



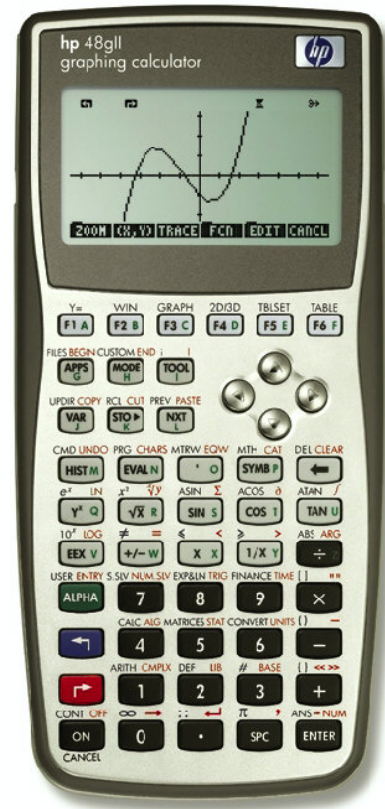
hp calculators

HP 48GII Finding Limits

The menu LIMITS AND SERIES

The basic commands for finding limits

Practice finding limits



The menu LIMITS AND SERIES

The menu LIMITS AND SERIES contains commands related to limits. To access it, press \leftarrow CALC . The calculus menu is then presented as a CHOOSE box:



Figure 1

The second menu item is 2.LIMITS AND SERIES... You can use such CHOOSE boxes much like menus of computer applications. You can move the selection using the arrow keys. You can also jump to a certain menu item by typing the first few letters of the command and the number at the left of the menu item. Pressing the ENTER key or the menu key $\left[\text{MENU}\right]$ will execute the selected item. In this CHOOSE box you press $\left[2\right]$ to select 2.LIMITS AND SERIES.. and then ENTER or $\left[\text{MENU}\right]$ to display the menu:



Figure 2

The command DIVPC needs two polynomials and an integer. It returns the increasing power quotient of the two polynomials up to an order indicated by the integer. The command lim takes an algebraic object and an equation of the form $variable=expression$. It returns the limit of the algebraic expression when the given variable approaches the expression at the right hand side of the equation. The command SERIES needs an algebraic expression, and equation of the form $variable=expression$, and an integer. It returns a list at stack level 2 and an equation at stack level 1. The list contains 4 items: The limit of the expression when the given variable approaches the expression at the right hand side of the equation. The equivalent value expression at that point. The power expansion at that point. And finally the order of the residual at that point. The equation on stack level 1 is of the form $h=variable-expression$, where $variable$ and $expression$ are the same as in the equation $variable=expression$ that we provided as argument for the command. The command TAYLOR0 performs a Maclaurin series expansion of an expression in the default independent variable, VX (usually 'X'). The expansion uses a 4-th order relative power, i.e., the difference between the highest and lowest power in the expansion is 4. The command TAYLR needs an algebraic expression, a variable and an integer. It produces a Taylor series expansion of the expression for the given variable x about a point $variable=0$ for the order specified by the integer. Finally the menu item 6.CALCULUS.. takes you back to the calculus menu.

The basic commands for finding limits.

To find limits of functions the command lim is used most often. But sometimes lim will not be successful. In such cases we can try to use SERIES since this command is more powerful in finding limits. The SERIES command is not covered in this training aid, however.

Practice finding limits

Example 1: Find the limit:



Figure 3

Solution: Assume algebraic exact mode, radians angle mode, and CHOOSE boxes. Start the equation writer and bring up the calculus menu.

\rightarrow EQW \leftarrow CALC



Figure 4

2 ENTER



Figure 5

2 ENTER

lim(♦,♦)



Figure 6

SIN X \uparrow \uparrow \div X \rightarrow X \rightarrow = 0

$\lim\left(\frac{\sin(X)}{X}, X=0\right)$



Figure 7

ENTER ENTER (find the limit)



Figure 8

Answer: 1

Example 2: Find the limit:



Figure 9

Solution: Assume RPN mode, CHOOSE boxes, and radians angle mode. Write the expression in the equation writer.

