hp calculators

HP 9s Statistics – Process Capability

Process Capability

Practice Solving Process Capability Problems

Process capability

Any manufacturing process is subject to variability due to both assignable (e.g. a defective machine, the operator's mistakes) and non-assignable causes (e.g. the limited precision of machines, the operator's *skill*). When the former are eliminated, then the process is said to be <u>in control</u>. Variability is then *predictable.*

We can determine whether an *in-control process* meets the quality specifications by measuring the <u>capability</u> of the process. A process is <u>capable</u> when it produces similar products for a sustained period of time, that is to say, when almost all pieces are fabricated within a given tolerance. Assuming that the performance measure data is *distributed normally*, then approximately 99.73% of all values will lie within three standard deviations on either side of the average –- this interval (6 sigma) is called the <u>natural spread</u> of the process. This is why the process capability index is defined as:
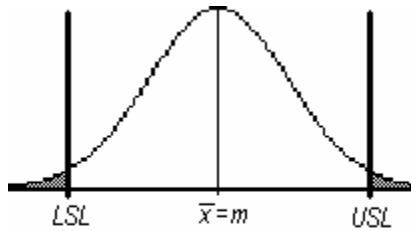

Figure 1

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL and LSL are the upper and lower specification limits, respectively. USL – LSL is called the <u>tolerance spread</u>. If C_p = 1, then 99.73% of pieces will be fabricated within the specification limits *provided that the process average is at the midpoint of the spec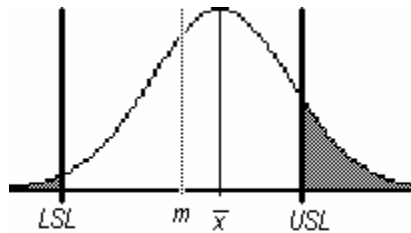ification range* $m = (USL + LSL)/2$ (see figure 1). The following index is used for assessing the capability of an off-center process (i.e. when $\overline{x} \neq m$, see figure 2):

Figure 2

$$C_{pk} = min\left(\frac{USL - \overline{x}}{3\sigma}, \frac{\overline{x} - LSL}{3\sigma}\right)$$

For a centered process (i.e. $\overline{x} = m$) $C_{pk} = C_p$, otherwise $C_{pk} < C_p$.

A process for which C_p = 1 might produce up to $(1 - 0.9973) \cdot 10^6 = 2700$ defective pieces *per million*. While such a process was considered to be right in the past, it might well be deemed unacceptable nowadays. Today's standards for both C_p and C_{pk} are that they must be greater than 1.33 for the process to be capable (that assures us of no more than 60 non-conforming parts per million) . But this value certainly depends on the product being fabricated.

Practice solving process capability problems

<u>Example 1:</u> In order to determine the capability of a manufacturing process, five samples of the resulting pieces are taken. Each sample consists of ten pieces. The following table shows the average and standard deviation of each sample:

$\overline{x}$	48	48	49	52	53
s	0.5	0.6	0.2	0.4	0.3

Determine whether the process is capable if the tolerance limits for this piece are 50 ± 2.

Solution: We'll assume that the process is in control and also that the length of the pieces are normally distributed. We have to calculate the *process* average and standard deviation:

Press (2ndF) STAT to enter STAT mode and then:

DATA DEL (i.e. (MR)) (+) (4) (8) (x) (2) DATA DEL (4) (9) DATA DEL (5) (2) DATA DEL (5) (3) (ENTER) x̄ Σx̄ (i.e. (x–M))

which returns the process average: $\bar{X} = 50$

Now, let's calculate the average of the standard deviations. Press (2ndF) STAT twice to clear the previous data (remember that leaving STAT mode deletes the statistical data), and then press:

DATA DEL (.) (5) DATA DEL (.) (6) DATA DEL (.) (2) DATA DEL (.) (4) DATA DEL (.) (3) (ENTER) x̄ Σx̄

which returns 0.4. This is not yet the process standard deviation. It has to be divided by a coefficient that depends on the number of items in each sample, and which is given in tables. For n = 10, this coefficient is 0.9227. Therefore the estimated standard deviation is (rounded to two decimal digits):

$$\sigma = \frac{\bar{S}}{c} = \frac{0.4}{0.9227} \cong 0.43$$

We can now calculate the process capability indices. But first, we need to find a way to input $\bar{X}$ and σ! This can be done by clearing the data again and entering the following two data items: $\bar{X} + \sigma$ and $\bar{X} - \sigma$, i.e. 50.43 and 49.57:

(2ndF) STAT (2ndF) STAT DATA DEL (5) (0) (.) (4) (3) DATA DEL (4) (9) (.) (5) (7) (ENTER) .

Note that the calculations 50+ 0.43 and 50 – 0.43 cannot be done *after* pressing the DATA DEL key. Once DATA DEL has been pressed, the HP 9s expects either a number (e.g. 25.5) or a number followed by its frequency (e.g. 25.5 * 4) . Now press x̄ Σx̄ and (2ndF) s σ to verify that the desired values of $\bar{X}$ and σ are indeed in memory.

Calculating C_p is as simple as pressing (2ndF) CP . The USL annunciator is lit and the display starts blinking. This is the way the HP 9s prompts you for the value of USL! Just press (5) (2) (ENTER) . Now, it is the LSL annunciator that is lit. Press (4) (8) (ENTER) . The answer is displayed: $C_p = 1.55$ (rounded to two decimal digits). To find C_{pk} press (2ndF) CPK (ENTER) and (ENTER) (the limits specified when calculating C_p are now displayed by default: just press (ENTER) to confirm you want to reuse them). $C_{pk} = 1.55$ too.

Answer: Since $C_p = C_{pk}$, the process is centered. And since $C_p = 1.55 > 1.33$, the process is capable.

Example 2:   As part of a process optimization effort, an industrial manager had to determine whether their manufacturing process, for which $\bar{X} = 19.35$ and $\sigma = 2.05$, was capable. The population of data values is assumed to be normally distributed. The upper and lower specification limits of the process are 23.3 and 11, respectively. How could the HP 9s help her?

Solution: By calculating the C_p and C_{pk} statistics to measure the process capability. First of all, let's clear any previous statistical data in memory by pressing (2ndF) STAT twice. No set of data is provided this time, only

the specification limits and the $\bar{X}$ and σ values. But we know that these values are the only ones needed to calculate the capability indices. Let's input the $\bar{X}$ and σ values using the trick described in the previous example:

DATA DEL (2) (1) (•) (4) DATA DEL (1) (7) (•) (3) (ENTER)

Once again, note that the calculations 19.35+ 2.05 and 19.35 – 2.05 must be done before pressing DATA DEL .

We can now calculate C$_p$: press (2ndF) CP (2) (3) (•) (3) (ENTER) (1) (1) (ENTER) , which returns 1.

Likewise, to find C$_{pk}$, press (2ndF) CPK (ENTER) (ENTER) . C$_{pk}$ = 0.6423 (rounded to four decimal digits).

Note that C$_p \neq$ C$_{pk}$, which means that the process is off-center:

$$(USL + LSL)/2 = (23.3 + 11)/2 = 17.15 \neq \bar{X} = 19.35$$

The meaningful index is therefore C$_{pk}$.

Answer: Since C$_{pk}$ is clearly smaller than 1.33, the process was incapable: the process spread exceeded the tolerance spread, which means that a significant amount of out-of-tolerance products were being made.