

hp calculators

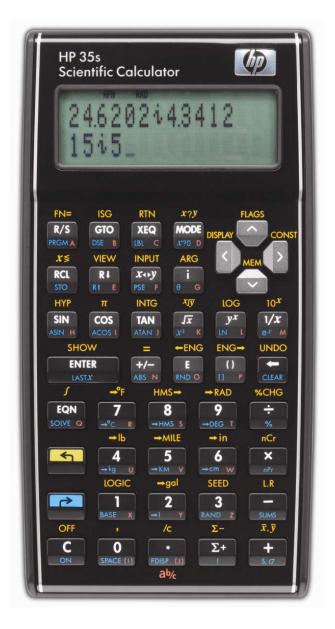
HP 35s Using the formula solver – part 2

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Overview of the Formula Solver

Given an expression of the form:

$$f(x) = y$$

The HP Solve Application searches for a value of **x** that gives:

$$f(x) = y = 0$$

A value of \mathbf{x} for which this is true is called a **root**, and it provides a **solution** of the equation $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. The graph in Figure 1 shows this graphically – there is a root at the value of \mathbf{x} where $\mathbf{f}(\mathbf{x})$ is zero.

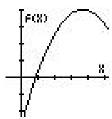


Figure 1

On the HP 35s, **f(x)** can be typed as a formula, in equation mode, or it can be typed as a program. When the Solver is used to find a root of a formula or equation typed in equation mode, it is referred to as the Formula Solver.

Part 1 of this training aid provided an introduction to the Formula Solver, using a few simple examples. This second part explains how the Solver works, and shows some more examples.

The Formula Solver works with f(x) as a formula containing x, for example

$$3x^2 - 3x - 15$$
 or $5\sin(x) - 7\log(x)$

If the Formula Solver is given an **equation** with terms on both sides of the equals sign, such as:

$$3x^2 + 4x = 7x + 15$$

then it begins by moving everything to one side of the equals sign, so the above equation would become the formula:

$$f(x) = 3x^2 - 3x - 15 = 0$$

The Formula Solver ignores the = 0 part, as it is trying to find a value for x to make the formula zero. So there is no need to type an equation with = 0 in it; it is enough to type the formula.

The variable **x** in the above is called the **unknown variable**. It can be represented by any of the HP 35s variables A through Z. A formula can contain more than one variable, the Solver will ask which is the unknown variable and will then ask for the **known values** of all the other variables.

The Formula Solver then tries to rearrange the equation f(x) = 0 to give a **direct solution** for x. An example will be shown later.

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HP 35s Using the formula solver – part 2

If a direct solution is not found, the Solver begins by first trying two guesses for the unknown variable. The user can give one or two guesses for the Solver to start from. The first part showed that this can be very useful, either to direct the Solver **towards** one root of several, or to direct the Solver **away** from values that would cause an error. Good guesses can also speed up the search for a root. In some cases the function varies very slowly over some values, and a good guess is needed to direct the Solver away from them and towards the range of values where a solution is expected.

Beginning from the values obtained for the first two guesses, the Solver searches for values of the unknown variable that make f(x) smaller. If two guesses have the opposite sign, the Solver tries to narrow down the region between them until it finds where the sign changes and f(x) is zero. If two guesses have the same sign, the Solver uses the difference between them to pick the direction in which to change x to look for a third value closer to zero.

The process is repeated until one of the following happens.

A value of \mathbf{x} is found for which $\mathbf{f}(\mathbf{x})$ is exactly zero.

Two neighboring values of \mathbf{x} are found, differing by 1 or 2 in the twelfth significant digit, such that $\mathbf{f}(\mathbf{x})$ changes sign between them. An example of this was given in part 1.

No value can be found, but the Solver finds a minimum. An example was given in part 1.

No value can be found because the Solver is looking at values of x for which f(x) is constant.

No value can be found because f(x) is decreasing asymptotically towards a non-zero value.

Two more cases arise because the HP 35s works with a finite range of numbers, negative numbers between -10⁵⁰⁰ and -10⁻⁴⁹⁹, 0 and positive numbers between +10⁻⁴⁹⁹ and +10⁵⁰⁰. This range is sufficient to cover all physical measurements, and even all numbers in government finances, but problems in number theory and in combinatorial operations may require numbers outside this range. (To be exact, the largest absolute value that the HP 35s can work with is 9.999999999E499.)

No value can be found because the root is at a value of \mathbf{x} that is not zero but lies between -10⁻⁴⁹⁹ and +10⁻⁴⁹⁹. In such a case the Solver gives a result of 0.

No value can be found because the root is at \mathbf{x} that is more negative than, or equal to, -10⁵⁰⁰ or greater than or equal to +10⁵⁰⁰. In such a case the Solver stops with an OVERFLOW error.

To help the user distinguish between the above, the Solver returns the last value it tried for x, the last but one value tried, and the value of f(x) at the last value. Part 1 showed how these values can be seen in RPN mode and in Algebraic mode.

The following examples show some of the features that were not included in part 1.

Practice Example: A formula with several variables

Example 1: A factory is to produce tin cans with a volume of 100 cubic centimeters. The designer estimates that the height should be 10 cm and the radius about 2 cm. Calculate the exact volume of this can, and if it is not close to 100 cubic centimeters then recalculate the radius to give the required volume.

Solution: The equation for a cylinder's volume V, given its radius r, and height h, is $V = \pi r^2 h$. Enter this as the formula $\pi r^2 h - V$ in equation mode and then use the Solver.

Go to equation mode by typing EQN]. If necessary, put the new equation in a particular place in the list of equations by moving up or down through the list with the up and down cursor keys below the HP 35s screen.

