hp calculators

HP 9s Solving Problems Involving Complex Numbers

Basic Concepts

Practice Solving Problems Involving Complex Numbers

Basic concepts

There is *no* real number *x* such that $x^2 + 1 = 0$. To solve this kind of equations a new set of numbers must be introduced. A complex number is a number of the form $a + bi$ where *a* and *b* are real numbers and *i* is the square root of –1, i.e. $i^2 = -1$, and is called the imaginary unit. Since *i* is used for representing the intensity of current in electromagnetism, engineers often write the imaginary unit as *j*. The real *a* is called the real part of the complex number, and *b*, also real, is the imaginary part. When both *a* and *b* are integers, the complex number $a + bi$ is called a Gaussian integer (e.g. $-4 + 3i$). Notice that real numbers can be thought as the subset of complex numbers whose imaginary part is zero. Here are the most basic rules:

- $a + bi = c + di$ if and only if a = c and b = d
- $(a + bi) + (c + di) = (a + c) + (b + d)i$
- $(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$
- $\dfrac{a + bi}{c + di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{bc - ad}{c^2 + d^2} i$

The conjugate complex number of $a + bi$ is $a - bi$. Note that the product of a pair of conjugate numbers is a real number ($a^2 + b^2$). The modulus or absolute value of the complex number $a + bi$ is defined as $\sqrt{a^2 + b^2}$.

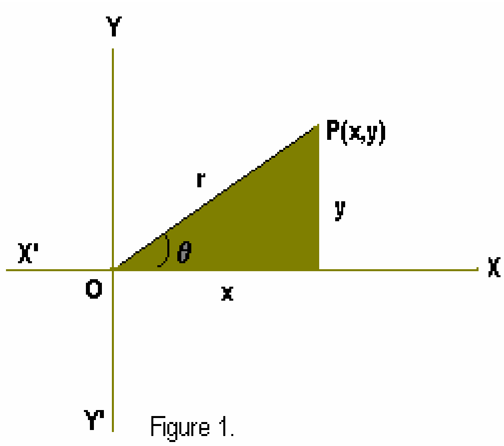

Figure 1.

Figure 1 shows the complex plane, also known as the *Argand diagram*. It is a representation of the complex number $z = x + yi$. XX' is the real axis, and YY' is the imaginary axis. The point P whose cartesian coordinates are (x, y) is called the affix of the complex number z. Note that r (the distance of the affix from the origin O) is equal to the modulus of z. The angle θ is called the argument of z. From the figure: $\tan\theta = \dfrac{y}{x}$

A complex number $x + yi$ can be represented in various ways:

- (x, y) i.e. as cartesian coordinates – the rectangular form.
- $r\cos\theta + ir\sin\theta$. This is the modulus-argument form or polar form, often written as $(r, \angle\theta)$, i.e. the polar coordinates.
- $re^{i\theta}$. It is the exponential form, and is based on Euler's formula $\cos\theta + i\sin\theta = e^{i\theta}$ (where θ is expressed in radians). The function $\cos\theta + i\sin\theta$ is sometimes referred to as cis θ. Note that if $\theta = \pi$ then we obtain the well-known relationship $e^{i\pi} = -1$.

Practice solving problems involving complex numbers

The HP 9s provides an operating mode in which complex calculations can be done. Let's select this mode now by pressing 2ndF CPLX. Notice the CPLX annunciator in the display. The calculator will stay in this mode until another mode is selected or until power is turned off. The following examples illustrate the various calculations with complex numbers that your HP 9s can perform.

Example 1: Find the modulus and the argument of the complex number 5 + 6·i.

Solution: The R→P function ([2ndF] [R–P]) returns the modulus and the argument of a complex number that is expressed in rectangular form. The R→P and P→R functions (described in greater detail in the HP 9s learning module *Polar/Rectangular Coordinate Conversions*) are available in the normal operating mode but also in Complex mode, where they are very useful in converting between exponential and rectangular forms. Remember that the operands (inputs) and the results of R→P and P→R are stored in the special registers *a* and *b* when the [a] and [b] keys are pressed. Let's find the modulus:

[5] [a] [6] [b] [2ndF] [R–P]

The modulus (i.e. *a*) is displayed. To find the argument simply press the [b] key. Bear in mind that the argument of a complex number is an angle, and therefore its value depends on the angular unit. The R→P function always returns the angle in the current angular mode.

