# hp calculators

HP 48GII Numeric Differentiation


Methods used

The differentiation commands

The substitution commands

The numeric evaluation commands

Practice solving numeric differentiation problems

## Methods used

Many methods are possible on the HP48GII for performing numeric differentiation. The primary steps for the process are:

1) Use one of the differentiation commands to find a symbolic derivative.

2) Substitute a numeric value for the variable of differentiation (and perhaps for other variables that appear in the derivative) either using one of the substitution commands, or by storing the value in the variable of differentiation (and perhaps other values in the rest of the variables that appear in the derivative).

3) Then manually evaluate, if needed.

## The differentiation commands

The HP48GII provides three basic commands for differentiation which we can use for finding numeric values of derivatives. The simplest provided commands for differentiation are: DERIV, DERVX and ∂. The command ∂ is available from the keyboard by pressing the keys ↱ ∂ . The other two commands are available in several menus. For example, to display the calculus menu, press ↰ CALC .

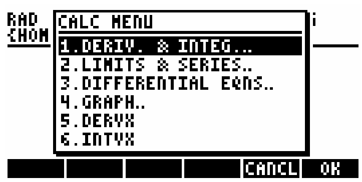

Figure 1

Its first menu item is 1.DERIV & INTEG.... and if ENTER is pressed, a menu which contains differentiation and integration commands is displayed.

Figure 2

The menu items 2.DERIV and 3.DERVX are the other two basic differentiation commands. Any of these three commands can be used for symbolic differentiation. The commands DERIV and ∂ both take two arguments: The expression to be differentiated and the variable with respect to which we want to differentiate. DERVX is provided as a shorter way to perform differentiation when the variable of differentiation is the same with the CAS variable VX (usually X). The command DERIV will also accept a scalar expression and a vector of names, or even a vector of algebraic objects and a vector of names as arguments, allowing the user to find gradients or even hessian matrices.

## The substitution commands

The HP48GII provides two basic commands for substitution, SUBST and | (where). They can be used for value substitution of a variable in any expression. The command | (where) is accessed through pressing ↱ | on the keyboard. The command SUBST is in several menus. For example if you have CHOOSE boxes on you can access it pressing ↱ ALG 8 ENTER . Both commands take two arguments in algebraic syntax: The expression in which we want to carry out the substitution and an equation that defines the substitution. In RPN syntax the second argument of the command | is a list. This list contains pairs of names (variables) and values or expressions, making the command | more flexible in RPN mode.

## The numeric evaluation commands

For numeric evaluation, any of the commands EVAL, →NUM, or EXPAND will work. These commands will evaluate some given expression to a number if possible. The commands EVAL and →NUM are available directly on the keyboard: [↱] →NUM and [EVAL]. The command EXPAND is in several menus, such as the algebra menu which is accessed pressing [↱] ALG .

## Practice solving numeric differentiation problems

Example 1: Find the slope of the function shown below at 0, π/2 and 3π/2.

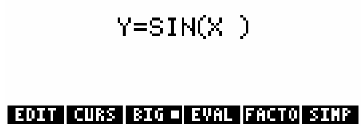


Figure 3

Solution: [↱] EQW [↱] ∂ [X] [▶] [SIN] [X]

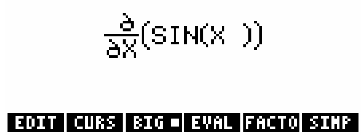


Figure 4

[↱] [▲] [↱] | [X] [▶] [0]

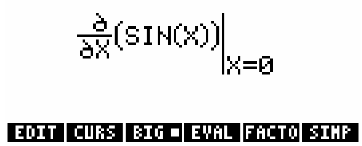


Figure 5

[ENTER] [ENTER]

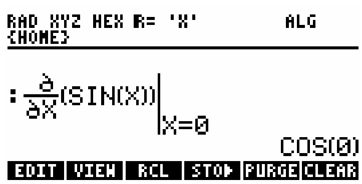


Figure 6

[↱] →NUM [ENTER]


Figure 7

To evaluate the derivative at the other points, press [▲] [▲] [▲] [▲] to enter the history stack and select the derivative. Press [ENTER] to place it on the command line. Then press [◀] [⬅] to delete the 0 as shown below.

