



## hp calculators

### HP 48GII Numeric Integration

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## Methods used

The HP 48GII provides several integration commands that we can use symbolic or numeric integration. This training aid considers two methods of numeric integration. The first is to directly evaluate numerically a given integral, while the second is to find a symbolic antiderivative, then substitute values for the variables, and evaluate it to a number.

## Integration commands

The provided integration commands are INT, INTVX, RISCH and  $\int$ . Any of these commands can be used for symbolic integration in combination with substitution, expansion, and so on. The command INT is accessible using the built-in command catalogue of the HP48GII. Press  $\rightarrow$   $\text{CAT}$  to open the catalogue. From the catalogue you can select and execute any of the existing commands. The catalogue is much like a menu of an application, where you can use the arrow keys to select menu items, or jump to the items typing the first few letters of them. While the catalogue is active, press  $\text{ALPHA}$   $\text{ALPHA}$   $\text{I}$   $\text{N}$   $\text{T}$  to jump to the command INT. Pressing the key  $\text{ENTER}$  or the menu key  $\text{MENU}$  will put the selected item on the command line (or execute the selected item if RPN mode is on). Pressing  $\text{EXIT}$  will quit the command catalogue without executing the selected item. The command INT needs three arguments: The expression to be integrated, the variable of integration, and the value of the variable of integration where the antiderivative will be evaluated.

The commands INTVX and RISCH are available in the menu "Derivatives and Integrals" This menu is accessed pressing  $\leftarrow$   $\text{CALC}$  to open the "Calculus" menu.



Figure 1

The first menu item is 1.DERIV & INTEG... and it is highlighted (selected). In this CHOOSE box selecting 1.DERIV & INTEG... and pressing  $\text{ENTER}$  or  $\text{MENU}$  takes you to a new menu which contains differentiation and integration commands:



Figure 2

The commands INTVX and RISCH are in the second page of the menu, so you must press  $\text{7}$  to have the CHOOSE box scroll down and see them. The command INTVX is provided as a shorter way to perform integrations as it only needs one argument, the expression to be integrated, and uses automatically the current CAS variable VX (usually X) as the variable of integration. RISCH needs two arguments: the expression to be integrated and the variable of integration.

Finally, the command  $\int$  is accessible from the keyboard pressing  $\rightarrow$   $\int$ . It needs four arguments: the lower and upper limit of integration, the expression that must be integrated, and the variable of integration. In many cases, this will be the command that is the best choice for numeric integration.

## The substitution commands

The commands for substitution are SUBST, | (where) , and PREVAL. The command PREVAL allows for the substitution and evaluation of the difference  $g(x_2)-g(x_1)$ , where  $g(x)$  is the antiderivative of some function  $f(x)$  that we want to integrate between the limits  $x_1$  and  $x_2$ . This command resides in the menu 1.DERIV & INTEG.... The command SUBST allows for the substitution of the variable of integration, since it will take care of altered integration limits and other necessary substitutions in the integral. This command resides in (the second page) of the menu "Algebra" which you access by pressing  $\rightarrow$   $\text{ALG}$ .

### Expansion commands

The command EXPAND can be used to find the symbolic anti derivative of a function (if possible) out of an expression that contains some of the integration commands. It needs one argument: The expression to be expanded. The HP48GII will try to perform the integrations and if it succeeds it will return an expression with no integrals present. The command is available in several menus, like for example in the menu "Algebra". You access this menu pressing  $\boxed{\rightarrow}$   $\boxed{ALG}$ .

### The numeric evaluation commands

For numeric evaluation, any of the commands EVAL,  $\rightarrow$ NUM, or EXPAND will work. These commands will evaluate some given expression to a number if possible. The commands EVAL and  $\rightarrow$ NUM are available directly on the keyboard:  $\boxed{\rightarrow}$   $\boxed{\rightarrow NUM}$  and  $\boxed{EVAL}$ . The command EXPAND is in several menus, such as the algebra menu which is accessed pressing  $\boxed{\rightarrow}$   $\boxed{ALG}$ .

### Practice solving numeric integration problems

Example 1: Integrate the expression below numerically from X = 0 to X =  $\pi$ .

$$\frac{\sin(x)}{x}$$

Solution: Assume algebraic exact mode, radians angle measure, and standard format for displaying numbers.

