

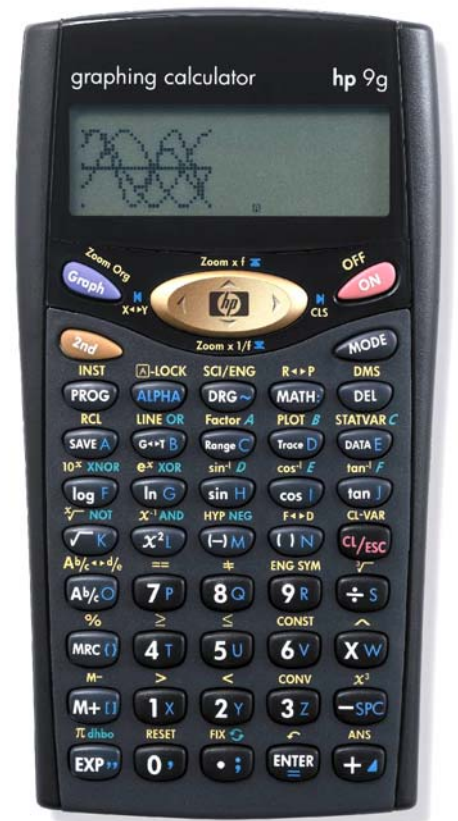


hp calculators

HP 9g Hyperbolic Functions

Hyperbolic Functions

Practice Using Hyperbolic Functions



Hyperbolic functions

Trigonometric functions are often called “circular” functions, because the value for the cosine and sine of an angle lie on the unit circle defined by $X^2 + Y^2 = 1$ (points on the unit circle will have the X and Y coordinate of (Cosine(theta), Sine(theta))). Hyperbolic trigonometric functions have a similar relationship, but with the hyperbola defined by the equation $X^2 - Y^2 = 1$.

Given a value for Z, the hyperbolic functions are defined by:

$$\sinh Z = \frac{e^Z - e^{-Z}}{2} \quad \cosh Z = \frac{e^Z + e^{-Z}}{2} \quad \tanh Z = \frac{\sinh Z}{\cosh Z}$$

Assume that Z is 3. The position on the unit hyperbola $X^2 - Y^2 = 1$ is defined by the point (COSH(Z), SINH(Z)), where COSH is the hyperbolic cosine and SINH is the hyperbolic sine. The value for the SINH(3) is equal to 10.0179 and the value of COSH(3) is 10.0677. When $10.0677 \times 10.0677 - 10.0179 \times 10.0179$ is evaluated, the value is 1, so the point falls on the unit hyperbola. The hyperbolic tangent is defined as the hyperbolic sine divided by the hyperbolic cosine.

Hyperbolic functions are similar to circular trigonometric functions in many ways. In fact, if an identity is true for circular trigonometric functions, there is a corresponding identity in hyperbolic functions in which the sign of any product of two sines is changed (including tangent which is sine/cosine), this is known as Osborne’s rule. For example,

	gives	
$\cos(A+B) = \cos A \cos B - \sin A \sin B$		$\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$
and $\sin^2 A + \cos^2 A = 1$	gives	$\cosh^2 A - \sinh^2 A = 1$

Another interesting property is that for all $Z > 0$, $\tanh Z < Z < \sinh Z < \cosh Z$. One of the examples in the HP 9g learning module *Graphing Functions – Part 1* verifies these inequalities by plotting the hyperbolic functions on the HP 9g .

If $x = \sinh y$, then y is defined to be the inverse hyperbolic sine (abbreviated as \sinh^{-1} or arcsinh). The other inverse hyperbolic functions are similarly defined, but notice that $y = \cosh^{-1} Z$ is only defined for $y \geq 0$.

Hyperbolic functions are useful in integration and have applications in many areas of engineering. For example, the shape formed by a wire freely hanging between two points (known as a catenary or chainette curve) is described by the hyperbolic cosine (COSH). Hyperbolic functions are also used in electrical engineering applications.

On the HP 9g, hyperbolic functions are accessed by pressing the 2nd HYP/NEG keys and then the appropriate trigonometric key or inverse trigonometric key. Once 2nd HYP/NEG has been pressed, the HYP annunciator is displayed; to cancel this mode just press 2nd HYP/NEG again. Notice that hyperbolic functions are *not* affected by the angle mode.

Practice using hyperbolic functions

Example 1: Find the Guadermannian of 2.

