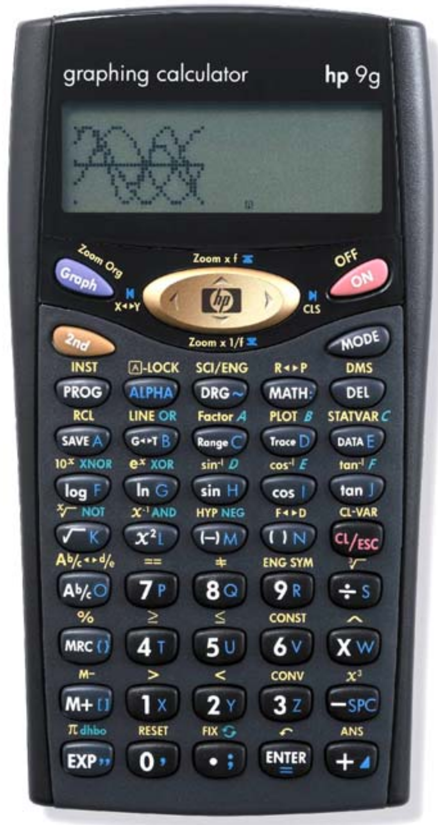
hp calculators

HP 9g Solving Problems Involving Fractions



Basic Concepts

Fractions on the HP 9g

Practice Working Problems Involving Fractions

Basic concepts

Those numbers that can be written as one integer over another, i.e. $\frac{a}{b}$, are called *rational* numbers. Note that b can't be zero. When written as the quotient of two integers, rational numbers are called fractions. In arithmetic there are three basic rules for fractions:

- $\frac{a}{b} > \frac{c}{d}$ if ad – bc > 0 (same for < and =)
- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
- $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b}$ is also referred to as a *vulgar* fraction when *a* and *b* are positive integers (the sign is considered apart) . *a* is called the **numerator** (corresponds to the dividend in a division) and *b* is the **denominator** (which corresponds to the divisor in a division). When the numerator is 1 (or –1), it is a unit fraction. A **proper** fraction is a fraction in which the numerator (apart from the sign, remember) is *less than* the denominator. Therefore, proper fractions always lie between –1 and 1. If the numerator is greater than the denominator, the fraction is called **improper**.

Those vulgar fractions that have the same value are called **equivalent** fractions: for example $\frac{3}{4}$ and $\frac{6}{8}$. **Reducing** a vulgar fraction to its lowest terms means to find the simplest equivalent fraction, which can be done by dividing the numerator and the denominator by the same number. This process is also called **cancellation**.

Mixed numbers are those improper fractions written as an integer followed by a proper fraction. For example: 4¾ or 2½. It is important to understand that there's no implicit multiplication in $a\,{}^{b}\!/_{c}$ (also written as $a\frac{b}{c}$). In fact, it is an *addition* that is implicit:

$$a\frac{b}{c} = a + \frac{b}{c} = \frac{ac + b}{c}$$

When the numerator and the numerator of a fraction are not both integers then the fraction is called **complex**, for example: $\frac{13}{3\frac{3}{4}}$. Finally, a number where the part which is a proper fraction is expressed as a set of digits placed after a decimal point, is called a **decimal (**also known as decimal fraction) e.g. $3\frac{7}{50}$ = 3.14.

Fractions on the HP 9g

The HP 9g has three keys to handle fractions, namely [A b/c] , [2nd] [A b/c ↔ d/e] and [2nd] [F↔D] . The symbol used by the HP 9g to show a fraction (i.e. the equivalent to the symbol "/", sometimes called solidus) is "⌟" and is entered into the entry line by pressing [A b/c] . Thus, 7⌟8 means $7/8$ and is entered by pressing [7] [A b/c] [8] . Mixed numbers are also keyed in using the [A b/c] key twice, for instance: 7⌟8⌟9 means $7\frac{8}{9}$ and is entered into the entry line by pressing [7] [A b/c] [8] [A b/c] [9] but this number is displayed in the *result* line (i.e. once [ENTER] has been pressed) as 7⊔8⌟9.