Enter the formula by typing:

 π × RCL R y^x 2 × RCL H – RCL V ENTER

As was explained in part 1, to enter a variable into an equation, press the RCL key and then one of the letter keys. As with STO, the symbol A..Z at the top of the screen is shown as a reminder that one of the keys marked A through Z must be pressed. For example press the 5 key to enter the variable V.

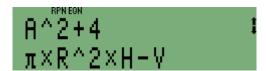


Figure 2

To solve the equation, press the Solve key. The Solver asks which variable to solve for:



Figure 3

The symbol A..Z is at the top of the screen again. The variable in this formula is V so press **5** again. The Solver now knows that V is the unknown variable and it asks for the values of the known variables.



Figure 4

The value that is already stored in R is shown too. If this is the required value then it is enough to press \mathbb{R}/\mathbb{S} . If the variable R has not been used before, then its value is zero. In this example, type the radius 2 and press \mathbb{R}/\mathbb{S} .



Figure 5

The Solver asks for the other known variable. Type the height, 10, and press **R/S** again. The HP 35s displays SOLVING for a moment, then the result.



Figure 6

The volume is over 125 cubic centimeters, considerably more than the intended 100. Repeat the calculation, but this time use the known volume of 100, and solve for the radius. Solve the equation again by pressing EQN SOLVE. The Solver asks for the unknown variable, press R. The Solver then asks for the known variables, first H.



Figure 7

The present value of H is the value previously given. As this is to remain the same, just press **R/S** again. The Solver now asks for the other variable, V.



Figure 8

The present value of V is shown; this is the volume just calculated. As the volume should be 100, type 100 and press **R/S**. The Solver calculates and displays the radius needed to give the required volume.

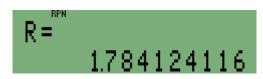


Figure 9

Answer: The cans should have a radius of 1.78 cm.

Practice Example: A direct solution

Example 2: To show that the HP 35s looks for a direct solution before starting to search for a root, try to solve ln(z) = 0 beginning from a negative number for the guess.

Solution: Store -5 in Z. Then store LN(z) as the formula to solve. This means that a solution is wanted for the equation LN(z) = 0.

5 +/_ STO Z EQN PLN RCL Z ENTER

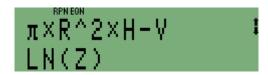


Figure 10

To solve the equation, press SOLVE Z. The Solver immediately displays the answer:



Figure 11

Answer:

Z = 1 is the solution to LN(z) = 0. This is obvious, the point of this example is that the answer was found immediately, and the negative guess was not tried. If the negative guess had been tried, it would have caused a LOG(NEG) error, as in Example 3 of part 1. The Formula Solver recognized that Z appears only once in the formula, and that LN(Z) = 0 can therefore be rewritten as Z = exp(0) to solve for Z directly. Such direct solutions can speed up the use of Solver, specially when a complicated formula with several variables is being solved several times for different variables.

Note: Where more than one solution is possible, for example ASIN(Y)=0, the direct solution is the "principal" value. For example, for ASIN(Y)=0, this is 0 degrees, not 180 degrees, or -180 degrees, or any other possible value. In the same way, an equation such as $X^2=4$ is solved directly and returns the positive root 2. To find other roots, it is necessary to write the expression in such a way that the Solver does not find a direct solution. An easy way to achieve this is to add $0\times$ the unknown variable into an expression, for example ASIN(Y) + $0\times$ Y = 0 or X^2 + $0\times$ X = 0. This is because the Solver stops looking for a direct solution as soon as it sees the unknown variable more than once in an expression.

Practice Example: Where two functions intersect

The Formula Solver can also be used to solve problems of the form:

$$g(x) = h(x)$$

This requires a value of x at which one function g(x) is equal to another function h(x). In other words, the problem is to find x at which these functions intersect.

The equation can be rewritten as:

$$f(x) = g(x) - h(x) = 0$$

Solving the formula g(x) - h(x) will give the value of x at which the two functions cross over.

Example 3: The factory from Example 1 is interested in designing spherical containers with the same volume and the same radius as their tin cans. This means that they want to find a radius r such that:

$$V = \pi r^2 h = 4/3 \pi r^3$$

Solution: Modify the formula from Example 1 to find r such that π r² h – 4/3 π r³ is zero.

A quick look at this expression shows that one solution is r = 0. This is not a useful solution so it would be helpful to provide a guess to direct the Solver away from zero. One guess is the value already in R. Type 8 on the lower line of the display as the second guess.

Go to equation mode by typing EQN. Find the old equation by moving up or down through the equation list with the \(\triangle \) and \(\subseteq \) keys.

Begin editing the formula by pressing the left cursor key . The cursor appears at the end of the formula.

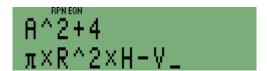


Figure 12

Press ___ to delete the V. Then type the formula for the volume of a sphere.

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4 \div 3 \times 5 \pi \times RCL R y^x 3 ENTER
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Press the right-arrow \longrightarrow key a few times to see the last part of the changed formula. It should look as in Figure 13.

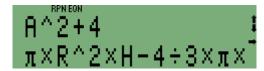


Figure 13

To solve the equation, press the SOLVE key. The Solver asks which variable to solve for: The unknown variable is R so press R. The Solver now asks for the value of the known variable H.



Figure 14

This value is to stay unchanged, so press **R/S**. The Solver looks for a solution.



Figure 15

Answer:

If both the radius of the sphere and that of the base of the cylindrical can are 7.5 then the sphere and the can will have the same volume.

The Solver can be used for many kinds of problems, further information is given in the HP 35s manual, and a detailed description of the Solver is provided in Appendix D of the manual.