Solution: The Guadermannian function (gd) of Z is defined as $\tan^{-1} \sinh Z$. Press

$$\text{2nd} \text{tan}^{-1} \text{F} \text{2nd} \text{HYP/NEG} \text{sin H} \text{2Y} \text{ENTER}$$

HP 9g Hyperbolic Functions

The result depends on the current angular mode (DRG) because of the arctan function. The Gudermannian function is used to calculate hyperbolic functions from trigonometric functions since if $y = \operatorname{gd} x$, then $\tan y = \sinh x$.

Answer: 1.301760336 assuming the angular mode is set to Radians.

Example 2: Find the hyperbolic cosecant and the inverse hyperbolic cosecant of 3.2

Solution: Inverse hyperbolic functions should not be mistaken for the reciprocal of the hyperbolic functions, which are sometimes referred to as:

$$\text{Hyperbolic cosecant} = \frac{1}{\sinh} \quad \text{Hyperbolic secant} = \frac{1}{\cosh} \quad \text{Hyperbolic cotangent} = \frac{1}{\tanh}$$

These functions also have their inverse functions, which can be written as:

$$\operatorname{csch}^{-1} Z = \sinh^{-1} \frac{1}{Z} \quad \operatorname{sech}^{-1} Z = \cosh^{-1} \frac{1}{Z} \quad \operatorname{coth}^{-1} Z = \tanh^{-1} \frac{1}{Z}$$

To find the hyperbolic cosecant of 3.2 press

$\boxed{1} \boxed{X} \boxed{\div} \boxed{5} \boxed{2nd} \boxed{HYP} \boxed{NEG} \boxed{\sin} \boxed{H} \boxed{3} \boxed{Z} \boxed{\cdot} \boxed{;} \boxed{2} \boxed{Y} \boxed{ENTER}$

and to find the inverse hyperbolic cosecant of 3.2 press

$\boxed{2nd} \boxed{HYP} \boxed{NEG} \boxed{2nd} \boxed{\sin^{-1}D} \boxed{1} \boxed{X} \boxed{\div} \boxed{5} \boxed{3} \boxed{Z} \boxed{\cdot} \boxed{;} \boxed{2} \boxed{Y} \boxed{ENTER}$

Answer: The hyperbolic cosecant is 0.081660091 and the inverse hyperbolic cosecant is 0.307625067.

Example 3: A tram carries tourists between two peaks that are the same height and 437 meters apart. Before the tram latches onto the cable, the angle from the horizontal to the cable at its point of attachment is 63 degrees. How long does it take the tram to travel from one peak to the other, if the tram moves at 135 meters per minute?

Solution: The travel time in minutes is given by the following formula:

$$t = \frac{437 \times \operatorname{TAN}(63^\circ)}{135 \times \operatorname{ASINH}(\operatorname{TAN}(63^\circ))}$$

$\boxed{4} \boxed{T} \boxed{3} \boxed{Z} \boxed{7} \boxed{P} \boxed{X} \boxed{W} \boxed{\tan} \boxed{J} \boxed{6} \boxed{V} \boxed{3} \boxed{Z} \boxed{2nd} \boxed{DMS} \boxed{ENTER} \boxed{\div} \boxed{5} \boxed{(} \boxed{)} \boxed{N} \boxed{1} \boxed{X} \boxed{3} \boxed{Z} \boxed{5} \boxed{U} \boxed{X} \boxed{W} \boxed{2nd} \boxed{HYP} \boxed{NEG} \boxed{2nd} \boxed{\sin^{-1}D} \boxed{\tan} \boxed{J} \boxed{6} \boxed{V} \boxed{3} \boxed{Z} \boxed{2nd} \boxed{DMS} \boxed{ENTER} \boxed{ENTER}$

If the angular unit is degrees (DEG), you can omit the $^\circ$ in the argument for the tangent functions.

Answer: 4.452687994. The travel time between the peaks is just under four and one half minutes.

HP 9g Hyperbolic Functions

Example 4: A cable is strung between two poles that are 40 feet apart, with the cable attached to each pole at a height of 30.436 feet above the level ground. At the midpoint between the poles, the cable is 18.63 feet above the level ground. What is the length of the cable required between the two poles?

Solution: The length of the cable is described by the formula below, where a is the lowest height of the cable and D is the distance between the two poles:

$$L = 2 a \operatorname{SINH} \frac{D/2}{a}$$

2Y XW 1X 8O .: 6V 3Z XW 2nd HYP NEG sin H 4T 0> ÷S 2Y ÷S 1X 8O .: 6V 3Z ENTER

Answer: The length of the cable will be 48.14 feet.