2nd Ab/c↔d/e is used for converting a mixed number to an improper fraction and vice versa, and 2nd F↔D for converting a decimal to and from a fraction. Let's illustrate all this with various examples.

Practice working problems involving fractions

Example 1: Enter the proper fractions $\frac{3}{9}$ and $\frac{21}{124}$

Solution: Note that pressing 3 ÷ 9 ENTER does not return a fraction but a decimal number which is the result—within the accuracy of the calculator—of dividing 3 by 9. As stated above, fractions are entered with the Ab/c key, which separates the numerator from the denominator. Press:

3 Ab/c 9

The entry line now reads 3⌋9. Press ENTER to display the result line. The fraction now displayed is 1⌋3, which is equivalent to the entered fraction but reduced to its simple form. The HP 9g always tries to find the simplest equivalent fraction. Let's enter the second fraction by pressing:

2 1 Ab/c 1 2 4 ENTER

No reduction is possible this time, therefore the fraction displayed in the result line is 21⌋124.

Example 2: Enter the improper fraction $\frac{1000}{101}$ and the complex fraction $\frac{1.8}{9}$

Solution: These fractions are entered exactly as in the previous example, but the results displayed after pressing the ENTER key are different. Let's enter the first fraction by pressing:

1 EXP 3 Ab/c 1 0 1 ENTER

The fraction displayed in the entry line (1E3⌋101) changes to 9␣91⌋101 which means $9\,{}^{91}\!/_{101}$. Improper fractions are always converted to mixed numbers (i.e. an integer plus a proper fraction) after pressing ENTER. As with proper fractions, the HP 9g tries to give the simplest form. As to the complex fraction, press:

1 • 8 Ab/c 9 ENTER

It returns 0.2 and not the fraction ${}^{1}\!/_{5}$. This is because calculations containing both fractions and decimals are calculated in decimal format. As long as both the numerator and the denominator evaluate to an integer, the result will be in fraction format, though. For example √(4) ⌋3 will return 2⌋3.

Example 3: Enter the mixed numbers $7\frac{2}{18}$, $-1\frac{57}{125}$ and $3\frac{19}{5}$.

Solution: When entering mixed numbers, remember that the Ab/c key is used for separating both the integer from the proper fraction and the numerator from the denominator. To enter the first fraction press:

7 a b/c 2 a b/c 1 8 ENTER

The fraction returned is 7⊔1⌋9, i.e. $7\frac{1}{9}$ which is the same as $7\frac{2}{18}$ after doing a cancellation.

Let's now enter the second fraction $-1\frac{57}{125}$. On the HP 9g negative fractions and mixed numbers are keyed in by pressing the (−) key before entering any part (integer, numerator or denominator). It is usually pressed just before the integer of a mixed number or the numerator of a fraction. In this example press:

(−) 1 a b/c 5 7 a b/c 1 2 5 ENTER

No cancellation is possible this time, so the result is –1⊔57⌋125.

Note that the third number is not actually a mixed number strictly speaking because its fraction part is not proper. Nevertheless, your HP 9g can handle it as well and will return the reduced, proper form. Press:

3 a b/c 1 9 a b/c 5 ENTER

which returns the mixed number $6\frac{4}{5}$.

Fractions can be entered and displayed wherever ordinary decimal numbers can be used. For example fractions can be used in ordinary arithmetic, in calculations with logarithmic and trigonometric functions and also in programs. But fractions cannot be used in Base-N mode – operations in this mode only work with integers.

Example 4: Add $1\frac{3}{4}$ to $2\frac{5}{8}$

Solution: We will enter the two mixed numbers as explained above. No parenthesis is necessary because fractions ("⌋") take priority over the addition. Press:

1 a b/c 3 a b/c 4 + 2 a b/c 5 a b/c 8

The entry line now reads 1⌋3⌋4 + 2⌋5⌋8. Press ENTER to carry out the calculation.