$\left(\rightarrow\right) \text{EQW} \left(\leftarrow\right) \text{CALC} \left(2\right) \text{ENTER} \left(2\right) \text{ENTER} \text{SIN} \text{X} \left(\uparrow\right) \left(\uparrow\right) \left(\div\right) \text{X} \left(\rightarrow\right) \text{X} \left(\rightarrow\right) \text{=} \left(\leftarrow\right) \infty$

$$\lim\left(\frac{\text{SIN}(X)}{X}, X=\infty\right)$$

EDIT CURS BIG EVAL FACTO SIMP

Figure 10

$\left(\text{ENTER}\right)$ (put the expression on the stack)

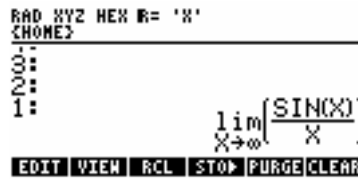


Figure 11

$\left(\rightarrow\right) \text{ALG} \left(2\right) \text{ENTER}$ (find the limit)



Figure 12

Answer: 0

Example 3: Find the limit:

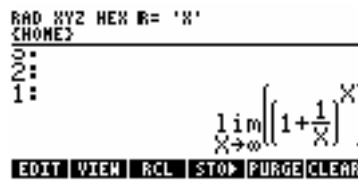


Figure 13

Solution: Assume RPN mode and CHOOSE boxes. Write the expression in the equation writer.

$\left(\rightarrow\right) \text{EQW} \left(\leftarrow\right) \text{CALC} \left(2\right) \text{ENTER} \left(2\right) \text{ENTER} \left(1\right) \left(+\right) \left(1\right) \left(\div\right) \text{X} \left(\uparrow\right) \left(\uparrow\right) \left(\uparrow\right) \text{Y} \text{X} \left(\rightarrow\right) \text{X} \left(\rightarrow\right) \text{=} \left(\leftarrow\right) \infty$

$$\lim\left(\left(1+\frac{1}{X}\right)^X, X=\infty\right)$$

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Figure 14

$\left(\text{ENTER}\right) \left(\rightarrow\right) \text{ALG} \left(2\right) \text{ENTER}$ (find the limit)



Figure 15

Answer: e

Example 4: The concentration $c(t)$ of a chemical substance that is produced during a reaction is given by:

$$c(t) = c_0(1 - e^{-k \cdot t})$$

as a function of time. What will be the final concentration of the substance when the process of the reaction reaches its end (in other words, when t approaches infinity)?

Solution: Assume RPN mode and CHOOSE boxes. Enter the expression from the command line.

$\text{[C] [ALPHA] [ALPHA] [C] [0] [X]}$ (Note: this is the multiply key)
 $\text{[←] [)] [I] [-] [ALPHA] [←] [e^x] [←] [ALPHA] [ALPHA] [←] [K] [X]}$ (Note: multiply key again)
 [←] [T] [ENTER]



Figure 16

Enter the equation for the point at which we get the limit.

$\text{[←] [ALPHA] [←] [T] [→] [=] [←] [∞] [ENTER]}$



Figure 17

$\text{[←] [CALC] [2] [ENTER] [2] [ENTER]}$ (find the limit)



Figure 18

The calculator errors and the error message gives provides information about the command that caused the error and about the reason for the error. The command that caused the error was lim. It errored out because it found an unsigned infinity trying to obtain the limit. This type of error can occur quite frequently in calculations with limits. In this example, it occurs because the Computer Algebra System (CAS) has no way to know if variable k is positive or negative. When $k > 0$, the product $-kt$ will be negative, but when $k < 0$, it will be positive. Therefore, the limit will depend on k being positive or negative. So additional information about k , in CAS terms, an assumption, is necessary. So, the inequality that represents the assumption must be entered.

\leftarrow ALPHA \leftarrow K \leftarrow \geq 0 ENTER

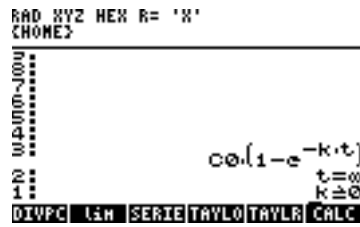


Figure 19

Make the assumption and try again.

ALPHA ALPHA A S S U M E ENTER \leftarrow \leftarrow CALC 2 ENTER 2



Figure 20

Remove the assumption.

\leftarrow ALPHA \leftarrow K ENTER ALPHA ALPHA U N A S S U M E ENTER \leftarrow

Answer: C₀

Note that even simple looking limits can cause errors if you omit making the appropriate assumptions for participating variables in the expression.