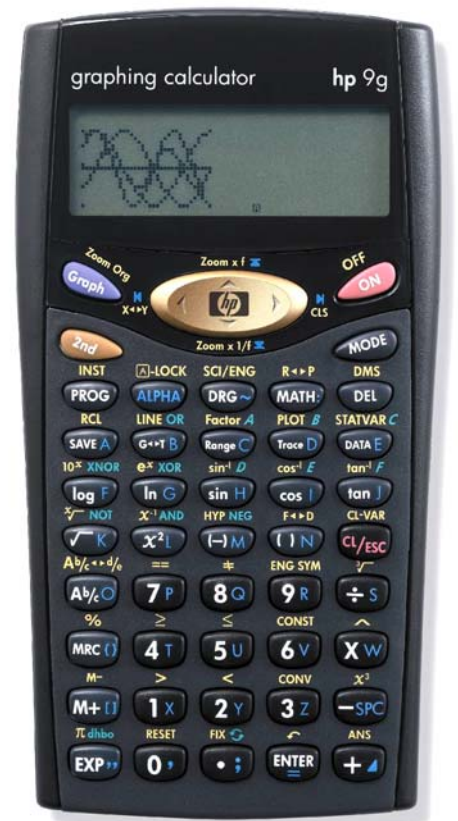**hp calculators**

HP 9g Logarithmic Functions

Log and Antilog Functions

Practice Solving Problems Involving Logarithms

Log and antilog functions

The logarithm of x to the base a (written as $\log_a x$) is defined as the inverse function of : $x = a^y$ The most usual values for a are 10 and e, which is the exponential constant and is defined by the infinite sum: $1 + 1/1! + 1/2! + 1/3! + ... + 1/n! + ...$ Its value is approximately 2.718 and is a transcendental number, that is, it cannot be the solution of a polynomial equation with rational coefficients.

Logarithms to base 10 are called common logarithms and also Briggsian logarithms. They are usually symbolized as $\log_{10} x$ or simply log 10, and on the HP 9g, they correspond to the ⬭log F key. These logarithms are used in calculations. Logarithms to base e are called natural logarithms, Naperian logarithms and also hyperbolic logarithms. Their symbol is ln x or $\log_e x$. They are calculated with the ⬭ln G key on the HP 9g. This kind of logarithms is the most used in mathematical analysis. There is still another kind of logarithms, though less frequently used; they are the binary logarithms, which are logarithms with base 2 ($\log_2 x$). In general the following formula is very useful to change logarithms from one base to another:

$$\log_n x = \frac{\log_m x}{\log_m n}$$

The denominator, $\log_m n$, is known as the modulus.

The inverse function of the logarithm is called the antilogarithm. If $y = \log_a x$, then $x = a^y$ is the antilogarithm of y. If the base is e then the inverse function is also called the exponential function, e^x, which is also known as the compound interest function and the growth or decay function. Perhaps the most important property of the exponential function is that its derivative is also e^x, that is, it's the solution of the differential equation dy/dx = y for which y=1 when x=0.

On the HP 9g, the keys that perform these functions are ⬭ln G , ⬭log F , ⬭2nd 10ˣ XNOR and ⬭2nd eˣ XOR . The function y^x (y ⬭2nd △ x ENTER) can be seen as the generic antilog function. If 10^x is the inverse of $\log_{10} x$ and e^x is the inverse of $\log_e x$, then y^x is the inverse of $\log_y x$. Refer to the HP 9g learning module *Solving Problems Involving Powers and Roots* for more information.

Practice solving problems involving logarithms

<u>Example 1:</u>   Find the common logarithm of 2

<u>Solution:</u>   ⬭log F ⬭2 Y ENTER

<u>Answer:</u>   0.301029996

<u>Example 2:</u>   What is the numerical value of the base of the natural logarithms?

<u>Solution:</u>   ⬭2nd eˣ XOR ⬭1 X ENTER

<u>Answer:</u>   2.718281828 Notice that the pattern 18-28-18-28 is easy to remember!

Example 3:   Verify that LN(8) + LN(5) = LN(8×5)

Solution: Before calculators like the HP 9g became easily available, logarithms were used to simply multiplication and division, since the logarithm of a product is equal to the sum of logarithms of the multiplicand and the multiplier.

(ln G) (8 O) ▸ (+.) (ln G) (5 U) (ENTER)   and (ln G) (8 O) (×W) (5 U) (ENTER)

Answer: Both expressions evaluate to 3.688879454

Example 4:   Find the log to base 3 of 5. Confirm the result using the ^ function.

Solution: Using the formula given above, the log to base 3 of 5 can be calculated as $\log_{10} 5$ / $\log_{10} 3$:

(log F) (5 U) ▸ (÷S) (log F) (3 Z) (ENTER)

Let's confirm this result by pressing

(3 Z) (2nd) (△) (2nd) (ANS) (ENTER)

We obtain 5, which means that the logarithm was correctly calculated.

Answer: 1.464973524

Example 5:   What is the value of x in the equation $18^x = 324$?

Solution: To solve this example, we will apply one of the properties of logarithms which states that the logarithm of a number raised to a power is equal to the power multiplied by the logarithm of the number. This involves taking the logarithm of both sides of the equation. The original equation would then look like this:

$$\log 18^x = \log 324 \Rightarrow x \log 18 = \log 324$$

and x is therefore equal to:

$$x = \frac{\log 324}{\log 18}$$

(log F) (3 Z) (2 Y) (4 T) ▸ (÷S) (log F) (1 X) (8 O) (ENTER)

Answer: 2. Note that the same answer will be found using natural logarithms or common logarithms.

Example 6:   A rare species of tree has a trunk whose cross-section changes as 1/x with the height x. (Obviously this breaks down at ground level and at the tree top.) The cross section for any such tree is given by A/x, where A is the cross-section calculated at 1 meter above the ground. What is the volume of the trunk between 1 meter and 2 meters above ground?

Solution: The volume is obtained by integrating the cross-section along the length, so it is given by the integral:

$$\int_1^2 \frac{A}{x}dx$$

Even though the HP 9g has no built-in integration function, this integral can be easily evaluated if we remember that indefinite integral of 1/x is LN x. The result is therefore:

$$V = A \times LN(2) - LN(1)$$

Since LN(1) = 0, it simplifies to:

$$V = A \times LN(2)$$

Since no one is likely to measure tree heights to an accuracy of more than three significant digits, set your HP 9g to display the answer with just 3 digits after the decimal point, by pressing

[2nd] FIX⟳ [3 Z] .

Now find LN(2) by pressing

[ln G] [2 Y] [ENTER]

Answer: The log to base e of 2 is close to 0.693, so the volume is 0.693A cubic meters. Press [2nd] FIX⟳ [· ;] to return to the default display format, once you have finished this example.

Example 7: An activity of 200 is measured for a standard of Cr^{51} (with a half-life of 667.20 hours). How much time will have passed when the activity measured in the sample is 170?

Solution: This is the formula for half-life computations:

$$A = A_0 \times \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$$

Let's rearrange the equation to solve for t:

$$t = \tau \frac{\ln \dfrac{A}{A_0}}{\ln \dfrac{1}{2}}$$

[6 V] [6 V] [7 P] [· ;] [2 Y] [X W] [ln G] [1 X] [7 P] [0 ›] [÷ S] [2 Y] [0 ›] [0 ›] [❭] [÷ S] [ln G] [1 X] [÷ S] [2 Y] [ENTER]

Answer: 156.4352172 hours.