hp calculators

HP 48GII Symbolic integration of polynomials

Methods used

The integration commands

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Practice integrating polynomials symbolically

Methods used

The HP48GII provides large selection of methods for performing symbolic integration and for finding antiderivatives. Several methods for the symbolic integration of expressions involving polynomials are considered here.

Integration commands

The provided integration commands are INT, INTVX, RISCH and ∫. Any of these commands can be used for symbolic integration in combination with substitution, expansion, and so on. The command INT is accessible using the built-in command catalogue of the HP48GII. Press ↱ CAT to open the catalogue. From the catalogue you can select and execute any of the existing commands. The catalogue is much like a menu of an application, where you can use the arrow keys to select menu items, or jump to the items typing the first few letters of them. While the catalogue is active, press ALPHA ALPHA I N T to jump to the command INT. Pressing the key ENTER or the menu key [ ] will put the selected item on the command line (or execute the selected item if RPN mode is on). Pressing [ ] will quit the command catalogue without executing the selected item. The command INT needs three arguments: The expression to be integrated, the variable of integration, and the value of the variable of integration where the antiderivative will be evaluated.

The commands INTVX and RISCH are available in the menu "Derivatives and Integrals" This menu is accessed pressing ↰ CALC to open the "Calculus" menu.

Figure 1

The first menu item is 1.DERIV & INTEG.... and it is highlighted (selected). In this CHOOSE box selecting 1.DERIV & INTEG... and pressing ENTER or [ ] takes you to a new menu which contains differentiation and integration commands:

Figure 2

The commands INTVX and RISCH are in the second page of the menu, so you must press 7 to have the CHOOSE box scroll down and see them. The command INTVX is provided as a shorter way to perform integrations as it only needs one argument, the expression to be integrated, and uses automatically the current CAS variable VX (usually X) as the variable of integration. RISCH needs two arguments: the expression to be integrated and the variable of integration.

Finally, the command ∫ is accessible from the keyboard pressing ↱ ∫ . It needs four arguments: the lower and upper limit of integration, the expression that must be integrated, and the variable of integration. In many cases, this will be the command that is the best choice for numeric integration.

The substitution commands

The commands for substitution are SUBST, | (where) , and PREVAL. The command PREVAL allows for the substitution and evaluation of the difference $g(x_2)-g(x_1)$, where g(x) is the antiderivative of some function f(x) that we want to integrate between the limits x_1 and x_2. This command resides in the menu 1.DERIV & INTEG..... The command SUBST allows for the substitution of the variable of integration, since it will take care of altered integration limits and other necessary substitutions in the integral. This command resides in (the second page) of the menu "Algebra" which you access by pressing ↱ ALG .

Practice integrating polynomials symbolically

Example 1: Find the antiderivative of the polynomial: $X^4 - 5X^3 + 18X^2 - 60X + 120$

Solution: Assume algebraic exact mode, CHOOSE boxes, and X as the current CAS variable VX. Start the equation writer and call the calculus menu.

[→] EQW [←] CALC

Figure 3

ENTER (Choose the menu 1.DERIV & INTEG.....)

Figure 4

8 (Select the command INTVX)

Figure 5

ENTER (Put the command INTVX with its place holders in the equation writer)

Figure 6

X y^x 4 ▶ − 5 × X y^x 3 ▶ ▶ ▶ +
1 8 × X y^x 2 ▶ ▶ − 6 0 × X + 1 2 8 (Enter the polynomial)

and perform the integration.

ENTER ENTER

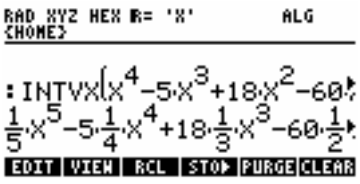


Figure 7

Answer: $\frac{1}{5}\cdot x^5-5\cdot\frac{1}{4}\cdot x^4+18\cdot\frac{1}{3}\cdot x^3-60\cdot\frac{1}{2}\cdot x^2+120\cdot x$

Note that since the antiderivative of a function is only determined up to an additive constant, the above result is only one of the possible antiderivatives. The general result is:

$\frac{1}{5}\cdot x^5-5\cdot\frac{1}{4}\cdot x^4+18\cdot\frac{1}{3}\cdot x^3-60\cdot\frac{1}{2}\cdot x^2+120\cdot x+C$

where C is the additive constant.

Example 2: Find the antiderivative of:

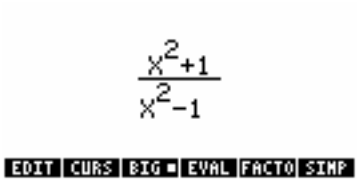


Figure 8

Solution: Assume RPN exact mode, CHOOSE boxes and X as current variable VX.

→ EQW X y^x 2 ▶ + 1 → ▲ ÷ X y^x 2 ▶ − 1 ENTER

Figure 9

← CALC ENTER 8 ENTER

Figure 10

→ ALG 4 ENTER (Collect the logarithms)

Answer:

Figure 11

Example 3: Integrate symbolically for Y

Figure 12

Solution: Assume RPN exact mode with CHOOSE boxes on.

Enter the expression using the EquationWriter.

↱ EQW √x ALPHA Y yˣ 2 ▶ ÷ ALPHA Y yˣ 2 ▶ + 2 ENTER

Figure 13

' ALPHA Y ENTER ↰ CALC ENTER / ▼ ENTER (to find the antiderivative – this takes a few seconds).

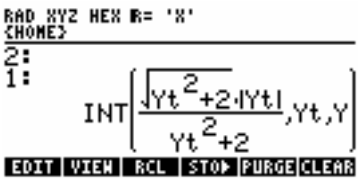
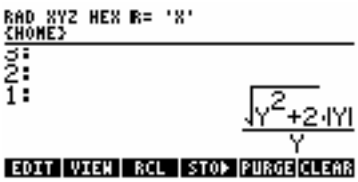


Figure 14

↱ ALG 2 ENTER



Answer:

Figure 15