



hp calculators

HP 33S Using the formula solver – part 2

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Overview of the Formula Solver

Given an expression of the form:

$$f(x) = y$$

The HP Solve Application searches for a value of x that gives:

$$f(x) = y = 0$$

A value of x for which this is true is called a **root**, and it provides a **solution** of the equation $f(x) = 0$. The graph in Figure 1 shows this graphically – there is a root at the value of x where $f(x)$ is zero.

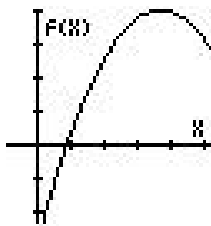


Figure 1

On the HP 33S, $f(x)$ can be typed as a formula, in equation mode, or it can be typed as a program. When the Solver is used to find a root of a formula or equation typed in equation mode, it is referred to as the Formula Solver.

Part 1 of this training aid provided an introduction to the Formula Solver, using a few simple examples. This second part explains how the Solver works, and shows some more examples.

The Formula Solver works with $f(x)$ as a **formula** containing x , for example

$$3x^2 - 3x - 15 \text{ or } 5\sin(x) - 7\log(x)$$

If the Formula Solver is given an **equation** with terms on both sides of the equals sign, such as:

$$3x^2 + 4x = 7x + 15$$

then it begins by moving everything to one side of the equals sign, so the above equation would become the formula:

$$f(x) = 3x^2 - 3x - 15 = 0$$

The Formula Solver ignores the $= 0$ part, as it is trying to find a value for x to make the formula zero. So there is no need to type an equation with $= 0$ in it; it is enough to type the formula.

The variable x in the above is called the **unknown variable**. It can be represented by any of the HP 33S variables A through Z. A formula can contain more than one variable, the Solver will ask which is the unknown variable and will then ask for the **known values** of all the other variables.

The Formula Solver then tries to rearrange the equation $f(x) = 0$ to give a **direct solution** for x . An example will be shown later. (This is different from the predecessor of the HP 33S, the HP32SII, which does not try to find a direct solution.)

If a direct solution is not found, the Solver begins by first trying two guesses for the unknown variable. The user can give one or two guesses for the Solver to start from. The first part showed that this can be very useful, either to direct the Solver **towards** one root of several, or to direct the Solver **away** from values that would cause an error. Good guesses can also speed up the search for a root. In some cases the function varies very slowly over some values, and a good guess is needed to direct the Solver away from them and towards the range of values where a solution is expected.

Beginning from the values obtained for the first two guesses, the Solver searches for values of the unknown variable that make $f(x)$ smaller. If two guesses have the opposite sign, the Solver tries to narrow down the region between them until it finds where the sign changes and $f(x)$ is zero. If two guesses have the same sign, the Solver uses the difference between them to pick the direction in which to change x to look for a third value closer to zero.

The process is repeated until one of the following happens.

A value of x is found for which $f(x)$ is exactly zero.

Two neighboring values of x are found, differing by 1 or 2 in the twelfth significant digit, such that $f(x)$ changes sign between them. An example of this was given in part 1. The two values can be on either side of a root, or on either side of a pole or a discontinuity in $f(x)$. An example of finding a pole is looking for a solution of $\tan(b) = 0$ between 89 degrees and 91 degrees, as in Example 3 below.

No value can be found, but the Solver finds a minimum. An example was given in part 1.

No value can be found because the Solver is looking at values of x for which $f(x)$ is constant.

No value can be found because $f(x)$ is decreasing asymptotically towards a non-zero value.

Two more cases arise because the HP 33S works with a finite range of numbers, negative numbers between -10^{500} and -10^{-499} , 0 and positive numbers between $+10^{-499}$ and $+10^{500}$. This range is sufficient to cover all physical measurements, and even all numbers in government finances, but problems in number theory and in combinatorial operations may require numbers outside this range. (To be exact, the largest absolute value that the HP 33S can work with is 9.999999999999E499.)

No value can be found because the root is at a value of x that is not zero but lies between -10^{-499} and $+10^{-499}$. In such a case the Solver gives a result of 0.

No value can be found because the root is at x that is more negative than, or equal to, -10^{500} or greater than or equal to $+10^{500}$. In such a case the Solver stops with an OVERFLOW error.

To help the user distinguish between the above, the Solver returns the last value it tried for x , the last but one value tried, and the value of $f(x)$ at the last value. Part 1 showed how these values can be seen in RPN mode and in Algebraic mode.

The following examples show some of the features that were not included in part 1.

Practice Example: A formula with several variables

Example 1: A factory is to produce tin cans with a volume of 100 cubic centimeters. The designer estimates that the height should be 10 cm and the radius about 2 cm. Calculate the exact volume of this can, and if it is not close to 100 cubic centimeters then recalculate the radius to give the required volume.

Solution: The equation for a cylinder’s volume V , given its radius r , and height h , is $V = \pi r^2 h$. Enter this as the formula $\pi r^2 h - V$ in equation mode and then use the Solver.