$\boxed{\rightarrow}$   $\boxed{EQW}$   $\boxed{\rightarrow}$   $\boxed{0}$   $\boxed{\rightarrow}$   $\boxed{\leftarrow}$   $\boxed{\pi}$   $\boxed{\rightarrow}$   $\boxed{SIN}$   $\boxed{X}$   $\boxed{\uparrow}$   $\boxed{\uparrow}$   $\boxed{\div}$   $\boxed{X}$   $\boxed{\rightarrow}$   $\boxed{X}$

$$\int_0^{\pi} \frac{\sin(x)}{x} dx$$

**EDIT CURS BIG = EVAL FACTO SIMP** Figure 3

$\boxed{ENTER}$   $\boxed{ENTER}$  (Submit for processing)

The HP48GII tries to solve the integral symbolically. It can't find a symbolic solution and so it leaves the integral on the history area:

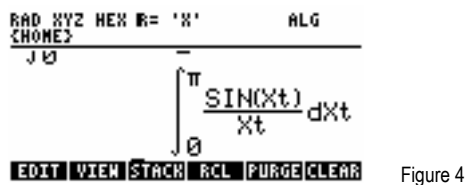


Figure 4

The replacement of variable X with Xt was carried out by the CAS while the HP48GII was trying to solve the integral symbolically.

Find a numeric approximation.

$\boxed{\rightarrow}$   $\boxed{\rightarrow NUM}$   $\boxed{ENTER}$

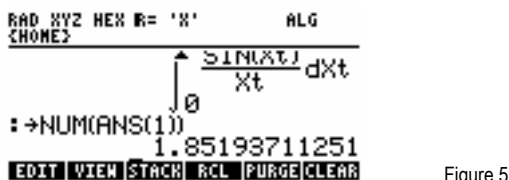


Figure 5

Answer: 1.85193705198

**Example 2:** Find the numeric value of the integral:

$$\int_0^{\pi} e^{-x^2} dx$$

**Solution:** Assume RPN mode, CHOOSE boxes, variable X as the current VX and standard display format of numbers. Enter the integral using the command catalogue. Press

$\left[ \rightarrow \right]$   $\left[ \text{EQW} \right]$   $\left[ \rightarrow \right]$   $\left[ \text{CAT} \right]$   $\left[ \text{ALPHA} \right]$   $\left[ \text{ALPHA} \right]$   $\left[ \text{I} \right]$   $\left[ \text{N} \right]$

to jump to the first command that starts with "INT". Now the command INCR should be selected. The fourth command after INCR is the command INT. Press  $\left[ \downarrow \right]$   $\left[ \downarrow \right]$   $\left[ \downarrow \right]$   $\left[ \downarrow \right]$  to select it.



Figure 6

Press  $\left[ \text{ENTER} \right]$  to put the command INT in the EquationWriter. Now continue the formula.

$\left[ \leftarrow \right]$   $e^x$   $\left[ + \right]$   $\left[ \text{ALPHA} \right]$   $\left[ \text{Y} \right]$   $\left[ \text{Y}^x \right]$   $\left[ 2 \right]$   $\left[ \rightarrow \right]$   $\left[ \text{ALPHA} \right]$   $\left[ \text{Y} \right]$   $\left[ \rightarrow \right]$   $\left[ \text{X} \right]$

$$\text{INT}(e^{-Y^2}, Y, X)$$

**EDIT CURS BIG EVAL FACTO SIMP**

Figure 7

We use Y instead of X in the function and in the variable of integration, and we keep X as the value that will be used to supply the limits of integration. Actually any other variable name could be used in the function and in the variable of integration. If your current VX is not X but, say A, you just supply that variable name as the third argument of INT. Press  $\left[ \text{ENTER} \right]$  to put the expression on stack level 1.

Enter the limits of integration.

$\left[ 0 \right]$   $\left[ \leftarrow \right]$   $\left[ \pi \right]$



Figure 8

Evaluate the integral at the limits of integration and build-up the difference.

$\left[ \leftarrow \right]$   $\left[ \text{CALC} \right]$   $\left[ \text{ENTER} \right]$   $\left[ \text{I} \right]$   $\left[ \text{ENTER} \right]$

The command PREVAL evaluates an expression of the current variable VX at two particular values and then calculates the difference between the two evaluated expressions. It takes three arguments: The expression on stack level 3, the lower value at stack level 2 and the higher value at stack level 1. The fact that PREVAL substitutes values only for the current VX was the reason for using variable Y instead of X and leaving variable X (usually the current VX) as a variable that PREVAL can use. The result of this operation is an unevaluated integral, which can be evaluated numerically. The

HP 48GII displays now:

