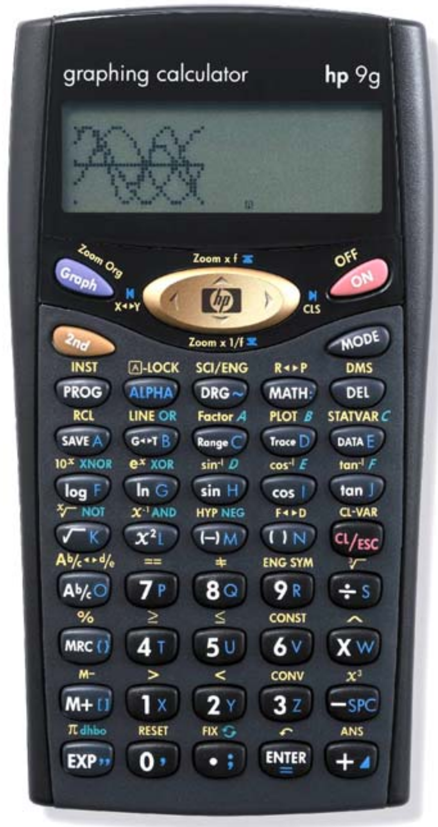
hp calculators

HP 9g Statistics – Process Capability

Process Capability

Practice Solving Process Capability Problems



Process capability

Any manufacturing process is subject to variability due to both assignable (e.g. a defective machine, the operator's mistakes) and non-assignable causes (e.g. the limited precision of machines, the operator's *skill*). When the former are eliminated, then the process is said to be in control. Variability is then *predictable.*

We can determine whether an *in-control process* meets the quality specifications by measuring the capability of the process. A process is capable when it produces similar products for a sustained period of time, that is to say, when almost all pieces are fabricated within a given tolerance. Assuming that the performance measure data is *distributed normally*, then approximately 99.73% [1] of all values will lie within three standard deviations on either side of the mean –- this interval (6 sigma) is called the natural spread of the process. This is why the process capability index is defined as:

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL and LSL are the upper and lower specification limits, respectively. USL – LSL is called the tolerance spread. If C_p = 1, then 99.73% of pieces will be fabricated within the specification limits *provided that the process mean is at the midpoint of the specification range* ($m = (USL + LSL)/2$). The following index is used for assessing the capability of an off-center process (i.e. when $\mu \neq m$):

$$C_{pk} = \min(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma})$$

C_{pk} can be expressed in terms of C_p by defining another index, C_a, which is the scaled distance between the mean of the process and the midpoint of the specification range:

$$C_a = \frac{|m - \mu|}{(USL - LSL)/2}$$

Thus, $C_{pk} = C_p\,(1 - C_a)$. Note that $0 \leq C_a \leq 1$, which implies that $C_{pk} \leq C_p$. For a centered process (i.e. μ = m) $C_a = 0$, and therefore $C_{pk} = C_p$.

A process for which C_p = 1 might produce up to $(1 - 0.9973) \cdot 10^6 = 2700$ defective pieces *per million*. While such a process was considered to be right in the past, it might well deemed unacceptable nowadays. Today standards for both C_p and C_{pk} are that they must be greater than 1.33 for the process to be capable (that assures us of no more than 60 non-conforming parts per million) . But this value certainly depends on the kind of product being fabricated.

The Hp 9g provides functions to calculate the three statistics described above, as well as an estimation of defection per million opportunities (ppm).

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[1] On the HP 9g, this value can be calculated as 1 – 2·R(3), when $\bar{x}$ = 0 and σx = 1: MODE 1 X (D-CL) ENTER MODE 1 X (1-VAR) ENTER DATA E ENTER 1 X ▼ ▼ (–) M 1 X ▼ DATA E DATA E ▲ ENTER 3 Z ENTER 2nd STATVAR C ▲ (–) M 2 Y + 1 X ENTER .

Practice solving process capability problems

<u>Example 1:</u> As part of a process optimization effort, an industrial manager had to determine whether their manufacturing process, for which μ = 19.35 and σ = 2.05, was capable. The population of data values is assumed to be normally distributed. The upper and lower specification limits of the process are 23.3 and 11, respectively. How could the HP 9g help her?

