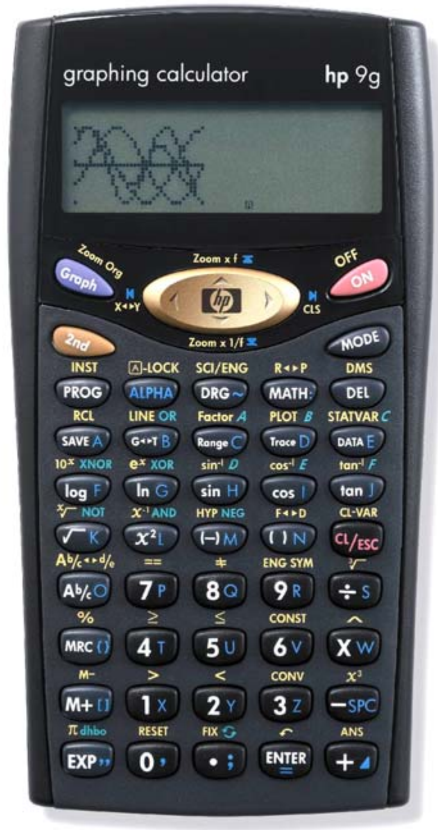
hp calculators



HP 9g Probability – Random Numbers

Random Numbers

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Random numbers

Strictly speaking, random numbers are those numbers the digits of which are chosen with replacement so that it is one of a set of numbers that can be chosen, all of which are equally likely. "Chosen with replacement" means that once a digit has been chosen it may be chosen again with the same probability as the other digits. When it is a computer or calculator that generates the random numbers, they are also known as pseudo-random numbers or quasi-random numbers because they are not exactly random, but can be treated as if they were random for all practical purposes.

The HP 9g provides two commands to generate random numbers, namely RAND (MATH: 1 X) and RANDI (MATH: 2 Y). The former returns a random number between 0 and 1, and the latter takes two integers A and B and returns a random integer n such as $A \le n \le B$ (assuming $A \le B$). Random numbers are internally generated using the previous random number generated, and the first one is generated using a seed – for each seed, a different series of numbers is generated. On the HP 9g, there is no way of changing this seed, but it is based on the value of the internal clock, so it changes at every reset.

Among the most important uses of random numbers are games and simulations: they are used to decide what the next piece will be in the Tetris game we are playing, as well as in stock market simulations.

Simulation

In mathematics, a simulation is a mathematical model of a system or a process which often involves the element of chance. Simulations help to explain some physical behavior (e.g., the motion of waves, the flip of a coin, etc.). This is certainly an extensive topic and the following examples cannot be exhaustive, but they show a few ways random numbers can be used on your HP 9g.

Note. The values returned by RAND and RANDI on your HP 9g will probably be different than the ones in these examples because a new series of random numbers is created for each seed.

Practice using random numbers for simulations

Example 1: Simulate flipping a coin four times.

Solution: When a coin is flipped, there are only two possible events: the probability of heads is 0.5 and of tails also 0.5. Let the number 0 equate to observing a "heads" and 1 equate to observing a "tails." Therefore, RANDI(0,1) is a suitable simulation of the flip of a coin, since it returns either 0 or , and both results are equally likely. On your HP 9g, press:

MATH: 2 Y 0 , ALPHA 0 , 1 X ENTER

Notice that to separate the two arguments, you have to use the comma, which is entered by ALPHA 0 , . It returns 1 (tails). To simulate the other three flips, you have to repeat the calculation, which is as easy as pressing ENTER three times: 1, 1, and 0 are returned which means that "heads" happened only once.

Answer: The result of flipping the coin four times is three tails and a heads.

Example 2: Simulate rolling 2 dice.

Solution: When a die is rolled, the result is equally likely to be a 1, 2, 3, 4, 5, or 6. Since these are all integer numbers, we can use the RANDI command for this purpose: RANDI(1,6).

Press MATH 2 1 ALPHA 0 6 ENTER jot down the result and press ENTER again to simulate the second dice.

Answer: The value of the first die was a 6 and the second was a 5, for a total on the two dice of 11.

Example 3: Vladimir's Newstand sells newspapers and has experienced demand for newspapers as follows over the last 50 days: 10 newspapers on 5 of the days; 15 newspapers on 20 of the days; 20 newspapers on 15 of the days; and 25 newspapers on 10 of the days. Using random numbers simulate demand for the next 5 days.

Solution: The first step will be to translate the past demand into ranges for our random numbers for the simulation. Out of past 50 days, demand was 10 on 5 of these days, or 10% of the time. Out of the past 50 days, demand was 15 on 20 of these days, or 40% of the time. Out of the past 50 days, demand was 20 on 15 of these days, or 30% of the time. Finally, out of the past 50 days, demand was 25 on 10 of the days, or 20% of the time. This information can be summarized in a table as shown below.

