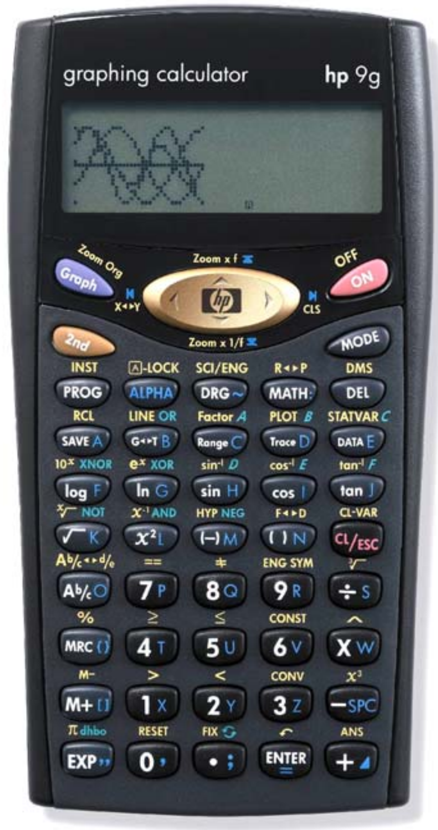
hp calculators



HP 9g Graphing Functions – Part 2

Tracing Graphs

The Zoom Function

Examples of Statistical Graphs

In this second part we will introduce two very useful graphing functions: tracing and zooming, which will allow us to find the roots of a polynomial, verify trigonometric identities, estimate derivatives, find discontinuities, etc. Additionally, we will plot a few examples of statistics graphs, even though they are described in greater detail in the statistics modules.

Tracing graphs

Example 1: Identify the number of x-axis intercepts of the expression: $x^3 - 2x^2 - x + 2$.

Solution: First of all, clear the graph display by pressing [2nd] CLS . We now have to specify our own range and scale. As we don't know the behavior of this function yet, let's try using the default values, already mentioned in the first part of this learning module. Press:

[CL/ESC] [Range C] [CL/ESC] [Range C]

The ranges and scales are now set to: Xmin = –3.4, Xmax = 3.4, Xscl = 1, Ymin = –2.2, Ymax = 2.2, and Yscl =1. To plot the cubic polynomial, press:

[Graph] [ALPHA] [1 X] [2nd] x^3 [–SPC] [2 Y] [ALPHA] [1 X] [x^2 L] [–SPC] [ALPHA] [1 X] [+∠] [2 Y] [ENTER]

Answer: Figure 1 shows the resulting graph.[1] There are three x-axis intercepts, therefore the three solutions of the equation $x^3 - 2x^2 - x + 2 = 0$ (remember that an *nth*-degree polynomial has exactly *n* roots in the complex number system) are all real numbers.

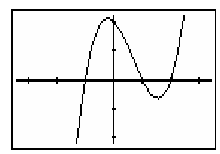


Figure 1

Example 2: Using graphic techniques, find the roots of the above polynomial.

Solution: With the graph in figure 9 still displayed on your HP 9g, press the [Trace D] key. You can now move a cursor (a blinking pixel) along the graph by pressing the ◀ and the ▶ keys. The X-coordinate (abscissa) of the cursor is displayed in the result line. The Y-coordinate (ordinate) can be displayed by the [2nd] X↔Y key, which acts as a toggle. Let's now find the approximate values of the three roots. Press [2nd] X↔Y to display the ordinates, and the ▶ key until Y is equal to zero, then display the X-coordinate and jot down the displayed value. Repeat this process for the next two roots.

Answer: The values of X for which Y=0 are –1, 1 and 2. The default setting proved to be perfect. In many cases, we won't be able to find the exact value, because of poorly chosen settings, or simply because of the limited precision of the graphic display. In such cases try changing the range and using the zoom function, which is described later in this document.

Example 3: Graphically, show that $\tan^2 x + 1 = \sec^2 x$.

Solution: We'll plot the function $f(x) = \cos^{-2} x - \tan^2 x$. If the graph is the same as the function y = 1, then the given identity holds. As usual, press [2nd] CLS first. The current settings (that is, the default ones) are a priori appropriate for this exercise (we expect a line at y = 1, for all values of x). To plot f(x) press:

[1] Although figures in this document are not an exact replica of the actual HP 9g graph display, we hope they will help you follow the examples.