<u>Solution:</u> By calculating the C_p and C_{pk} statistics to measure the process capability. First of all, let's clear any previous statistical data in memory (press [MODE] [1] , then select D-CL and finally press [ENTER]). Since it is one single process, we'll carry out all calculations in 1-VAR mode (i.e. [MODE] [1] , select 1-VAR and press [ENTER]). No data set is provided. Therefore, in order to enter μ and σ into the calculator, we'll have to use the method described in the HP 9g learning module *Statistics – Normal Distribution.*, that is to say: create a data set consisting of two values $\mu + \sigma$ and $\mu - \sigma$. To do this press:

[DATA] [ENTER] [1] [9] [.] [3] [5] [+] [2] [.] [0] [5] [▼] [▼] [1] [9] [.] [3] [5] [−] [2] [.] [0] [5] [▼]

It is very important to bear in mind that, when using this trick, those variables shown in the STATVAR menu that depend on the actual data or the number of data, n, will be ***<u>meaningless</u>***. Only those which only depend on the average and the population standard deviation (that is, in which n is not explicitly used in their calculation) will be correct; namely: $\bar{x}$, $\bar{y}$, σx, σy, C_{px}, C_{pkx}, C_{ax}, C_{py}, C_{pky}, C_{ay}, P(t), Q(t), R(t) and t . Any other variable involves the value of n or the data in their calculation. Particularly, note that 'ppm' is calculated by the HP 9g using the actual value of n, not the one 'embedded' in μ.

The next step is to input the LSL and USL values. Press:

[DATA] , select LIMIT, [ENTER] [1] [1] [▼] [2] [3] [.] [3] [▼]

Let's determine if the process is located at the midpoint of the specification range. We can do it graphically by plotting the process control chart: press [2nd] [STATVAR] [Graph] [2] . The x-axis range is automatically set to (LSL, USL). The resulting bell-shaped curve is clearly off-center. Alternatively, we can do it numerically:

$$\frac{\text{USL} + \text{LSL}}{2} = \frac{23.3 + 11}{2} = 17.15 \neq \mu = 19.35$$

Or simply by finding the value of C_a , which is displayed by pressing:

[2nd] [STATVAR] [▼] [▼] [▼] [▼]

C_a =0.3577 $\neq$ 0. Therefore, the capability index that we must find is C_{pk} because C_{pk}, unlike C_p, accounts for a shift in the mean of the process to either specification limit. All these statistics are in the STATVAR menu, so press [2nd] [STATVAR] [▼] [▼] [▼] [▶] [▶] to display C_{pk}.

<u>Answer:</u> C_{pk} = 0.6423 (rounded to four decimal digits). Since C_{pk} is clearly smaller than 1.33 (even than 1, in fact) the process is incapable: the process spread exceeds the tolerance spread, which assures that out-of-tolerance products will be made.

Example 2: The length of pieces made by certain machine is found to be normally distributed ($\mu = 3$ and $\sigma = 0.001$). The established upper and lower specification limits for this particular piece are 3.005 and 2.995, respectively. Assuming the manufacturing process is in control, determine whether the process is capable or incapable.

Solution: Let's input the process average and standard deviation, which will overwrite the ones used in the previous example (remember to clear the statistical data first, if there are more than two values):

[DATA] [ENTER] [3] [+] [.] [0] [0] [1] [▼] [▼] [3] [−] [.] [0] [0] [1] [▼]

And now let's input USL and LSL

[DATA] [DATA] [▼] [2] [.] [9] [9] [5] [▼] [3] [.] [0] [0] [5] [▼] [ENTER]

The process capability indices are all shown in the STATVAR menu: [2nd] [STATVAR] :

$C_a = 2.842170943 \cdot 10^{-12}$, $C_p = 1.666666621$ and $C_{pk} = 1.666666621$.

Answer: Since C_a is almost zero, the process is centered. And since C_p is greater than 1.33, the process is capable.

Example 3: If the mean of the previous process shifts two standard deviations to the USL (μ is now 3.002), will the process still be capable? How many pieces will be defective?

Solution: Let's input the new average by pressing:

[DATA] [ENTER] [3] [.] [0] [0] [2] [+] [.] [0] [0] [1] [▼] [▼] [3] [.] [0] [0] [2] [−] [.] [0] [0] [1] [▼]

The process capability indices are now: ([2nd] [STATVAR])

$C_a = 0.4$, $C_p = 1.666666724$ and $C_{pk} = 1.000000035$.

Defective pieces are those bigger than 3.005 and smaller than 2.995. The probability that a piece is defective is: R(3.005) + P(2.995),[2] which can be calculated by pressing:

[DATA] [▲] [ENTER] [3] [.] [0] [0] [5] [ENTER] [2nd] [STATVAR] [▲] [SAVE] [9] [ENTER] , which stores R(3.005) in R)
[DATA] [▲] [ENTER] [2] [.] [9] [9] [5] [ENTER] [2nd] [STATVAR] [▲] [▲] [+] [ALPHA] [9] [ENTER]

R(3.005) + P(2.995) = 0.0013 + 0.0 = 0.0013, or expressed in pieces per million: 1300.

Answer: The process is now clearly off-center, and incapable because $C_{pk} \le 1.33$. The estimated number of defective pieces is 1300 each million of pieces fabricated.

[2] Refer to the Hp 9g learning module *Statistics – Normal Distribution*.