Go to equation mode by typing $\left[\rightarrow \right]$ $\left[\text{EQN} \right]$. If necessary, put the new equation in a particular place in the list of equations by moving up or down through the list with the up and down cursor keys below the HP 33S screen.

Enter the formula by typing:

$\left[\rightarrow \right]$ $\left[\pi \right]$ $\left[\times \right]$ $\left[\text{RCL} \right]$ $\left[R \right]$ $\left[y^x \right]$ $\left[2 \right]$ $\left[\times \right]$ $\left[\text{RCL} \right]$ $\left[H \right]$ $\left[- \right]$ $\left[\text{RCL} \right]$ $\left[V \right]$ $\left[\text{ENTER} \right]$

As was explained in part 1, to enter a variable into an equation, press the $\left[\text{RCL} \right]$ key and then one of the letter keys. As with $\left[\text{STO} \right]$, the symbol A..Z at the top of the screen is shown as a reminder that one of the keys marked A through Z must be pressed. For example press the $\left[6 \right]$ key to enter the variable V.



Figure 2

To solve the equation, press the $\left[\text{SOLVE} \right]$ key. The Solver asks which variable to solve for:

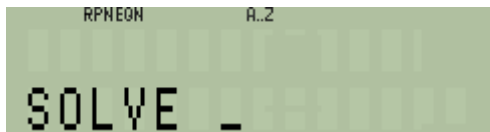


Figure 3

The symbol A..Z is at the top of the screen again. The variable in this formula is V so press $\left[6 \right]$ again. The Solver now knows that V is the unknown variable and it asks for the values of the known variables.



Figure 4

The value that is already stored in R is shown too. If this is the required value then it is enough to press $\left[\text{R/S} \right]$. If the variable R has not been used before, then its value is zero. In this example, type the radius 2 and press $\left[\text{R/S} \right]$.



Figure 5

The Solver asks for the other known variable. Type the height, 10, and press **[R/S]** again. The HP 33S displays SOLVING for a moment, then the result.

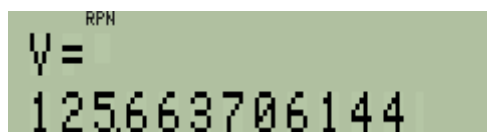


Figure 6

The volume is over 125 cubic centimeters, considerably more than the intended 100. Repeat the calculation, but this time use the known volume of 100, and solve for the radius. Solve the equation again by pressing **[>] [EQN] [SOLVE]**. The Solver asks for the unknown variable, press **[R]**. The Solver then asks for the known variables, first H.



Figure 7

The present value of H is the value previously given. As this is to remain the same, just press **[R/S]** again. The Solver now asks for the other variable, V.

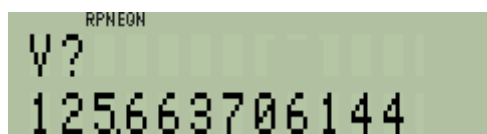


Figure 8

The present value of V is shown; this is the volume just calculated. As the volume should be 100, type 100 and press **[R/S]**. The Solver calculates and displays the radius needed to give the required volume.

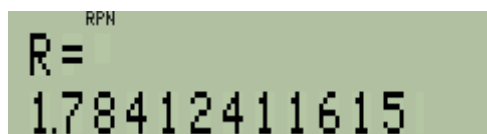


Figure 9

Answer: The cans should have a radius of 1.78 cm.

Practice Example: A direct solution

Example 2: To show that the HP 33S looks for a direct solution before starting to search for a root, try to solve $\ln(z) = 0$ beginning from a negative number for the guess.

Solution: Store -5 in Z. Then store LN(z) as the formula to solve. This means that a solution is wanted for the equation $\ln(z) = 0$.

[5] [+/-] [STO] [Z] [R] [EQN] [LN] [RCL] [Z] [R] [)] [ENTER]

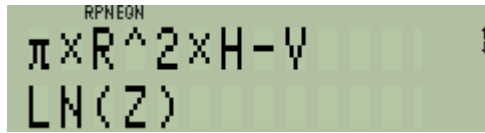


Figure 10

To solve the equation, press **SOLVE** **Z**. The Solver immediately displays the answer:



Figure 11

Answer: $Z = 1$ is the solution to $\text{LN}(z) = 0$. This is obvious, the point of this example is that the answer was found immediately, and the negative guess was not tried. If the negative guess had been tried, it would have caused a LOG(NEG) error, as in Example 3 of part 1. The Formula Solver recognized that Z appears only once in the formula, and that $\text{LN}(Z) = 0$ can therefore be rewritten as $Z = \exp(0)$ to solve for Z directly. Such direct solutions can speed up the use of Solver, specially when a complicated formula with several variables is being solved several times for different variables.

