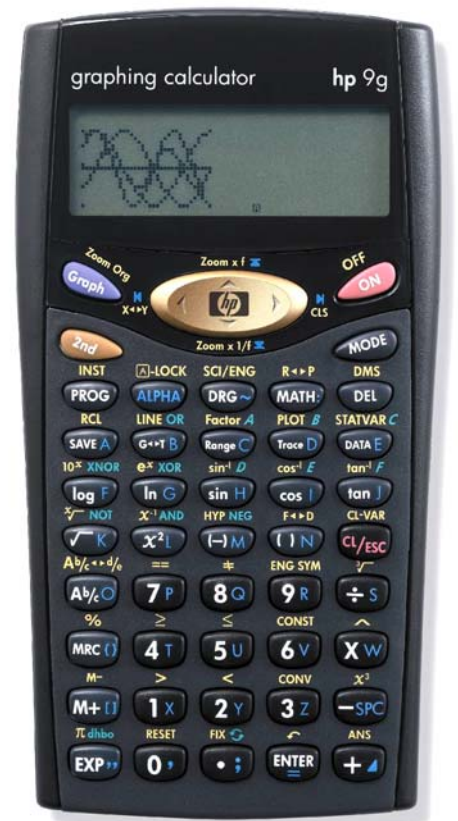# hp calculators

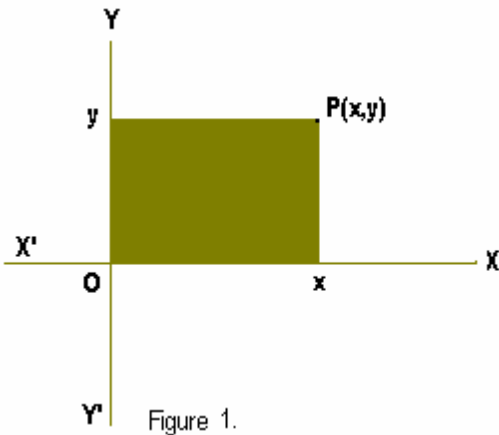**HP 9g** Polar/Rectangular Coordinate Conversions

Rectangular and Polar Coordinates

Practice Solving Problems Involving Coordinate Conversions

## Rectangular and polar coordinates



Figure 1.

Coordinates define the position of a point in space. In two dimensions, they are an ordered pair $(a, b)$. There are two primary coordinate systems: rectangular and polar. The **rectangular** system is the commonest coordinate system and uses perpendicular lines in order to measure distances to the base lines or axes, which are also perpendicular to each other. It is often referred to as rectangular cartesian system, and even as cartesian system, even though cartesian coordinates may not be rectangular (i.e. when axes are not perpendicular to each other, coordinates are called oblique.) Figure 1 shows the plane rectangular cartesian coordinates of a two dimensional point P. They are written as $(x, y)$. The x-coordinate is measured along or parallel to the XX′-axis and is called **abscissa**. Likewise, the y-coordinate is measure along the YY′-axis and is called **ordinate**.

**Polar** coordinates describe the position of a point P by its distance to a fixed point O (the pole) and the angle that OP makes with the base line (XX′ in figure 2). The angle is measure in the positive or counterclockwise direction from the base line. Coordinates are written as $(r, \theta)$. OP is known as the **radius vector** and θ as **vectorial angle**.
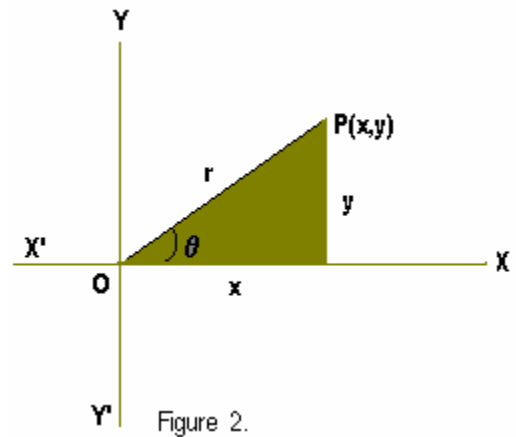
The HP 9g provides four functions for converting between polar and cartesian coordinates. They are in the R↔P menu ($\boxed{2nd}$ $\boxed{R↔P}$) : R ▶ Pr, R ▶ Pθ, P ▶ Rx, P ▶ Ry. All these functions take two arguments e.g. R ▶ Pθ (x , y), which are separated by commas ($\boxed{ALPHA}$ $\boxed{0,}$). It is worth learning what these functions actually do; thus, we will be able to use them in different contexts. Figure 2 shows the relationship between rectangular and polar coordinates. Note that:



Figure 2.

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan\frac{y}{x}, \quad x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

## Practice solving problems involving coordinate conversions

<u>Example 1:</u>  Convert the rectangular coordinates $(-7.5, 13)$ into polar coordinates.

<u>Solution:</u>  To find the corresponding radius vector and vectorial angle, we can use the R ▶ Pr and R ▶ Pθ functions respectively. Press:

$\boxed{2nd}$ $\boxed{R↔P}$ $\boxed{ENTER}$ $\boxed{(-)M}$ $\boxed{7P}$ $\boxed{.;}$ $\boxed{5U}$ $\boxed{ALPHA}$ $\boxed{0,}$ $\boxed{1X}$ $\boxed{3Z}$ $\boxed{ENTER}$ which returns r, and
$\boxed{2nd}$ $\boxed{R↔P}$ ▶ $\boxed{ENTER}$ $\boxed{(-)M}$ $\boxed{7P}$ $\boxed{.;}$ $\boxed{5U}$ $\boxed{ALPHA}$ $\boxed{0,}$ $\boxed{1X}$ $\boxed{3Z}$ $\boxed{ENTER}$ which returns θ.

θ is expressed in the current angle unit. Note that the latter keystroke sequence can be shortened if we edit the previous calculation instead of entering it anew. If *r* has just been calculated, then to find θ press:

(ALPHA) ◄ (DEL) (2nd) R↔P ▶ (ENTER) (DEL) (ENTER)

Answer:     Shown to four decimal digits, r = 15.0083 and θ = 119.9816º

Example 2:  Express the point whose polar coordinates are $(\sqrt{3}, \dfrac{2\pi}{3})$ in rectangular coordinates.

Solution:   The functions we will now use are P ▶ Rx and P ▶ Ry, which will return the abscissa and the ordinate
            respectively. Its arguments are $r$ and θ. Since θ is an angle, you must make sure that the appropriate angle
            unit is set. Alternatively, you can specify the angle unit just after keying in the value. Let's now find the
            abscissa by pressing:

(2nd) R↔P ▼ (ENTER) (√ K) (3 Z) ▶ (ALPHA) (0 ') (( ) N) (2 Y) (2nd) π dhbo (÷ S) (3 Z) ▶ (2nd) DMS ▶ ▶ ▶ (ENTER) (ENTER)

To find the ordinate just replace P ▶ Rx with P ▶ Ry in the above calculation:

(ALPHA) ◄ (DEL) (2nd) R↔P ▲ (ENTER) (DEL) (ENTER)

Answer:     $(-0.8660, 1.500)$ to four decimal places.

Example 3:  Find the hypotenuse of a right triangle whose catheti are 9 and 40.

Solution:   Pythagoras' theorem states that the hypotenuse is given by $\sqrt{x^2 + y^2}$ (x and y being the catheti of a right
            triangle). But this is exactly what the R ▶ Pr(x , y) function returns.  Let's compare both methods:

(√ K) (9 R) (x² L) (+ ↲) (4 T) (0 ') (x² L) (ENTER)
(2nd) R↔P (ENTER) (9 R) (ALPHA) (0 ') (4 T) (0 ') (ENTER)

The former is one keystroke shorter, but the latter takes two steps fewer in a program!

Answer:     41. (The set 9, 40 and 41 is another example of Pythagorean triples. Refer to the HP 9g learning module
            *Powers and Roots*).