[Graph] [cos I] [ALPHA] [1 X] [›] [2nd] [^] [(−) M] [2 Y] [−SPC] [tan J] [ALPHA] [1 X] [›] [x^2 L] [ENTER]

Answer: Figure 2 shows the resulting graph. It's indeed a horizontal line, let's check that it's at y =1. Press [Trace D] to trace the function, and [2nd] X↔Y to display the y-coordinate: For all x, y is always 1, therefore the identity is true, regardless of the angle unit.

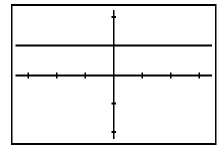


Figure 2

Example 4: Study the continuity of the function:

$$f(x) = \frac{x^2 - 1}{x^2 - x - 2}$$

in the interval (–3 , 3).

Solution: There's no need to change the range for the x-axis, which is currently (–3.4 , 3.4). Clear the graph display ([2nd] CLS) and press:

[Graph] [( ) N] [ALPHA] [1 X] [x^2 L] [−SPC] [1 X] [›] [÷ S] [( ) N] [ALPHA] [1 X] [x^2 L] [−SPC] [ALPHA] [1 X] [−SPC] [2 Y] [ENTER]

Figure 3 shows the graph displayed. Notice that there's a "hole" in the graph of the function. Let's use the trace function to find out where it is. Press [Trace D] and [›] ten times: the hole is between x = –1.2 and x = –0.8. Since each press of the [›] key corresponds to 0.2 units, the hole is by (–1, 0.66) Remember that the y-value is displayed by pressing [2nd] X↔Y . By x = 2 we can see another discontinuity: an asymptote. All this can be verified analytically. The roots of the denominator are –1 and 2, therefore:

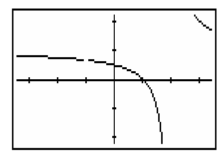


Figure 3

$$f(x) = \frac{x^2 - 1}{x^2 - x - 2} = \frac{(x+1)(x-1)}{(x+1)(x-2)} \underset{\text{if } x \neq -1}{\overset{\uparrow}{=}} \frac{x-1}{x-2}$$

which tends to $\pm\infty$ when x approaches 2. Also, note that f(x) is not equal to the simplified expression *at* x = –1, but as x *approaches* –1, both functions are the same. In other words, the limit of f(x) as x tends to –1 is the value returned by the simplified function at x = –1, i.e. $\frac{-2}{-3} = 0.66...$

Answer: f(x) has a removable discontinuity at x = –1 (f(x) can be made continuous by defining f(–1) = $2/3$) and a non-removable discontinuity (a vertical asymptote) at x = 2.

The zoom function

The zoom function lets you look at a plot in more detail (by zooming in: [2nd] Zoom x f) or look at more of the plot that is currently displayed (by zooming out: [2nd] Zoom x 1/f). Pressing [2nd] ZoomOrg returns the plot to its original size. The x and y zoom factors are set by the factor function ([2nd] Factor A), which works much like the range function. The default value for both factors is 2. If tracing is active, the new graph is recentered at the cursor position.

Example 5: Using the zoom function, examine the derivative at the origin of the functions $f(x) = x^2 - x$ and $g(x) = |x|$.

Solution: Let's plot f(x) using the default setting once again. After clearing the previous graph, press:

[Graph] [ALPHA] [1 X] [x^2 L] [−SPC] [ALPHA] [1 X] [ENTER]

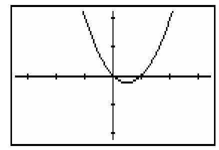


Figure 4

Figure 4 shows the resulting plot.

Based on the idea that differentiable functions are *locally* linear, we can study the derivative of a function at a point by zooming in repeatedly until the graph appears linear. Both zoom factors must be equal in order to find the slope at that point.

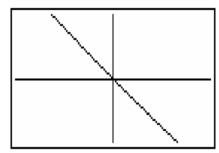


Figure 5

Press [2nd] Zoom x f six times and the plot will look like figure 5. We can estimate the derivative at x = 0 by finding the slope of this line With the trace function, we find the following points of the line: (–0.028125, 0.028916016) and (0.025, –0.024375). The slope is then:

$$-\frac{0.028916016 + 0.024375}{0.02815 + 0.025} = -1.002...$$

which is quite similar to the actual value (–1).