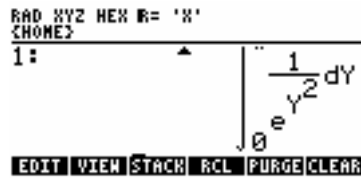


Figure 9

$\rightarrow$   $\rightarrow$ NUM (Calculate the numeric value)

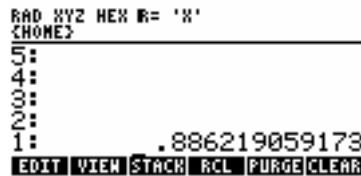
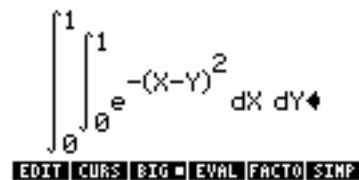


Figure 10

Answer: .886219059173

Note: Even if the HP48GII can integrate symbolically some function, the expression INT(function, vvariable, value) is only determined up to an additive constant. The anti derivative of a function is not a single function but a family of functions that differ from each other by some constant. It is the difference  $INT(\text{function}, \text{variable}, \text{value}_2) - INT(\text{function}, \text{variable}, \text{value}_1)$  that is unambiguous.

Example 3: Find the numeric value of:



Solution: Assume algebraic mode. Enter the integral.

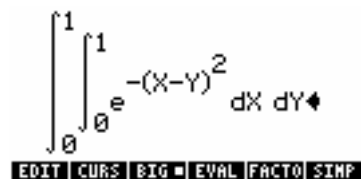


Figure 11

$\rightarrow$   $\rightarrow$  (Submit for processing. Note: this takes a few seconds)

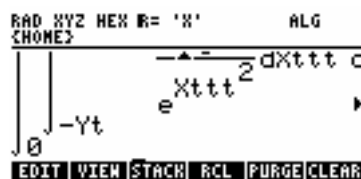


Figure 12

The HP48GII can't solve the integral symbolically. It has transformed it to a double integral in which there is only a single variable in the exponential.

**MODE** **NUM** **4** (Set a numeric accuracy to 10<sup>-4</sup>)

```

RAD XYZ BIN R= 'X'      ALG
{HOME}
-----
CALCULATOR MODES
Operating Mode..Algebraic
Number Format...Fix  4  _FN,
Angle Measure...Radians
Coord System....Rectangular
✓Beep  _Key Click  ✓Last Stack
Choose decimal places to display
FLAGS CHOOSE CAS DISP CANCL OK
    
```

Figure 13

Press **NUM** to accept these settings and leave the screen.

**→** **NUM** **ENTER** (calculate the numeric answer – note: this takes a few seconds)

```

RAD XYZ HEX R= 'X'      ALG
{HOME}
-----
∫01 -Yt e-Yt
:→NUM(ANS(1))
0.8615
EDIT VIEW STACK RCL PURGE CLEAR
    
```

Figure 14

Answer: 0.8615

Of course it is not necessary to have integrals that can't be solved symbolically in order to do numeric integration. All integrals that the HP48GII can solve symbolically can also be numerically evaluated. This case is demonstrated in the next examples.

Example 5: Find a numeric value for:

$$\int_0^{\pi} X \sin(X) - \cos(X) dx$$

Solution: Assume RPN mode, CHOOSE boxes, standard numeric formatting, and X as the current variable VX.

Put the expression on stack level 1.

**→** **EQW** **←** **CALC** **ENTER** **8** **ENTER** **X** **X** **SIN** **X** **→** **→** **-** **COS** **X** **ENTER**

```

RAD XYZ HEX R= 'X'
{HOME}
-----
5:
4:
3:
2:
1: INTVX(X*SIN(X)-COS(X))
EDIT VIEW STACK RCL PURGE CLEAR
    
```

Figure 15

**→** **ALG** **2** **ENTER** (Solve the integral symbolically.)

```

RAD XYZ HEX R= 'X'
{HOME}
-----
5:
4:
3:
2:
1: -(X-COS(X))
EDIT VIEW STACK RCL PURGE CLEAR
    
```

Figure 16

Calculate the difference of the anti derivative at the limits of integration.

$\boxed{0}$   $\boxed{\leftarrow}$   $\pi$   $\boxed{\leftarrow}$   $\boxed{CALC}$   $\boxed{ENTER}$   $\boxed{/}$   $\boxed{ENTER}$



Figure 17

The answer is  $\pi$ . Use  $\boxed{\rightarrow}$   $\rightarrow$ NUM to get a numeric approximation of  $\pi$ .

Answer: 3.14159265359

Note: Any of the four integration commands (not only INTVX) can be used in such cases where symbolic solutions of an integral can be found by the HP48GII.