



hp calculators

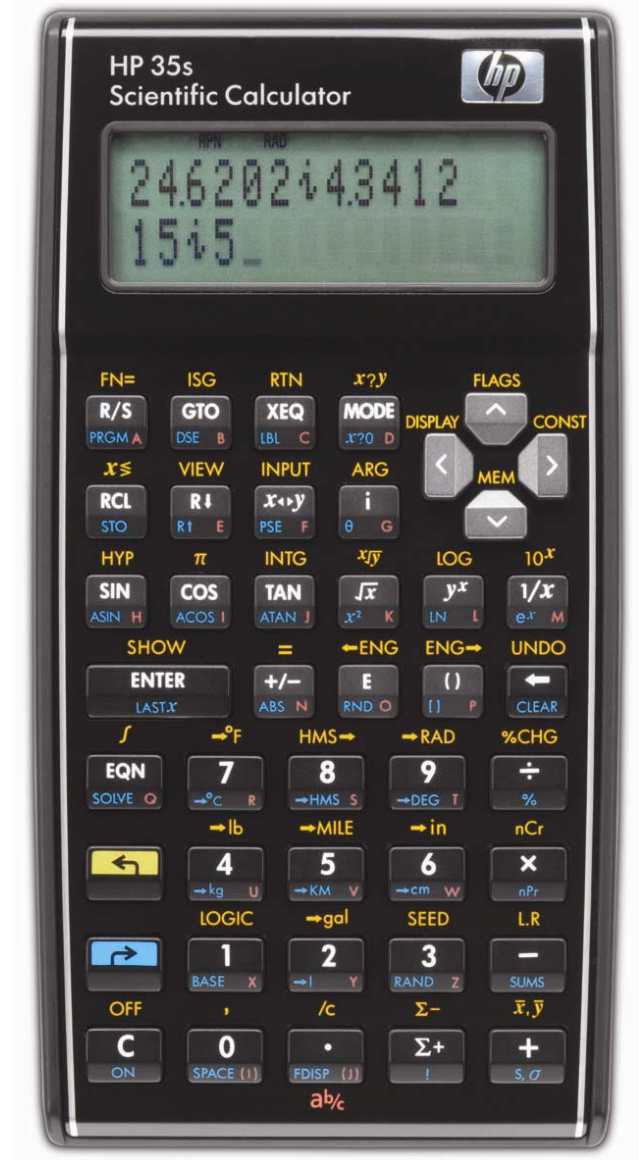
HP 35s Roots of polynomials

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Polynomials

A polynomial is an expression containing one or more terms called monomials. These terms contain one or more variables multiplied by a constant coefficient. Each of these variables will have a positive exponent. A term may have an exponent of zero, in which case it is a constant term. A polynomial will generally not contain negative exponents or division by a term or expression containing a variable.

The degree of a polynomial is determined by the largest exponent of a variable within the expression.

Roots of a polynomial are values that when substituted into the expressions variable cause the polynomial's value to be zero. These would correspond to the X-intercept of a polynomial's graph. Some polynomials do not have roots that are real numbers. However, from the fundamental theorem of algebra, every polynomial has at least one root, if the allowable values are expanded to include complex numbers.

Roots of a polynomial

The roots of a polynomial are values of X where the value of the function of x (or the value of the polynomial) is equal to zero. For example, the polynomial $f(x) = X - 2$ has a real root at the value +2. The polynomial $X^2 - 9$ has real roots at the values of + 3 and - 3. Not every polynomial has roots that are real numbers. For example, the $X^2 + 4$ has no real roots, meaning there are no real values for X that will cause $X^2 + 4$ to equal zero.

Using the SOLVE function

The HP 35s has a very powerful root finding capability built into its SOLVE function. As applied in this training aid, the SOLVE function, accessed by pressing the **SOLVE** key, will be used to find roots from user-written programs computing the value of a function. This will involve entering a small program, keying in a small equation into the program using a variable, indicating to the HP 35s which variable is being considered as the current function, and then solving for the value of that variable when the function is equal to zero. The HP 35s knows which variable to solve for by setting the value of the function under consideration using the **↵** **FN** **=** **X** function. To indicate to the HP 35s that the variable X is to be used, press **↵** **FN** **=** **X**.

This training aid cannot begin to illustrate the wide range of applications available using the built-in solver, but it can illustrate some of the more common uses.

Practice solving for roots of polynomials

Example 1: Solve for the roots of $4X^2 - 2X - Y = 12$

Solution: First, rearrange the equation so that the variable Y is isolated. This is necessary to use the SOLVE function as we are doing in this training aid. The rearranged equation is $Y = 4X^2 - 2X - 12$. This is a polynomial.

We're looking for values of X such that $4X^2 - 2X - 12 = 0$. First, we'll enter a program that computes the value of the function. If a program already exists in program memory with the name of X, then it will need to be cleared. This can be done by pressing **↵** **MEM** **>** **ENTER** to have the HP 35s display the list of programs in the calculator and then press **↵** to step through the program labels. When the label of the program to be deleted is shown in the display, pressing **↵** **CLEAR** will delete that program from the calculator's memory. Pressing **C** will then clear the display and allow you to proceed.