Demand	Probability
10	0.1
15	0.4
20	0.3
25	0.2

Next, we need to assign a range for each level of demand that corresponds to the relative probability for that demand. It is this range that will be used to classify each random number as a specific simulated demand.

Demand	Probability	Range
10	0.1	0.0 < random number ≤ 0.1
15	0.4	0.1 < random number ≤ 0.5
20	0.3	0.5 < random number ≤ 0.8
25	0.2	0.8 < random number < 1.0

Note that each range corresponds to the probability of each outcome (the range between 0.1 and 0.5 is 40% of the possible outcomes of the random numbers and therefore reflects the 40% chance that a demand of 15 will occur). We can now generate the five random numbers. Let's evaluate each random number as it is generated.

On your HP 9g, press MATH 1 ENTER The number returned is: 0.430603027. This corresponds to a demand of 15 newspapers. To repeat the calculation and generate a new random number, press ENTER : the number displayed is now 0.239334106, which also corresponds to a demand of 15 newspapers. We have to repeat this calculation three times more:

Pressing [ENTER] returns 0.564239502. This corresponds to a demand of 20 newspapers.
Pressing [ENTER] returns 0.59501648. This corresponds to a demand of 20 newspapers.
Pressing [ENTER] returns 0.780151367. This corresponds to a demand of 20 newspapers.

<u>Answer:</u> The results were demands of 15, 15, 20, 20 and 20 newspapers. If the simulation were carried out for a longer period (which could be done by writing a program), other levels of demand would be generated.

<u>Example 4:</u> Write a program that generates a large number of random digits and returns how many 0's, 1's, 2's, ..., 8's and 9's have been generated.

<u>Solution:</u> This is one of the simplest ways of testing a random number generator, it is called the frequency test. The ten digits should occur about 10% each.

This is the program:

```
INPUT N
FOR ( Z = 1; Z ≤ N ; Z++ )  {
        ++A [RANDI( 0 , 9 )]
        PRINT Z }
PRINT “PRESS RCL”
END
```

Line #	Command	Explanation	Keys
Line 1	INPUT N	Asks for the number of random digits to be generated	[2nd] [INST] [▲] [▲] [▲] [0 ,] [ALPHA] [( ) N] [ENTER]
Line 2	FOR (Z = 1; Z ≤ N ; Z++) {	The following two instructions are done N times	[2nd] [INST] [3 Z] [ALPHA] [3 Z] [ALPHA] [ENTER] [1 X] [▶] [ALPHA] [3 Z] [2nd] [≤] [ALPHA] [( ) N] [▶] [ALPHA] [3 Z] [2nd] [INST] [▼] [▼] [▼] [2 Y] [▶] [▶] [ENTER]
Line 3	++A [RANDI(0 , 9)]	Generates a random digit i (=0...9). Var A[i] (A...J) is incremented by one	[2nd] [INST] [▼] [▼] [▼] [2 Y] [ALPHA] [SAVE A] [ALPHA] [M+ []] [MATH:] [2 Y] [0 ,] [ALPHA] [0 ,] [9 R] [▶] [▶] [ENTER]
Line 4	PRINT Z }	Displays the loop counter	[2nd] [INST] [▲] [▲] [▲] [3 Z] [ALPHA] [3 Z] [▶] [▶] [ENTER]
Line 5	PRINT “PRESS RCL”	Once finished, tells the user what to do next.	[2nd] [INST] [▲] [▲] [▲] [3 Z] [2nd] [A-LOCK] [EXP ”] [7 P] [9 R] [DATA E] [÷ S] [÷ S] [− SPC] [9 R] [Range C] [x² L] [ALPHA]

When running for the first time, press [2nd] [CL-VAR] to clear all the variables, it is not done by the program so that further executions can add their results to the existing values. The program starts by asking the user how many random digits have to be generated, say, 200. Since it takes some time, the program always displays the number of random digits that has generated thus far. At the end, the message “PRESS RCL” appears meaning that the results can be viewed in the menu of variables displayed by [2nd] [RCL] . A contains the number of 0's generated, in B are the number of 1's, and so on till J, which contains the number of 9's. To get the relative frequency instead, divide each variable by N.

The above table includes the keystrokes needed to enter the program once you are in Program Edit mode. (Refer to the learning module *Writing a Small Program* for more information on creating and editing programs).

<u>Answer:</u> Running the program for N=100,000 returns these values: (remember that your calculator may generate different random numbers)

	0's (A)	1's (B)	2's (C)	3's (D)	4's (E)	5's (F)	6's (G)	7's (H)	8's (I)	9's (J)
Freq.	9,936	10,003	10,042	9,994	9,962	10,043	10,036	10,045	9,886	10,053

All these values are quite close to the ideal value a "perfect" generator would return, i.e. 10,000. Below is a frequency histogram of the generated digits. Number 8 was the least frequent digit and 9 the most frequent. Try increasing the number of random digits generated in order to get more meaningful results.

Frequency of the numbers generated by the RANDI function