Let's now study the second function, but first press [2nd] ZoomOrg to reset the range values, and [2nd] CLS to clear the graph display. To plot g(x) press:

[Graph] [MATH] [MATH] [MATH] [3 Z] [ALPHA] [1 X] [ENTER]

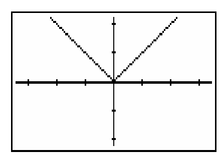


Figure 6

The plot of g(x) is shown in figure 6. As before, zoom in several times by pressing [2nd] Zoom x f . No matter how many times we press this key, the sharp corner persists, which means that the derivative at x = 0 does *not* exist.

Answer: $f'(0) = -1$ and $\nexists\ g'(0)$.

Example 6: Compare the period of the functions $f(x) = 2^{\sin x}$ and $g(x) = 2^{\sin^2 x}$.

Solution: We'll work in radian mode this time, so press [DRG~] , select RAD and press [ENTER] . As range settings, we'll try using the ones generated for the built-in graph sin(). Press:

[Graph] [sin H] [ENTER]

and once the graphing is complete, clear the graph display ([2nd] CLS) as we're interested in the range values only. Let's plot f(x):

Graph 2 2nd ^ sin ALPHA 1 ENTER

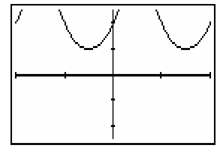


Figure 7

Figure 7 shows the result. Since part of the graph can't be seen, let's zoom out using the default factors (the x and y ranges will be multiplied by two), press:

2nd Zoom x 1/f

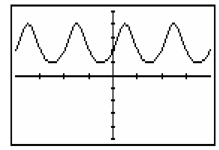


Figure 8

which results in the graph shown in figure 8. To find out the period (2π), use the trace function to find the difference between two consecutive local minimums (or two consecutive local maximums). The previous graph (redisplayed by 2nd ZoomOrg) will give a better approximation.

Let's now overlay the graph of g(x) over the one of f(x) so that we can compare their periods. Press:

Graph 2 2nd ^ ( ) sin ALPHA 1 ▶ x^2 ENTER

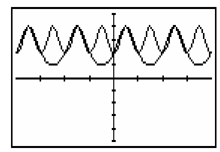


Figure 9

Answer: Figure 9 shows the resulting graph. We can clearly see that the period of g(x) is half the period of f(x), i.e. π.

Examples of statistical graphs

Example 7: In the HP 9g learning module *Probability – Random Numbers* is a frequency histogram of 100,000 random digits generated by the HP 9g RANDI function. Here's the data:

0's	1's	2's	3's	4's	5's	6's	7's	8's	9's
9936	10003	10042	9994	9962	10043	10036	10045	9886	10053

Plot the histogram of this data on your HP 9g.

Solution: Here's the keystroke sequence necessary to enter the data. (Since we're interested in the way of obtaining the plot, we won't go into details here, please refer to the corresponding learning module for more information).

MODE 1 select D-CL ENTER MODE 1 select 1-VAR ENTER DATA select DATA_INPUT ENTER

0 ▼ 9 9 3 6 ▼
1 ▼ 1 0 0 0 3 ▼
2 ▼ 1 0 0 4 2 ▼
3 ▼ 9 9 9 4 ▼
4 ▼ 9 9 6 2 ▼
5 ▼ 1 1 0 4 3 ▼
6 ▼ 1 0 0 3 6 ▼
7 ▼ 1 0 0 4 5 ▼
8 ▼ 9 8 8 6 ▼
9 ▼ 1 0 0 5 3 ▼

Now display the STATVAR menu (2nd STATVAR), and press Graph . (Note that the graph submenu is displayed by pressing Graph while in the STATVAR menu, not by selecting a menu option!). The HIST option is the second option in the graph menu. Press 1 X or ▼ ENTER .

Answer: The frequency histogram is displayed. (Ranges are automatically set to 'optimum' values).

Example 8: Plot the quadratic polynomial that best fits the points (0, 4), (2, 5) and (3, 6)

Solution: First clear the previous data by selecting D-CL in the STAT mode. Press: MODE 1 X select D-CL and ENTER . Let's now enter the given points. Press:

MODE 1 X select REG ENTER select QUAD ENTER DATA E select DATA_INPUT ENTER
0 , ▼ 4 T ▼
2 Y ▼ 5 U ▼
3 Z ▼ 6 V

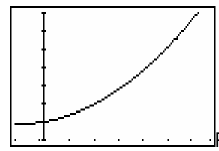


Figure 10

Display the STATVAR menu (2nd STATVAR) and press Graph .

Answer: The quadratic $y = \frac{1}{6}x^2 + \frac{1}{6}x + 4$ is then plotted (see figure 10), along with the three points, which are all on the curve.