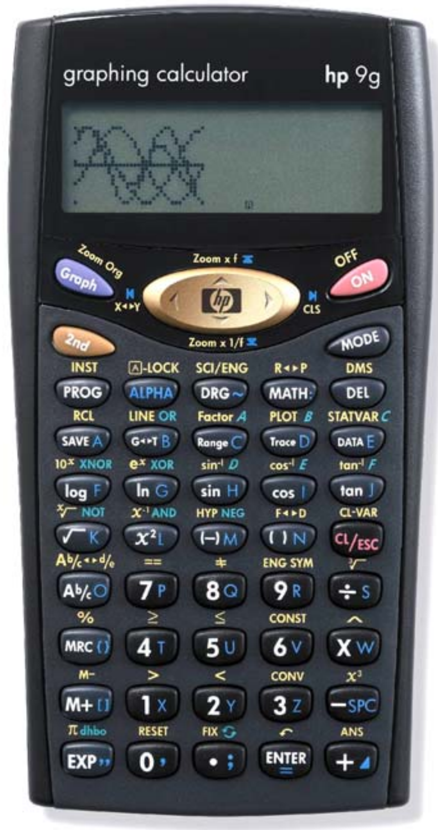
hp calculators

HP 9g Powers and Roots

Powers and Roots

Practice Solving Problems Involving Powers and Roots



Powers and roots

The number a in a^m is said to be raised to the power m, which is also called index or exponent. It obeys the so-called index laws, namely:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$

where a is a number and m and n are rational numbers. The process of raising to a power is often known as *involution*. The opposite of a power is called a root. For example, if $5^3 = 125$ then 5 is the third root of 125, and it is often written this way:

$$5 = \sqrt[3]{125}$$

The process of finding a root is known as *extraction* and also as *evolution*. Evolution is the inverse of the involution. Notice the following important relationships:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

The HP 9g has numerous functions to calculate powers and roots. These are: [x^2 L], [2nd] [10^x XNOR], the exponential function [2nd] [e^x XOR], [2nd] [x^3], [2nd] [x^{-1} AND] (same as 1/x), the power function [2nd] [^], [√ K] which calculates square roots (i.e. second roots), [2nd] [∛] that calculates cube roots(the third root), and the xth-root function [2nd] [$\sqrt[x]{}$ NOT].

Practice solving problems involving powers and roots

Example 1: Is e^{π} greater than π^e ?

Solution: We'll calculate the difference $e^{\pi} - \pi^e$:

[2nd] [e^x XOR] [2nd] [π dhbo] [⟩] [−SPC] [2nd] [π dhbo] [2nd] [^] [2nd] [e^x XOR] [1 X] [ENTER]

Even though no keystrokes will be saved in this example, remember that your HP 9g has a convenient 2nd-lock feature ([ALPHA] [2nd]):

[2nd] [e^x XOR] [2nd] [π dhbo] [⟩] [−SPC] [ALPHA] [2nd] [π dhbo] [^] [e^x XOR] [2nd] [1 X] [ENTER]

Answer: The difference is $0.681534914 = e^{\pi} - \pi^e > 0$, therefore $e^{\pi} > \pi^e$

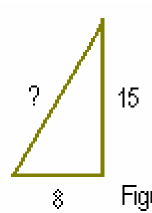


Figure 1.

Example 2: Find the hypotenuse of a triangle the catheti of which are 8 and 15 (see figure 1).

Solution: The hypotenuse of a right triangle is given by Pythagoras' theorem (even though the Babylonians already knew this relationship!) :

$$\text{Hypotenuse} = \sqrt{a^2 + b^2}$$

where a and b are the two catheti. In our example:

[√ K] [8 Q] [x² L] [+] [1 X] [5 U] [x² L] [ENTER]

Answer: 17 . The set of numbers 8, 15 and 17 is an example of a Pythagorean triple, i.e. integers that can be sides of the same right triangle. Some of the simpler sets were already known by the ancient Egyptians. Refer to the learning module *Polar/Rectangular Coordinate Conversions* to learn another way of calculating the hypotenuse.

Example 3: Calculate 0^0

Solution: Even though some calculators return 1, on the HP 9g, [0 ,] [2nd] [^] [0 ,] [ENTER] generates the error "DOMAIN Er" because 0^0 is mathematically an indeterminate (or undetermined) form, much like 0 / 0 or log 0.

Example 4: Calculate $9^{-0.27}$ Use the exponential function to confirm the result.