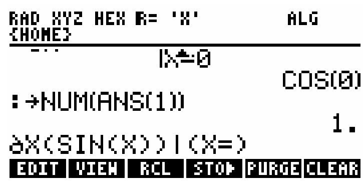

Figure 8

Now press [←] π [÷] [2] [ENTER] [→] →NUM [ENTER]

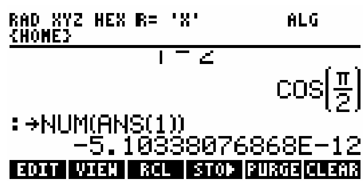

Figure 9

[▲] [▲] [▲] [▲] [ENTER] [◄] [◄] [◄] [3] [X] [ENTER] [→] →NUM [ENTER]

Figure 10

Answer: 1., -5.10338076868E-12 (or 0), -4.68985769397E-12 (or 0)

Example 2: Find the numeric value of the derivative

$$\frac{\partial^2 \text{ATAN}(X * Y)}{\partial X\, \partial Y}$$

at the point X = 2, Y = √2

Solution: Assume RPN mode and CHOOSE boxes on.

[→] EQW ATAN [X] [×] [ALPHA] [Y] [ENTER] (Note: That is X multiplied by Y)
[ ' ] [ALPHA] [Y] [ENTER] [←] CALC [ENTER] [▼] [ENTER]

Figure 11

[ ' ] [X] [ENTER] [←] CALC [ENTER] [▼] [ENTER]

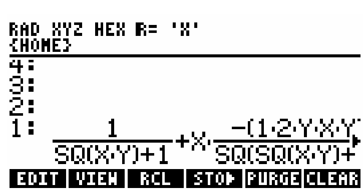

Figure 12

Now enter the substitution list containing the values of X and Y at which we wish to evaluate the derivative.

[←] {} [X] [SPC] [2] [SPC] [ALPHA] [Y] [SPC] ['] [√X] [2] [ENTER]


Figure 13

[→] | [ENTER]

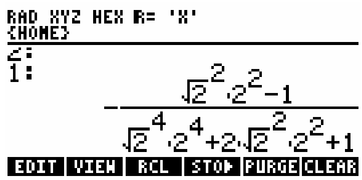


Figure 14

[→] →NUM

Answer: -8.64197530867E-2 or –0.086420, approximately.

Example 3: The potential energy of a mass-feather system is given by the formula below, where k is a constant describing the stiffness of the feather, and x is the distance of the mass from the equilibrium point. If k=2.8J/m² what force acts on the mass at x=1m ?

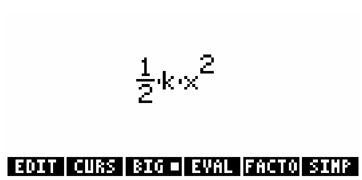


Figure 15

Solution: The force acting upon a mass can be found by calculating the negative of the first derivative of the potential energy with respect to x. Assume RPN mode.

[→] EQW [+/-] [→] ∂ [ALPHA] [←] [X] [▶] [1] [÷] [2] [▶] [×] [ALPHA] [←] [K] [×] [ALPHA] [←] [X] [$Y^x$] 2

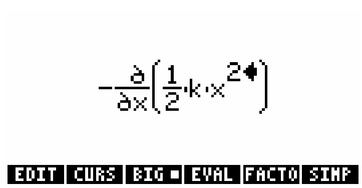


Figure 16

[ENTER] [→] ALG [2] [ENTER]


Figure 17

Now store values into x and k to find the solution.

2 • 8 ↱ _ ALPHA J ÷ ALPHA ↰ M Y^x 2 ENTER


Figure 18

' ALPHA ↰ K STO▸ to store the value in variable k. Press 1 ↱ _ ALPHA ↰ M ENTER and then ' ALPHA ↰ X STO▸ to store 1m in x. Now press EVAL

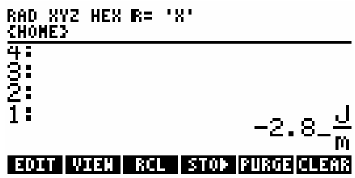


Figure 19

To convert this to Newtons, press 1 ↱ _ ↱ UNITS 8 ENTER ENTER ENTER

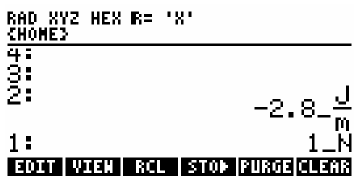


Figure 20

↱ UNITS ENTER ENTER

Answer: -2.8 Newtons