are called oblique.) Figure 1 shows the plane rectangular cartesian coordinates of a two dimensional point P. They are written as (x, y). The x-coordinate is measured along or parallel to the XX'-axis and is called **abscissa**. Likewise, the y-coordinate is measure along the YY'-axis and is called **ordinate**.

Polar coordinates describe the position of a point P by its distance to a fixed point O (the pole) and the angle that OP makes with the base line (XX' in figure 2). The angle is measured in the positive or counterclockwise direction from the base line. Coordinates are written as (r, θ). OP is known as the **radius vector** and θ as **vectorial angle**.

By learning what these functions actually do, we will be able to use them in different contexts. Figure 2 shows the relationship between rectangular and polar coordinates. Note that:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan\frac{y}{x}, \quad x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

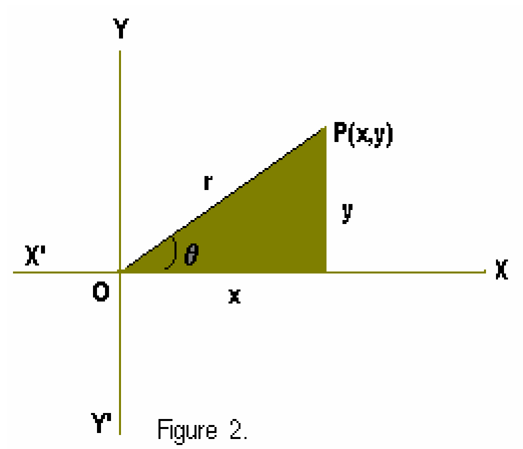

Figure 2.

The HP 9s provides two functions for converting between polar and rectangular coordinates. They are R→P (2ndF R-P) and P→R (2ndF P-R), which take two arguments that are input using the a and b keys. R→P and P→R return r and x, respectively; the θ and y values are shown by the b key, r and x can be reviewed by pressing the a key. The values stored in *a* and *b* are retained so long as no other calculation is started.

	R→P		P→R	
	INPUT	OUTPUT	INPUT	OUTPUT
a	x	r	r	x
b	y	θ	θ	y

Practice solving problems involving coordinate conversions

Example 1: Convert the rectangular coordinates $(-7.5, 13)$ into polar coordinates.

Solution: The function that we need to use in this example is [2ndF] [R–P], which converts the pair of rectangular coordinates (x, y) into the polar coordinates (r, θ). As shown in the above table, *x* and *y* must be stored into *a* and *b*, respectively:

[7] [•] [5] [+/–] [a] [1] [3] [b]

No ENTER is necessary. To perform the conversion, press:

[2ndF] [R–P]

r is displayed. To view the value of θ simply press *b*. Note that θ is expressed in the current angle unit. If you need to view *r* again, press *a*.

Answer: Rounding to four decimal digits, r = 15.0083 and θ = 119.9816º

Example 2: Express the point whose polar coordinates are $(\sqrt{3}, \frac{2\pi}{3})$ in rectangular coordinates.

Solution: The function we will now use is P→R, which will return the abscissa and the ordinate respectively. Its arguments are *r* and θ. Since θ is an angle, you must make sure that the appropriate angle unit is set before pressing *b*. Let's input the arguments:

[2] [x] [2ndF] [π A] [÷] [3] [ENTER] (and [DRG] as needed) [b] [3] [√] [a]

We have input *r* first because once *a* or *b* have been pressed, the following keys: [ENTER], [+], [–], [x], [÷], [x^y], [2ndF] [$\sqrt[x]{}$ B], [2ndF] [π A] and [2ndF] [RND] (but not the postfix functions such as [sin], [2ndF] [e^x E] and [√]) will cancel the conversion in progress, deleting the values stored in both *a* and *b*. That is to say, the following sequence won't return the expected results:

[3] [√] [a] [2] [x] [2ndF] [π A] [÷] [3] [ENTER] [b] [2ndF] [P–R]

We can always use the X→M function to store either coordinate, which can then be recalled by the [MR] key:

[2] [x] [2ndF] [π A] [÷] [3] [ENTER] [X–M] [3] [√] [a] [MR] [b]

Let's now find the rectangular coordinates:

[2ndF] [P–R]

Answer: $(-0.8660, 1.500)$ rounded to four decimal digits.

Example 3: Find the hypotenuse of a right triangle whose catheti are 9 and 40.

Solution: Pythagoras' theorem states that the hypotenuse is given by $\sqrt{x^2 + y^2}$ (x and y being the catheti of a right triangle). But this is exactly what the P→R function returns. Let's compare both methods:

[9] [x²] [+] [4] [0] [x²] [ENTER] [√] (8 keystrokes)

[9] [a] [4] [0] [b] [2ndF] [R−P] (7 keystrokes)

The latter method is one keystroke shorter.