Answer: The result line reads 4⊔3⌋8 which means $4\frac{3}{8}$. Since this calculation only contains fractions then the result is expressed as a fraction too.

Example 5: Express the previous result as a decimal.

Solution: F↔D (2nd F↔D) is a one-argument function, *a* F↔D, that converts a decimal to a fraction or vice versa. Since the result of the previous example is stored in ANS, we don't need to enter it again, just press:

2nd F↔D ENTER

Answer: 4.375. Since pressing the ENTER key repeats the most recent calculation, if you press ENTER now, the result line will contain the mixed number again. Press ENTER one more time to display the decimal fraction.

Example 6: Are the fractions $\frac{147}{489}$ and $\frac{539}{1793}$ equivalent to each other?

Solution: There are several ways of testing whether two fractions are equivalent. We can subtract one from the other and see if the result is zero, or we can calculate the cross multiplication since if a/b = c/d then ad = bc. Doing the division is another way. But let your HP 9g do the hard part, and just enter both fractions. Since the Hp 9g carries out an automatic simplification, if the fractions displayed in the result line are the same then they are equivalent. Press:

1 4 7 Ab/c 4 8 9 ENTER which returns 49⌟163
5 3 9 Ab/c 1 7 9 3 ENTER

Answer: Since the latter results in 49⌟163 too, they are equivalent fractions.

Example 7: Convert the mixed number $2\frac{89}{133}$ to an improper fraction.

Solution: We have seen how improper fractions are automatically converted to mixed numbers when evaluated. But the opposite is also possible. The $A\,{}^{b}\!/_{c} \leftrightarrow {}^{d}\!/_{e}$ function (2nd Ab/c↔d/e) carries out conversions between mixed numbers and improper fractions. Press:

2 Ab/c 8 9 Ab/c 1 3 3 2nd Ab/c↔d/e ENTER

Answer: 355⌟133, i.e. $\frac{355}{133}$.

Note. $A\,{}^{b}\!/_{c} \leftrightarrow {}^{d}\!/_{e}$ and F↔D must be the *last* functions entered into the entry line, otherwise a syntax error occurs. However, chain calculations are always possible using the ANS function. For example:

Example 8: Calculate $\sin(30^{\circ}) + \frac{3}{4}$ and express the result as a fraction.

Solution: The calculation in question is: ($\sin(30^{\circ}) + 3$⌟4 ▸ F↔D) ▸ $A\,{}^{b}\!/_{c} \leftrightarrow {}^{d}\!/_{e}$. Trouble is that F↔D cannot be placed in the middle of a calculation. We have to split it this way:

sin 3 0 2nd DMS ENTER ▸ + 3 Ab/c 4 2nd F↔D ENTER

We now have the mixed number $1\frac{1}{4}$ in ANS. (The sequence 2nd DMS ENTER inserts the symbol º after 30, so as not to be affected by the current angle mode). We can now convert it to an improper fraction by pressing:

2nd Ab/c↔d/e ENTER

Answer: 5⌋4, i.e. $\frac{5}{4}$

Example 9: Find a fraction which approximates π to four decimal places.

Solution: First set the number of displayed decimal places to 4 by pressing 2nd FIX 4 . Notice that π F↔D will *not* return a fraction, because π is calculated to 24 significant digits. Too many digits for F↔D to handle. (Also, bear in mind that denominators on the HP 9g must be less than 10000). π has to be rounded to four decimal places before attempting the conversion. To do so use the RND command in the MATH menu:

MATH 3 2nd π 2nd F↔D ENTER

The mixed number 3⌞177⌋1250 is returned. If a fraction is preferred, convert this number to an improper fraction by pressing:

2nd Ab/c↔d/e ENTER

Answer: $3\frac{177}{1250} = \frac{3927}{1250} = 3.1416$