Solution: A convenient way of computing x^y is as $e^{y\ln x}$ since $x^y = e^{\ln x^y} = e^{y\ln x}$ The HP-35, the world's first scientific electronic pocket calculator, used this method to save valuable space in ROM, which could be noticed by the fact that some results were not accurate to the last decimal place (e.g. 2^9). Therefore, $9^{-0.27}$ can be calculated with the ^ function ([2nd] [^]):

[9 R] [2nd] [^] [(−) M] [• ;] [2 Y] [7 P] [ENTER]

and as $e^{-0.27\times\ln 9}$. To evaluate the latter expression press

[2nd] [eˣ XOR] [(−) M] [• ;] [2 Y] [7 P] [ln G] [9 R] [ENTER]

Answer: Both methods return 0.552528294

Example 5: Assume that a body moves along a straight line according to the equation $S = \frac{1}{2}t^6 - 4t$. Determine its velocity ($V = 3t^5 - 4$) and acceleration ($A = 15t^4$) at t = 2 seconds.

Solution: $V = 3 \times 2^5 - 4$, which can be evaluated by pressing:

[3 Z] [× W] [2 Y] [2nd] [^] [5 U] [− SPC] [4 T] [ENTER]

and $A = 15 \times 2^4$, press:

[1 X] [5 U] [× W] [2 Y] [2nd] [^] [4 T] [ENTER]

Answer: V = 92 and A = 240

Example 6: Find the maximum shear stress on an element if the stress in the x –direction (S_x) is 25,000 psi, the stress in the y-direction (S_y) is –5,000 psi and the shear stress on the element for the Mohr circle input (τ_{xy}) is 4,000 psi.

Solution: The maximum shear stress is calculated by the following formula:

$$\tau_{max} = \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + \tau_{xy}{}^2}$$

[√] [( )] [( )] [2] [5] [EXP] [3] [−] [(−)] [5] [EXP] [3] [▸] [÷] [2] [▸] [x²] [+] [4] [EXP] [3] [x²] [ENTER]

Answer: 15,524.1747 psi.

Example 7: Find the geometric mean of the set of numbers { 2, 3.4, 3.41, 7, 11, 23 }

Solution: For a set of n positive numbers { a_1, a_2, ..., a_n } the geometric mean is defined by

$$G = (a_1 \cdot a_2 \cdot \ldots \cdot a_n)^{\frac{1}{n}}$$

To find G press

[( )] [2] [×] [3] [•] [4] [×] [3] [•] [4] [1] [×] [7] [×] [1] [1] [×] [2] [3] [▸] [2nd] [^] [( )] [1] [÷] [6] [ENTER]

Remember that we can write the above equation as:

$$G = \sqrt[n]{(a_1 \cdot a_2 \cdot \ldots \cdot a_n)}$$

which can be calculated in fewer keystrokes:

[6] [2nd] [ˣ√] [2] [×] [3] [•] [4] [×] [3] [•] [4] [1] [×] [7] [×] [1] [1] [×] [2] [3] [ENTER]

Answer: G =5.873725441

The following example has been taken from the HP-67 Owner's Handbook and Programming Guide. This formula was often used as an example of the value of the Hewlett-Packard RPN logic system, but we'll use it here to show that it can be computed on your HP 9g *exactly as written*.

Example 8: In a rather overoptimistic effort to break the speed of sound, high-flying pilot Ike Daedalus cranks open the throttle on his surplus Hawker Siddeley Harrier aircraft. From his instruments he reads a pressure altitude (PALT) of 25,500 feet with a calibration airspeed (CAS) of 350 knots. What is the flight mach number

$$M = \frac{\text{speed of aircraft}}{\text{speed of sound}}$$

if the following formula is applicable?

$$M = \sqrt{5\left[\left(\left\{\left[\left(1+0.2\left[\frac{350}{661.5}\right]^{2}\right)^{3.5}-1\right]\left[1-(6.875\times10^{-6})25500\right]^{-5.2656}\right\}+1\right)^{0.286}-1\right]}$$

Solution:

[√] [5] [(] [(] [(] [(] [(] [1] [+] [.] [2] [(] [3] [5] [0] [÷] [6] [6] [1] [.] [5] [▸] [x²] [▸]
[2nd] [^] [3] [.] [5] [−] [1] [▸] [(] [1] [−]
[(] [6] [.] [8] [7] [5] [EXP] [(−)] [6] [▸] [2] [5] [5] [0] [0] [▸] [2nd] [^] [(−)] [5] [.] [2] [6] [5] [6] [▸]
[+] [1] [▸] [2nd] [^] [.] [2] [8] [6] [−] [1]

Answer: M =0.835724535