

The trigonometric functions

The trigonometric functions, sine, cosine, tangent, and related functions, are used in geometry, surveying, and design. They also occur in solutions to orbital mechanics, integration, and other advanced applications.

The HP 33S provides the three basic functions, and their inverse, or “arc” functions. These work in degrees, radians and gradians modes. In addition, π is provided as a function on the right-shifted “cos” key, and the sign function is on the right-shifted “sin” key.

The secant, cosecant and cotangent functions are easily calculated using the **COS**, **SIN**, and **TAN** keys respectively, followed by **$\frac{1}{x}$** . To help remember whether the secant function corresponds to the inverse sine or cosine, it can be helpful to note that the first letters of “secant” and “cosecant” are inverted in relation to those of “sine” and “cosine”, just as the secant and cosecant are the inverted cosine and sine functions.

The two keys **$\leftrightarrow\theta,r$** and **$\leftrightarrow y,x$** also use trigonometric functions to convert between rectangular and radial coordinates. They can therefore be useful in some trigonometric calculations.

Trigonometric modes

The HP 33S can calculate trigonometric functions in any of these three modes: Degrees, Radians or Gradians.

Practice working problems involving trig functions

Example 1: Select the appropriate angle mode.

Solution: Press the **MODES** key below the screen.

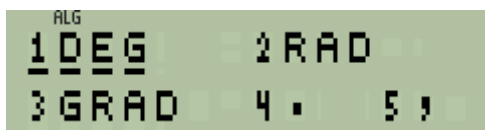


Figure 1

Press **1**, **2** or **3** to select DEGrees, RADians or GRADians mode, or use the arrow keys **←**, **→**, **↑** and **↓** to select the required mode and then press **ENTER**. For example, to select RAD, press **2**.



Figure 2

Answer: The selected trigonometric mode is displayed at the top of the screen if it is RAD or GRAD. If no angle mode is shown, then it is degrees. The **MODES** command works the same way in algebraic and in RPN modes.

There are 360 degrees, or 2π radians in a circle. Gradians mode divides each quarter of a circle into 100 parts, in a sort of decimal system, making 400 gradians in a circle.

Note: It is very easy to forget that one angle mode is set but angles are being entered in a different mode. It is a good policy to make it a habit to check the angle mode before every calculation. The commands DEG, RAD and GRAD can be entered into programs, and it is worth using them to ensure that a program will work as required.

Example 2: What is the sine of $\pi/2$ radians?

Solution: In RPN mode, calculate $\pi/2$, then press **SIN**.

⏎ **π** **2** **÷** **SIN**.

In algebraic mode, calculate $\pi/2$ in parentheses, then press **SIN**.

⏎ **(** **⏎** **π** **÷** **2** **⏎** **)** **SIN**.

Following this with **ENTER** is not necessary, but it completes the calculation by putting an equals sign at the end.

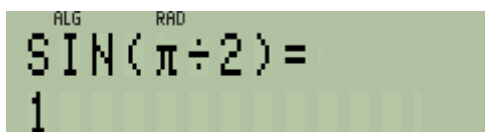


Figure 3

Answer: The sine of $\pi/2$ radians is calculated as exactly 1. Answers will not always be exact as in this case. The HP 33S works with 12 decimal digits, so trigonometric calculations can be expected to be accurate to 12 decimal places. For example the sine of π radians is calculated as 2×10^{-13} , displayed as 2E-13, which is correctly equal to zero to 12 decimal places.

Example 3: Show that the rule $\sin^2(x) + \cos^2(x) = 1$ applies correctly when x is 30° .

Solution: First, remember to set the required angle mode. Press **MODES** **1**.

In algebraic mode, work through the problem by calculating the sine, squaring it, then adding the square of the cosine:

3 **0** **SIN** **x²** **+** **3** **0** **COS** **x²** **ENTER**

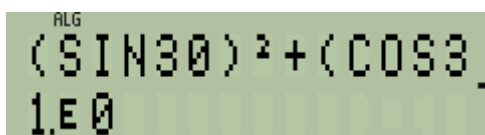


Figure 4

The same can be done in RPN mode, but the HP 33S provides some short cuts. First, calculate the sine and cosine of 30° .

3 **0** **SIN** **↵** **LAST X** **COS**

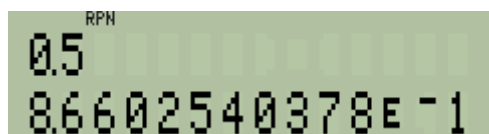


Figure 5

Now the $\boxed{\rightarrow\theta,r}$ key can provide a short cut. $\boxed{\rightarrow\theta,r}$ calculates “r” by squaring the number in the first level of the stack, squaring the number in the second level of the stack, adding them, and taking the square root, all at the pressing of one key. Calculate the sum of the squares of the numbers shown in Figure 5 by pressing:

$\boxed{\leftarrow}$ $\boxed{\rightarrow\theta,r}$ $\boxed{x^2}$



Figure 6

Answer: Both the algebraic and the RPN calculations confirm that the rule $\sin^2(x) + \cos^2(x) = 1$ applies correctly when x is 30° . In the RPN example, a shortcut was used, and another keystroke could have been saved since squaring 1 obviously gives 1.