Answer: Rounding to four decimal digits, r = 7.8102 and θ = 50.1944º

Example 2: Calculate $(-14-3\cdot \text{i})-(4+8\cdot \text{i})$

Solution: This is as simple as subtracting the real parts and then subtracting the imaginary parts. But, in Complex mode you don't need to do the two subtractions. In fact, adding, subtracting, multiplying and dividing two complex numbers in polar or rectangular form is what Complex mode is all about. When CPLX is lit, the [+], [–], [x] and [÷] keys no longer perform *real* number operations. The [a] and [b] keys are now used to store complex numbers: the real part or the modulus being in *a*, and the imaginary part or the argument in *b*:

[1] [4] [+/–] [a] [3] [+/–] [b] [–] [4] [a] [8] [b] [ENTER]

The result is in rectangular form. *a* appears in the display as soon as [ENTER] is pressed. Press [b] to view the imaginary part.

Answer: $-18-11\cdot \text{i}$

Example 3: The voltage in a circuit is 45 + 5·j volts and the impedance is 3 + 4·j ohms. Find the total current.

Solution: The current is given by the following formula:

$$I=\frac{E}{Z}=\frac{45+5j}{3+4j}$$

We have to divide two complex numbers here. One way of doing this calculation is to use the basic formula given on page 2, which divides two complex numbers expressed in rectangular form. But, once again Complex mode makes things easier:

[4] [5] [a] [5] [b] [÷] [3] [a] [4] [b] [ENTER]

The result is in rectangular form. If you are interested in the exponential form, simply press [2ndF] [R–P] to find the modulus and the argument of the result, as described in Example 1.

Answer: $9.0554e^{-j0.8166} = 6.2 - 6.6j$ amperes. (–0.8166 is the argument expressed in radians).

Example 4: Calculate $(14 + 3 \cdot i) \cdot 3.9$ and $i \cdot (5 - 5 \cdot i)$

Solution: As noted in Example 2, the [×] key expects complex numbers as its arguments, but note that the pure number 3.9 can be written as (3.9 , 0) or 3.9 + 0·i, and the pure imaginary number *i* is in fact (0, 1). The default values for *a* and *b* (even if the [×] key has been pressed) are 0, so there's no need to store 0 in *a* or *b* when entering pure numbers.[1] The following keystroke sequences:

[1] [4] [a] [3] [b] [×] [3] [•] [9] [a] [ENTER]
[1] [b] [×] [5] [a] [5] [+/–] [b] [ENTER]

perform the desired calculations.

Answer: $(14 + 3 \cdot i) \cdot 3.9 = 54.6 + 11.7 \cdot i$ and $i \cdot (5 - 5 \cdot i) = 5 + 5 \cdot i$

The main purpose of the HP 9s Complex mode is to perform basic arithmetic with complex numbers. In fact, certain keys are disabled in this mode, e.g. [a b/c] , [2ndF] [d/c] , [x^y] , [2ndF] [$\sqrt[x]{\ }$] . On the other hand, all postfix functions ([sin] ,[2ndF] [e^x] , [2ndF] [$\sqrt[3]{\ }$] , unit conversions, etc) are available so that complex numbers such as $(5, \sqrt{3})$ can be entered.[2] The following example shows how calculations involving disabled keys can be performed.

Example 5: Calculate $(5 + 3 \cdot i)^3$

Solution: Even though this calculation involves a complex number, it cannot be done in Complex mode because the [x^y] key is disabled. Let's exit Complex mode then: press [2ndF] [CPLX] . When expressed in exponential form, powers of complex numbers are very easy to calculate because *in radian mode*:

$$(a + b \cdot i)^n = \left(r \cdot e^{i\theta}\right)^n = r^n \cdot e^{i \cdot n \cdot \theta}$$

Therefore, the result in rectangular form is a complex number whose modulus and argument are r^n and $n \cdot \theta$ respectively, where r and θ are the modulus and the argument of the original complex number.