Example 4:  A vector has components –8 in the X direction and –5 in the Y direction. In what direction does it point?

Solution:   The angle is given (in the current angle unit) by R ▶ Pθ( –8, –5):

(2nd) R↔P ▶ (ENTER) (↔ M) (8 Q) (ALPHA) (0 ') (↔ M) (5 U) (ENTER)

Answer:     –147.9946º which is the same as the positive angle  212.0054º (because 147.9946º + 212.0054º = 360º).
            Notice that the calculation $\arctan(\dfrac{-5}{-8})$ won't give the right result – you have to add 180º because the
            arctangent function does not take into account the quadrant where the vector lies, unlike the coordinate
            conversion function R ▶ Pθ.

**HP 9g** Polar/Rectangular Coordinate Conversions

Example 5: Add two vectors having polar coordinates $(8,30^{\circ})$ and $(12,60^{\circ})$. Represent the sum in terms of magnitude and angle $(r,\theta)$.

Solution: Vectors can be easily added (or subtracted) when expressed in rectangular form. The components of the resultant will be the sum of the corresponding components of the vectors. In other words, the sum vector has the following components.

$$P \blacktriangleright Rx(8,30^{\circ}) + P \blacktriangleright Rx(12,60^{\circ}) \text{ and } P \blacktriangleright Ry(8,30^{\circ}) + P \blacktriangleright Ry(12,60^{\circ})$$

First of all, let's set the angle unit to degrees (press (DRG↝), select DEG and press (ENTER) ). And now we can find the components and store them in the variables X and Y by pressing:

(2nd) (R↔P) ⌄ (ENTER) (8 O) (ALPHA) (0 ,) (3 Z) (0 ,) ❭ (+↲) (2nd) (R↔P) ⌄ (ENTER) (1 X) (2 Y) (ALPHA) (0 ,) (6 V) (0 ,) ❭ (SAVE A) (1 X) (ENTER)
(2nd) (R↔P) ⌃ (ENTER) (8 O) (ALPHA) (0 ,) (3 Z) (0 ,) ❭ (+↲) (2nd) (R↔P) ⌃ (ENTER) (1 X) (2 Y) (ALPHA) (0 ,) (6 V) (0 ,) ❭ (SAVE A) (2 Y) (ENTER)

(Remember that you can also obtain the latter expression by editing the former). In order to give the magnitude and the angle of the resultant we have to express the resultant in polar coordinates. We have to calculate R ▶ Pr (X,Y) and R ▶ Pθ(X,Y):

(2nd) (R↔P) (ENTER) (2nd) (ALPHA) (1 X) (0 ,) (2 Y) (ALPHA) (ENTER) and (ALPHA) ◀ (2nd) (R↔P) ❭ (ENTER) (DEL) (DEL) (ENTER)

Answer: $(19.3462, 48.0675^{\circ})$ to four decimal places.

Example 6: Calculate $\sqrt{5 + 3i}$

Solution: Even though the HP 9g operates with real numbers only, its coordinate conversion functions make calculations with complex numbers easy to do. When expressed in exponential form, square roots of complex numbers are very easy to calculate because, *in radian mode*:

$$\sqrt{a + bi} = \sqrt{re^{i\theta}} = \sqrt{r}e^{i\frac{\theta}{2}}$$

Therefore, the result in rectangular form is given by:

$$P \blacktriangleright Rx(\sqrt{R \blacktriangleright Pr(5,3)}, \frac{R \blacktriangleright P\theta(5,3)}{2}) \text{ (abscissa) and } P \blacktriangleright Ry(\sqrt{R \blacktriangleright Pr(5,3)}, \frac{R \blacktriangleright P\theta(5,3)}{2}) \text{ (ordinate).}$$

Incidentally, the HP 9g coordinate conversion functions are not nestable, i.e. they cannot appear in their own arguments. Actually, they can be part of the first argument but not of the second. This means that the above calculations have to be split. But we can still enter the entire calculation in the same entry line by using the (ALPHA) (+↲) function: (remember to set the angle unit to radian first, if it's not already set)