Example 4: A designer wants to use triangular tiles with sides 3 inches, 5 inches and 7 inches long, to put a mosaic on a floor. What is the angle opposite the 7 inch side? Will it be possible to lay three tiles next to each other with this angle pointing inwards?

Solution: Use the cosine rule to calculate the angle. The cosine rule states that for any triangle with sides a , b and c , and angle A facing side a :

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(A)$$

Figure 7

From this, A can be calculated as:

$$A = \arccos\left(\frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}\right)$$

Figure 8

In RPN mode, the calculation can be done like this:

$\boxed{5}$ $\boxed{x^2}$ $\boxed{3}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{7}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{\div}$ $\boxed{\leftarrow}$ $\boxed{\text{ACOS}}$

In algebraic mode, calculate:

$\boxed{\rightarrow}$ $\boxed{5}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{3}$ $\boxed{x^2}$ $\boxed{-}$ $\boxed{7}$ $\boxed{x^2}$ $\boxed{\rightarrow}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{5}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{\rightarrow}$ $\boxed{\leftarrow}$ $\boxed{\text{ACOS}}$ $\boxed{\text{ENTER}}$

The $\boxed{\text{ENTER}}$ is not necessary, but it completes the calculation and puts an equals sign at the end.

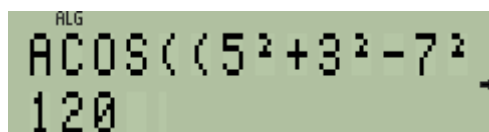


Figure 9

Answer: The angle opposite the 7 inch side is 120 degrees. This means that three tiles will fit together exactly with this angle pointing inwards, as they would make up 360 degrees.

Example 5: A ladder is leaning against a vertical wall. The ladder is 6 meters long and the foot of the ladder is 3 meters from the base of the wall. What is the angle between the top of the ladder and the wall?

Solution: In RPN mode, divide the side opposite the angle by the long side and get the arc sine:

3 **ENTER** **6** **÷** **ASIN**

In algebraic mode calculate 3/6 in parentheses, then the arc sine:

(**3** **÷** **6** **)** **ASIN**. Following this with **ENTER** is not necessary, but it completes the calculation by putting an equals sign at the end.

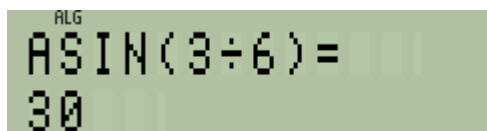


Figure 10

Answer: The ladder is at an angle of 30 degrees from the wall.

Example 6: A vector has components -5 in the X direction and -8 in the Y direction. In what direction does it point?

Solution: It would be possible to divide -5 by -8 and calculate the arc tangent, giving approximately 32 degrees, but this would not specify the quadrant in which the vector lies. Fortunately, the **→θ,r** function provides a complete arc tangent function that recognizes in which quadrant an angle lies.

In RPN mode, specify the Y and X magnitudes, then press **→θ,r** and then use **x↔y** to put the angle in the X register:

5 **+/-** **ENTER** **8** **+/-** **→θ,r** **x↔y**

In algebraic mode type the Y value, press **x↔y**, then type the X value, and finish with **→θ,r**. This displays the vector magnitude; to see the vector angle, press the down-arrow key **↓**. If necessary, press **ENTER** only after the required result, magnitude or direction, is displayed, to make that the final answer.

5 **+/-** **x↔y** **8** **+/-** **→θ,r** **↓**

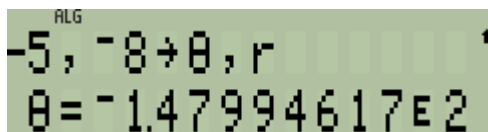


Figure 11

Answer: The vector direction is very nearly -148 degrees.

Example 7: A program is being written to automate calculations with vectors. The program needs to know whether the Y component of directions in calculations such as the above is in the +Y or the -Y direction. How can the direction be obtained?

Solution: The SIGN function gives the sign of a number, +1 or -1. Thus it is enough to obtain the sign of the angle calculated in the previous example and to check whether it is +1 or -1. Both in RPN mode and in algebraic mode, follow the above calculation with **SGN**.



Figure 12

Answer: The sign is -1 , so the vector direction is down, not up.