Let's convert $5 + 3i$ into polar form. Be sure the RAD annunciator is displayed and press:

[5] [a] [3] [b] [2ndF] [R–P]

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1 But, at least one component must be specified (i.e. either [a] or [b] must be pressed) when the complex number is (0,0) !

2 Since in Complex mode the four basic arithmetic keys only operate on complex numbers, a number such as $\left(3, \frac{8}{15}\right)$ must be entered by storing 8/15 in memory first and then—once in Complex mode—pressing [3] [a] [MR] [b] . Also, [2ndF] [π] and [2ndF] [RND] should not be used once [a] or [b] have been pressed because pressing those keys deletes the part of the complex number already stored.

We can now write: $5+3\cdot i=5.830951895\cdot e^{0.5404195\cdot i}$. We have to multiply 0.5404195 by 3:

[b] [X–M] [M+] [M+]

The M register now contains the triple of θ, which will be the new *b* . We have not used the [x^y] key because *a* will be lost (cleared) if any new calculation is started. But before storing M in *b* (if we store M in *b* now, then no two-argument function can be used in calculating the value of *a*) , let's calculate the modulus of the result i.e. 5.830951895^3:

[a] [x^y] [3] [ENTER]

which is the new *a*, so press [a] . 3θ can now be stored in *b*: [MR] [b]

The cube of $5+3i$ is now in *a* and *b* in polar form. To convert it into rectangular form press [2ndF] [P–R] .

Answer: $-10+198\cdot i$. Complex mode is assumed from now on, so press [2ndF] [CPLX] to activate it.

Example 6: Calculate $(5+3\cdot i)\cdot 3e^{-2.0123\cdot i}$

Solution: This is the multiplication of two complex numbers that are expressed in different forms. In general, we need both the multiplicand and the multiplier (or the augend and the addend in an addition, or the minuend and the subtrahend in a subtraction, or the dividend and the divisor in a division) to be expressed in rectangular form before performing the operation in the HP 9s Complex mode. This can be done easily because the conversion can be done *while* the multiplication is being entered. Make sure RAD is the current angular mode (remember that the argument of a complex number in exponential form is always given in radians) and press:

[5] [a] [3] [b] [x] [3] [a] [2] [•] [0] [1] [2] [3] [+/–] [b] [2ndF] [P–R] [ENTER]

Note that the P→R conversion function converts the second complex number into rectangular form before the multiplication is performed by the [ENTER] key press.

Answer: Rounding to four decimal digits, $1.7275-17.4073\cdot i$

Example 7: Calculate $\dfrac{(35,\angle 27^o)}{(3,\angle -70^o)}$

Solution: Both complex numbers in this operation are expressed in the same form, but arithmetic operations in Complex mode must be performed in rectangular form (results are always in rectangular form too). So, two conversions are needed this time. Select DEG as the angular mode and press:

[3] [5] [a] [2] [7] [b] [2ndF] [P–R] [÷] [3] [a] [7] [0] [+/–] [b] [2ndF] [P–R] [ENTER]

which returns the complex number –1.421809006 + 11.5797051·i. Since the numbers were given in polar form, we should express the result in polar form too. To do so, simply press [2ndF] [R–P]

Answer: $(11.66666667, \angle 97^{o})$

Example 8: Calculate $\left(\frac{3-8\cdot i}{8-9\cdot i}\right)\left(\frac{14-3\cdot i}{7+5\cdot i}\right)$

Solution: Although the parentheses keys are disabled in Complex mode, chain calculations are possible. Be warned, however, that the precedence is not the same as with real numbers, e.g. $1+2\times 3=7$ outside CPLX mode, but $(1,0)+(2,0)\times(3,0)=(9,0)$ in CPLX mode.

3 a 8 +/– b ÷ 8 a 9 +/– b x 1 4 a 3 +/– b ÷ 7 a 5 b ENTER

Answer: Rounded to two decimal digits, $0.43-1.10\cdot i$.