(2nd) (R↔P) ❭ (ENTER) (5 U) (ALPHA) (0 ,) (3 Z) ❭ (÷ S) (2 Y) (ALPHA) (+↲) (2nd) (R↔P) ⌄ (ENTER) (√ K) (2nd) (R↔P) (ENTER) (5 U) (ALPHA) (0 ,)
(3 Z) ❭ ❭ (ALPHA) (0 ,) (2nd) (ANS) (ENTER) (ENTER)

Note that two ⟨ENTER⟩ keys are necessary because the first one returns the partial result (up to the ⟨ALPHA⟩ ⟨+⟩ function) whereas the second finishes the calculation. To find the y-coordinate, just modify the above expression by pressing:

⟨ALPHA⟩ ◄ ⟩⟩⟩⟩⟩⟩⟩⟩ ⟨DEL⟩ ⟨2nd⟩ R↔P ⟨▲⟩ ⟨ENTER⟩ ⟨DEL⟩ ⟨ENTER⟩ ⟨ENTER⟩

For more examples of complex calculations, refer to the HP 9g learning module *Solving Problems Involving Complex Numbers.*

<u>Answer:</u>    $(2.3271, 0.6446)$  to four decimal places.

<u>Example 7:</u>    The coordinates of the point P (see fig. 3) after the axes have been rotated 30º are $(4.5, 1.5)$. Calculate the old coordinates.
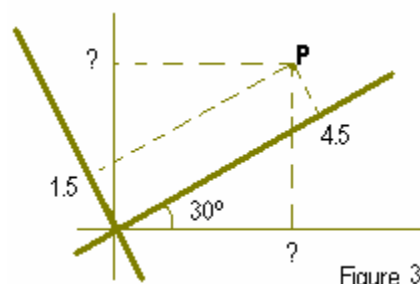
<u>Solution:</u>    The old coordinates are given by:

$x = 4.5\cos 30 - 1.5\sin 30$  and
$y = 4.5\sin 30 + 1.5\cos 30$

Make sure DEG is the current angle mode. Then press:

⟨4 T⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨cos I⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟩ ⟨–SPC⟩ ⟨1 X⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨sin H⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟨ENTER⟩  and
⟨4 T⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨sin H⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟩ ⟨+⟩ ⟨1 X⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨cos I⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟨ENTER⟩

Notice that the P ▶ Rx(r,θ) function can be used to calculate r·cosθ. Likewise, P ▶ Ry(r,θ) returns y·sinθ. Therefore, the equations that describe the transformation of coordinates can be written as:

$x = P ▶ Rx(4.5,30) - P ▶ Ry(1.5,30)$  and
$y = P ▶ Ry(4.5,30) + P ▶ Rx(1.5,30)$

which can be evaluated by pressing:

⟨2nd⟩ R↔P ⟨▼⟩ ⟨ENTER⟩ ⟨4 T⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨ALPHA⟩ ⟨0 ,⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟩ ⟨–SPC⟩ ⟨2nd⟩ R↔P ⟨▲⟩ ⟨ENTER⟩ ⟨1 X⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨ALPHA⟩ ⟨0 ,⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟨ENTER⟩
and ⟨2nd⟩ R↔P ⟨▲⟩ ⟨ENTER⟩ ⟨4 T⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨ALPHA⟩ ⟨0 ,⟩ ⟨3 Z⟩ ⟨0 ,⟩ ⟩ ⟨+⟩ ⟨2nd⟩ R↔P ⟨▼⟩ ⟨ENTER⟩ ⟨1 X⟩ ⟨• ;⟩ ⟨5 U⟩ ⟨ALPHA⟩ ⟨0 ,⟩ ⟨3 Z⟩ ⟨0 ,⟩
⟨ENTER⟩ , respectively.

<u>Answer:</u>    $(3.1471, 3.5490)$  to four decimal places.