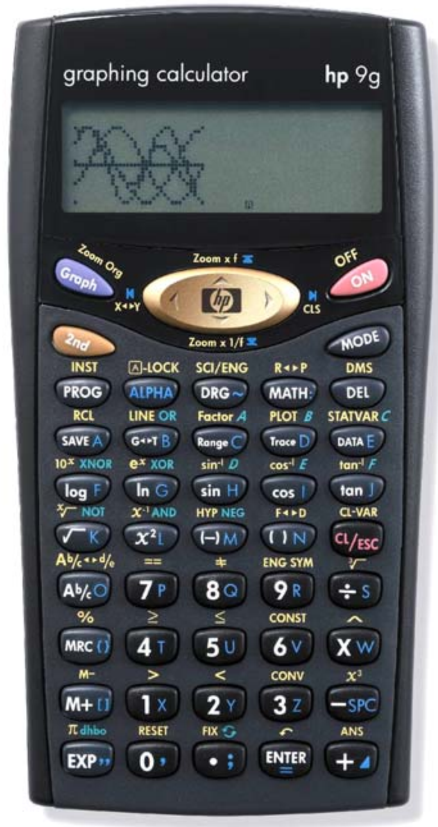
hp calculators

HP 9g Probability – Rearranging Items



Rearranging Items

Practice Solving Problems Involving Factorials, Permutations, and Combinations

Rearranging items

Counting the ways in which items can be rearranged is a frequent task when calculating probabilities, and we have three functions at our disposal for this purpose: factorial, permutations and combinations.

If *n* is a positive integer, its factorial (whose symbol is *n*!) is the product of all the positive integers up to and including *n*:

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$$

0! is defined as 1.

A permutation of *r* from *n* is a way in which a set of *r* elements may be chosen in order from a set of *n* elements. In other words, it's an *ordered* subset of a set of distinct objects. The number of possible permutations, each containing *r* objects, that can be formed from a collection of *n* distinct objects is given by:

$$^{n}P_r = P_r^n = {}_nP_r = P(n,r) = (n)_r = \frac{n!}{(n-r)!}$$

The above equation also shows the multiple ways of symbolizing permutations. When a permutation involves all the elements of a set (i.e. r = n) then it is called a rearrangement, and also as shuffle especially when cards are used. Notice that in this case:

$$P_r^n = P_n^n = \frac{n!}{0!} = n!$$

A combination is a selection of one or more of a set of distinct objects without regard to order. These are the different ways of denoting a combination and the number of possible combinations, each containing r objects, that can be formed from a collection of m distinct objects:

$$^{n}C_r = C_r^n = {}_nC_r = C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Vandermonde's theorem is an important theorem on combinations and states that:

$$C_r^n = C_{r-1}^{n-1} + C_r^{n-1}$$

As far as rearranging items is concerned, whether order is important or not is the only difference between combinations and permutations.

The factorial function was traditionally used for calculating permutations and combinations, which show up in many discrete probability distribution calculations, such as the binomial and hypergeometric distributions.

On the HP 9g, all these functions are in the menu MATH – just press MATH 0 to enter the factorial function, and MATH ⌃ 0 or MATH ⌃ 1 to enter the permutations function or the combinations function respectively. The HP 9g errors for factorials of numbers greater than 69, but this covers most of the problems you will find.

Practice solving problems involving factorials, permutations, and combinations

Example 1: Calculate the number of combinations and permutations of 15 objects taken 5 at a time.

Solution: The number of combinations is calculated by

[1 X] [5 U] [MATH] ^ [1 X] [5 U] [ENTER]

and the number of permutations by pressing

[1 X] [5 U] [MATH] ^ [0 ,] [5 U] [ENTER]

Answer: 3003 and 360360.

Example 2: Calculate the number of ways that six people can line up for a photograph.