In RPN or algebraic mode: $\boxed{\rightarrow}$ $\boxed{\text{PRGM}}$ $\boxed{\rightarrow}$ $\boxed{\text{LBL}}$ $\boxed{\text{X}}$ $\boxed{\text{EQN}}$
 $\boxed{4}$ $\boxed{\text{X}}$ $\boxed{\text{RCL}}$ $\boxed{\text{X}}$ $\boxed{y^x}$ $\boxed{2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{\text{X}}$ $\boxed{\text{RCL}}$ $\boxed{\text{X}}$ $\boxed{-}$ $\boxed{1}$ $\boxed{2}$ $\boxed{\text{ENTER}}$
 $\boxed{\leftarrow}$ $\boxed{\text{RTN}}$

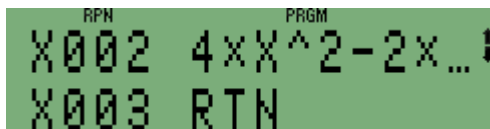


Figure 1

To show the checksum and length of this program, press the following in RPN or algebraic mode. Note that the symbol $\boxed{\rightarrow}$ means to press the right arrow direction of the cursor key.

In RPN or algebraic mode: $\boxed{\leftarrow}$ $\boxed{\text{MEM}}$ $\boxed{\rightarrow}$ $\boxed{\text{ENTER}}$ $\boxed{\leftarrow}$ $\boxed{\text{SHOW}}$



Figure 2

If the checksum of the program just entered does not equal 8BFA, then you have not entered it correctly.

To clear the checksum display press:

In RPN or algebraic mode: $\boxed{\rightarrow}$ $\boxed{\text{PRGM}}$

Then, to exit the program environment, press:

In RPN or algebraic mode: $\boxed{\rightarrow}$ $\boxed{\text{PRGM}}$

Store an initial guess for X of 10 into the variable X. Then set the function to X and solve for the value of X.

In RPN or algebraic mode: $\boxed{1}$ $\boxed{0}$ $\boxed{\rightarrow}$ $\boxed{\text{STO}}$ $\boxed{\text{X}}$ $\boxed{\leftarrow}$ $\boxed{\text{FN=}}$ $\boxed{\text{X}}$ $\boxed{\rightarrow}$ $\boxed{\text{SOLVE}}$ $\boxed{\text{X}}$



Figure 3

Since you feel this equation might have a root larger than this, store a new guess for X of 100 into the variable X. There is no need to set the function to X (since it has already been done). Then solve for the value of X.

In RPN or algebraic mode: $\boxed{1}$ $\boxed{0}$ $\boxed{0}$ $\boxed{\rightarrow}$ $\boxed{\text{STO}}$ $\boxed{\text{X}}$ $\boxed{\rightarrow}$ $\boxed{\text{SOLVE}}$ $\boxed{\text{X}}$

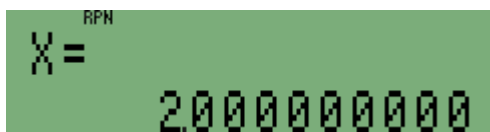


Figure 4

The same root is returned. This is a good indication (but certainly not foolproof) that there are no roots larger than +2 for this equation.

To see if there is a root less than +2 for this equation, store a new guess for X of -10 into the variable X. Then solve for the value of X.

In RPN or algebraic mode: **1 0 +/- ▸ STO X ▸ SOLVE X**



Figure 5

Answer: Roots found for the equation are -1.5 and +2. Note that the HP 35s owners manual provides much more information about providing initial guesses for the SOLVE feature.

Example 2: Solve for the roots of $f(x) = (1/5)x^3 + (4/5)x^2 - (7/5)x - 2$

Solution: We're looking for values of X such that $(1/5)x^3 + (4/5)x^2 - (7/5)x - 2 = 0$. We expect three roots.

First, we'll enter a program that computes the value of the function. If a program already exists in program memory with the name of X, then it will need to be cleared. This can be done by pressing **◀ MEM > ENTER** to have the HP 35s display the list of programs in the calculator and then press **▼** to step through the program labels. When the label of the program to be deleted is shown in the display, pressing **▸ CLEAR** will delete that program from the calculator's memory. Pressing **C** will then clear the display and allow you to proceed.

In RPN or algebraic mode: **▸ PRGM ▸ LBL X EQN**

**1 ÷ 5 × RCL X y^x 3 + 4 ÷ 5 × RCL X y^x 2 - 7 ÷ 5 × RCL X - 2
ENTER ▶ RTN**

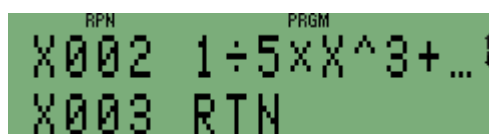


Figure 6

To show the checksum and length of this program, press the following in RPN or algebraic mode. Note that the symbol **▶** means to press the right arrow direction of the cursor key.

In RPN or algebraic mode: **◀ MEM ▶ ENTER ▶ SHOW**

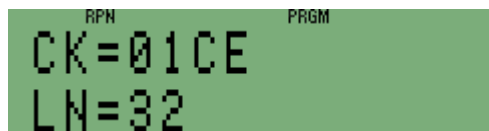


Figure 7

If the checksum of the program just entered does not equal 01CE, then you have not entered it correctly.

To clear the checksum display press: **▸ PRGM**

Then, to exit the program environment, press:  

Store an initial guess for X of 10 into the variable X. Then set the function to X and solve for the value of X.

In RPN or algebraic mode:        



Figure 8

Since you feel this equation might have a root larger than this, store a new guess for X of 100 into the variable X. There is no need to set the function to X (since it has already been done). Then solve for the value of X.

In RPN or algebraic mode:       



Figure 9

The same root is returned. This is a good indication (but certainly not foolproof) that there are no roots larger than +2 for this equation.

To see what the values of roots less than +2 for this equation might be, store a new guess for X of -10 into the variable X. Then solve for the value of X.

In RPN or algebraic mode:      



Figure 10






In RPN or algebraic mode:       



Figure 11

Answer: Roots found for the polynomial are 2, -1, and -5. Note that the HP 35s owners manual provides much more information about providing initial guesses for the SOLVE feature.