Answer: 41. (The set 9, 40 and 41 is another example of Pythagorean triples: refer to the HP 9s learning module *Powers and Roots*).

Example 4: A vector has components –8 in the X direction and –5 in the Y direction. In what direction does it point?

Solution: The angle is given (in the current angle unit) by R→P(–8, –5):

[8] [+/–] [a] [5] [+/–] [b] [2ndF] [R−P] [b] (press [2ndF] [DRG−] twice to express the angle in degrees, if the current mode is RAD).

Answer: –147.9946º which is the same as the positive angle 212.0054º (because 147.9946º + 212.0054º = 360º). Notice that the calculation $\arctan(\frac{-5}{-8})$ won't give the right result – you have to add 180º because the arctangent function does not take into account the quadrant where the vector lies, unlike the coordinate conversion function R→P.

Example 5: Add two vectors having polar coordinates $(8,30^o)$ and $(12,60^o)$. Represent the sum in terms of magnitude and angle (r,θ).

Solution: Vectors can be easily added (or subtracted) when expressed as complex numbers in rectangular form. The components of the resultant will be the sum of the corresponding components of the complex numbers. Since vectors are given in polar form, we'll use the P→R function to convert them into rectangular form. And since the coordinate conversion functions are available in Complex mode, this conversion can be done *while* adding the vectors.

Make sure DEG is the current angle mode, and then enter Complex mode: [2ndF] [CPLX] [1]. To input the first vector and convert it into rectangular coordinates press:

[8] [a] [3] [0] [b] [2ndF] [P−R]

Let's now add the second vector, which must also be converted into rectangular coordinates:

[+] [1] [2] [a] [6] [0] [b] [2ndF] [P−R] [ENTER]

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[1] The Complex mode is described in greater detail in the HP 9s learning module *Solving Problems Involving Complex Numbers.*

The result is expressed in rectangular coordinates, so press 2ndF R–P to convert it into polar form. The display now contains the magnitude of the sum. Press b to display the angle.

Answer: Rounding to four decimal places $(19.3462, 48.0675^{o})$. To quit Complex mode, press 2ndF CPLX or simply turn off your calculator.

Example 6: Calculate $\sqrt{5+3i}$

Solution: Even though this calculation involves a complex number, it can be done in the normal operating mode. When expressed in exponential form, square roots of complex numbers are very easy to calculate because *in radian mode*:

$$\sqrt{a+bi} = \sqrt{re^{i\theta}} = \sqrt{r}e^{i\frac{\theta}{2}}$$

Therefore, the result in rectangular form is a complex number whose modulus and argument are $\sqrt{r}$ and $\theta/2$ respectively, where r and θ are the polar coordinates of the original complex number.

Let's convert $5+3i$ to polar form:

5 a 3 b 2ndF R–P

Since we need to do calculations with these coordinates, let's store *a* (i.e. *r*) in memory. Press a X–M. And now, let's divide the angle by two:

b ÷ 2 ENTER (*a* and *b* have now been lost)

which is the new b: b

and the new a is:

MR √ a

The square root of $5+3i$ is now in *a* and *b* in polar form. To convert it to rectangular form press 2ndF P–R. Jot down the displayed number (x) and press b to view y.

Answer: $(2.3271, 0.6446)$ rounded to four decimal places.

(The next example is at the top of the next page)

Example 7: The coordinates of the point P (see fig. 3) after the axes have been rotated 30º are $(4.5, 1.5)$. Calculate the old coordinates.

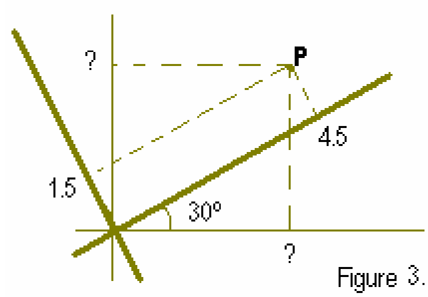

Figure 3.

Solution: The old coordinates are given by:

$x = 4.5\cos 30 - 1.5\sin 30$ and
$y = 4.5\sin 30 + 1.5\cos 30$

Make sure DEG is the current angle mode. Then press:

4 • 5 x 3 0 cos – 1 • 5 x 3 0 sin ENTER and
4 • 5 x 3 0 sin + 1 • 5 x 3 0 cos ENTER

Answer: $(3.1471, 3.5490)$ rounded to four decimal places.