

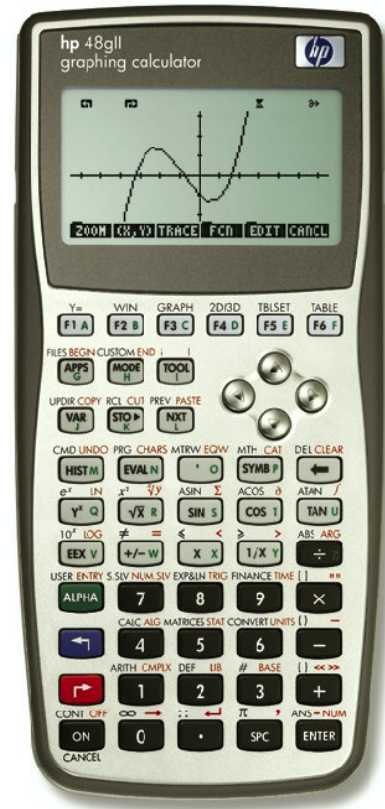


hp calculators

HP 48GII Using Taylor Series

The menu LIMITS AND SERIES

Practice using Taylor series



The menu LIMITS AND SERIES

The menu LIMITS AND SERIES contains commands related to limits. To access it you press \leftarrow CALC . You are presented then the calculus menu as a CHOOSE box:



Figure 1

Its second menu item is 2.LIMITS AND SERIES... You can use such CHOOSE boxes much like menus of computer applications. You can move the selection using the arrow keys. You can also jump to a certain menu item by typing the first few letters of the command or the number at the left of the menu item. Pressing the ENTER key or the menu key M will execute the selected item. In this CHOOSE box you press 2 to select 2.LIMITS AND SERIES.. and then ENTER or M to display the menu:



Figure 2

The command DIVPC needs two polynomials and an integer. It returns the increasing power quotient of the two polynomials up to an order indicated by the integer. The command lim takes an algebraic object and an equation of the form $variable=expression$. It returns the limit of the algebraic expression when the given variable approaches the expression at the right hand side of the equation. The command SERIES needs an algebraic expression, and equation of the form $variable=expression$, and an integer. It returns a list at stack level 2 and an equation at stack level 1. The list contains 4 items: The limit of the expression when the given variable approaches the expression at the right hand side of the equation. The equivalent value expression at that point. The power expansion at that point. And finally the order of the residual at that point. The equation on stack level 1 is of the form $h=variable-expression$, where $variable$ and $expression$ are the same as in the equation $variable=expression$ that we provided as argument for the command. The command TAYLOR0 performs a Maclaurin series expansion of an expression in the default independent variable, VX (usually 'X'). The expansion uses a 4-th order relative power, i.e., the difference between the highest and lowest power in the expansion is 4. The command TAYLR needs an algebraic expression, a variable and an integer. It produces a Taylor series expansion of the expression for the given variable x about a point $variable=0$ for the order specified by the integer. Finally the menu item 6.CALCULUS.. takes you back to the calculus menu.

Practice using Taylor series

Example 1: Expand the function $\text{SIN}(X)$ to a Taylor series of order 5 around $X=0$.

Solution: Assume algebraic exact mode, radians angle mode, and CHOOSE boxes. Press \leftarrow CALC to get the calculus menu:



Figure 3

Press 2 ENTER to get the menu LIMITS AND SERIES:



Figure 4

Build-up the command in the command line

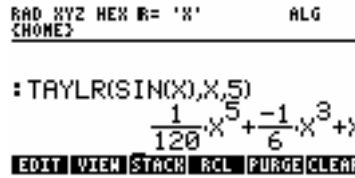


Figure 5

Answer: $\frac{1}{120} \cdot X^5 + \frac{-1}{6} \cdot X^3 + X$

Example 2: Expand the function SIN(X) to a Taylor series of order 5 around X=π.

Solution: Assume RPN mode and CHOOSE BOXES. Enter the function.



Figure 6

Enter the point at which we expand, X=π



Figure 7

Enter the order 5.



Figure 8

Perform series expansion.