Solution: Since the order does matter in this example, the problem is solved as a permutation: $P_6^6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$. Press

[6 V] [MATH] ^ [0 ,] [6 V] [ENTER] *or* [6 V] [MATH] [0 ,] [ENTER]

Answer: 720

Example 3: In how many different ways can five seats be filled by a group of ten persons?

Solution: Again, the order is important ,so the answer is P_5^{10}. Press

[1 X] [0 ,] [MATH] ^ [0 ,] [5 U] [ENTER]

Answer: 30,240

Example 4: Verify Vandermonde's theorem for n = 4 and r = 3.

Solution: Pressing [4 T] [MATH] ^ [1 X] [3 Z] [ENTER] returns the same result as [3 Z] [MATH] [1 X] [2 Y] [+] [3 Z] [MATH] [1 X] [3 Z] [ENTER]

Answer: C_3^4=4 and $C_2^3 + C_3^3$=4

Example 5: How many different hands of 5 cards could be dealt from a standard deck of 52 cards? Assume the order of the cards in the hand does not matter.

Solution: Since the order of the cards in the hand does not matter, the problem is solved as a Combination: C_5^{52}

[5 U] [2 Y] [MATH] ^ [1 X] [5 U] [ENTER]

Answer: 2,598,960 different hands.

Example 6: If five cards are dealt from a standard deck of 52 cards, calculate the probability of these five cards containing four-of-a-kind.

Solution: The probability is the number of favorable events divided by the number of possible events. There are 13 ranks of four-of-a-kind, the fifth card being any of the remaining 48 cards. The number of possible five-card hands has been calculated in the previous example: it is given by the combination the C_5^{52}. Therefore, the probability of being dealt four-of-a-kind is:

$$P = \frac{\text{possible four-of-a-kind hands}}{\text{possible five-card hands}} = \frac{13 \times 48}{\binom{52}{5}}$$

The keystroke sequence is:

1 3 × 4 8 ÷ 5 2 MATH ^ 1 5 ENTER

If the result of the previous example is still displayed (and therefore stored in Ans), then you can press this instead:

1 3 × 4 8 ÷ 2nd ANS ENTER

Answer: 0.000240096. Approximately, 1 in every 4165 cards (which is calculated by inverting the result).

Example 7: Evaluate the binomial density function f(x) at x=4 for a constant probability of success (p) of 0.49 and 6 Bernoulli trials (n).

Solution: $f(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$ In our case:

(6 MATH ^ 1 4) • 4 9 2nd ^ 4 (1 − • 4 9) 2nd ^ (6 − 4 ENTER

Notice that the first parenthesis is actually not mandatory because nCr and nPr have priority over the multiplication.

Answer: f(4) = 0.224913711

Example 8: If you flip a coin 10 times, what is the probability that it comes up tails exactly 4 times?

Solution: It is an application of the previous example. In this case, n = 10, x = 4 and p = 0.5:

1 0 MATH ^ 1 4 × • 5 2nd ^ 4 (1 − • 5) 2nd ^ (1 0 − 4 ENTER

Answer: 0.205078125. If you flip a coin 10 times, there is a 20.51% chance of seeing heads 4 times.

Example 9: An urn contains eight white, five red and six black balls. If three balls are drawn at random, find the probability that all three are white.

Solution: This problem can be solved using either using combinations or permutations with the same result:

$$\frac{C_3^8}{C_3^{19}} = \frac{P_3^8}{P_3^{19}}$$

[8] [MATH] ^ [1] [3] [÷] [1] [9] [MATH] ^ [1] [3] [ENTER] or
[8] [MATH] ^ [0] [3] [÷] [1] [9] [MATH] ^ [0] [3] [ENTER]

Answer: 0.057791538

Example 10: In the previous example, find the probability that one ball is black and two are white.

Solution: $\frac{C_1^6 \cdot C_2^8}{C_3^{19}}$

[6] [MATH] ^ [1] [1] [×] [8] [MATH] ^ [1] [2] [÷] [1] [9] [MATH] ^ [1] [3] [ENTER]

Answer: 0.173374613