\leftarrow CALC $\left[2\right]$ ENTER $\left[3\right]$ ENTER

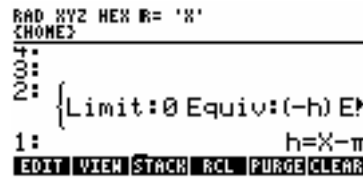


Figure 9

Substitute $h = X - \pi$ in all expressions of the list on stack level 2, expand the list and drop the list element count.

\rightarrow ALG $\left[8\right]$ ENTER \leftarrow PRG $\left[5\right]$ ENTER ENTER \leftarrow

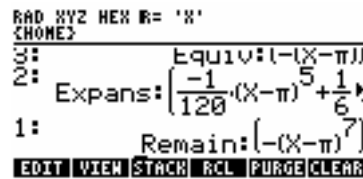


Figure 10

The series expansion was performed around $X - \pi$ and the remainder is of order 7. Press \leftarrow to drop the remainder. Delete the tag from the series.

\leftarrow PRG $\left[5\right]$ ENTER $\left[1\right]$ $\left[1\right]$ ENTER

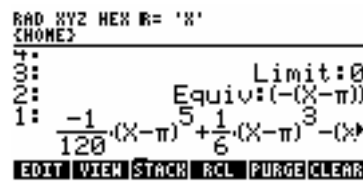


Figure 11

The series is ordered in order of decreasing powers of $X - \pi$. Order in decreasing powers of X .

\leftarrow X \rightarrow ALPHA ALPHA $\left[R\right]$ $\left[E\right]$ $\left[O\right]$ $\left[R\right]$ $\left[D\right]$ $\left[E\right]$ $\left[R\right]$ ENTER

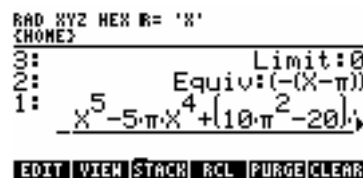


Figure 12

Answer:

$$\frac{X^5 - 5\pi X^4 + (10\pi^2 - 20)X^3 - (10\pi^3 - 60\pi)X^2 + (5\pi^4 - 60\pi^2 + 120)X - (\pi^5 - 20\pi^3 + 120\pi)}{120}$$

Example 3:

In special relativity the kinetic energy of a body with mass m moving with speed v is given by the formula:

$$m \cdot c^2 \cdot \left(\sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} - 1 \right)$$

where c is the speed of light in vacuum. Expand this formula for the case $v \ll c$ (i.e. around $v=0$) to a series of order 6. What can be said about the relativistic formula for the kinetic energy in this case?

Solution: Assume RPN mode and CHOOSE boxes. Enter the formula.

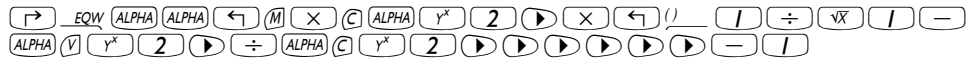


Figure 13

ENTER Then enter the variable.



Enter the order of the series.

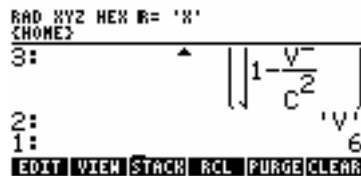
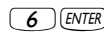


Figure 14

Perform series expansion.

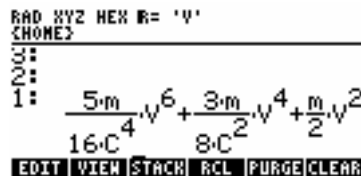


Figure 15

Answer: The series:

$$\frac{5 \cdot m}{16 \cdot C^4} \cdot v^6 + \frac{3 \cdot m}{8 \cdot C^2} \cdot v^4 + \frac{m}{2} \cdot v^2$$

shows that the relativistic kinetic energy of a body in the case of small speeds is essentially the same with its kinetic energy in Newtonian mechanics, since the members of the series starting at the 4th power of v are negligible (since they are divided by the value of the speed of light to the 2nd and 4th powers, etc.) compared to the member $mv^2/2$.