Note: Where more than one solution is possible, for example $\text{ASIN}(Y)=0$, the direct solution is the “principal” value. For example, for $\text{ASIN}(Y)=0$, this is 0 degrees, not 180 degrees, or -180 degrees, or any other possible value. In the same way, an equation such as $X^2=4$ is solved directly and returns the positive root 2. To find other roots, it is necessary to write the expression in such a way that the Solver does not find a direct solution. An easy way to achieve this is to add $0 \times$ the unknown variable into an expression, for example $\text{ASIN}(Y) + 0 \times Y = 0$ or $X^2 + 0 \times X = 0$. This is because the Solver stops looking for a direct solution as soon as it sees the unknown variable more than once in an expression.

Practice Example: a Formula with a Pole

The Solver generally looks for a root such as that shown in Figure 1, where negative and positive values converge towards zero. Just the same, it can also find a pole, where negative and positive values diverge at the point where the function sign changes.

Example 3: Try finding a root of the tan function, shown in Figure 12. This has a pole at 90 degrees, where the value changes from very large positive to very large negative numbers. Although the value goes from a large positive number to a large negative number, it is not zero at 90 degrees.

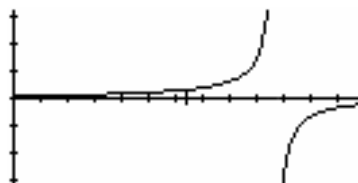


Figure 12

Solution: Store a guess of 80 degrees in X and use 100 as the second guess. Then select equation mode and enter the new equation $\tan(x)$, and solve for the variable X .

8 0 **STO** **X** **1 0 0** **EQN** **TAN** **RCL** **X** **1** **ENTER**

To solve the equation, press **SOLVE** **X**. The Solver correctly states that no root was found.



Figure 13

Press the back-arrow key \leftarrow to remove the message. In RPN mode, the HP 33S will display the last two values tried for X. In Algebraic mode, press \rightarrow to see a menu, as described in part 1.

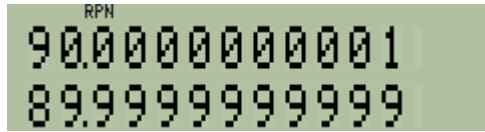


Figure 14

The two numbers are very close together, so the Solver did find a change of sign or a minimum between them. Note that the Solver recognized that exactly 90 is an unacceptable value and returned the two closest values on either side. In RPN mode, press \rightarrow to see the last value of the function. In algebraic mode, use the right arrow key to see the value in menu item x3.

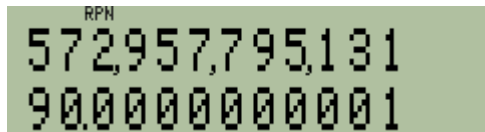


Figure 14

Answer: The last value calculated for the function was a very large number. This means that the Solver did not find a root, but did find a pole at 90 degrees. Note that the last number tried and the one before are on either side of 90, not exactly on 90, which the Solver rejects because it would give an infinite value. Starting instead with a guess close to zero would have found a root at 0 degrees.

Practice Example: Where two functions intersect

The Formula Solver can also be used to solve problems of the form:

$$g(x) = h(x)$$

This requires a value of **x** at which one function **g(x)** is equal to another function **h(x)**. In other words, the problem is to find **x** at which these functions intersect.

The equation can be rewritten as:

$$f(x) = g(x) - h(x) = 0$$

Solving the formula $g(x) - h(x)$ will give the value of x at which the two functions cross over.

Example 4: The factory from Example 1 is interested in designing spherical containers with the same volume and the same radius as their tin cans. This means that they want to find a radius r such that:

$$V = \pi r^2 h = 4/3 \pi r^3$$

Solution: Modify the formula from Example 1 to find r such that $\pi r^2 h - 4/3 \pi r^3$ is zero.

A quick look at this expression shows that one solution is $r = 0$. This is not a useful solution so it would be helpful to provide a guess to direct the Solver away from zero. One guess is the value already in R. Type 8 on the lower line of the display as the second guess.

Go to equation mode by typing EQN . Find the old equation by moving up or down through the equation list with the \uparrow and \downarrow keys below the HP 33S screen.

Begin editing the formula by pressing the back-arrow key \leftarrow . The black editing cursor appears at the end of the formula.

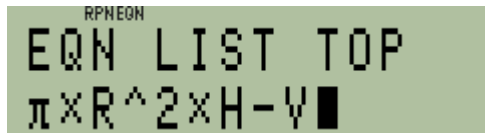


Figure 15

Press \leftarrow again to delete the V. Then type the formula for the volume of a sphere.

$4 \div 3 \times \pi \times R^3$ \leftarrow RCL R \rightarrow ENTER

Press the right-arrow \rightarrow key a few times to see the last part of the changed formula. It should look as in Figure 16.



Figure 16

To solve the equation, press the SOLVE key. The Solver asks which variable to solve for: The unknown variable is R so press R . The Solver now asks for the value of the known variable H.



Figure 17

This value is to stay unchanged, so press R/S . The Solver looks for a solution.



Figure 18

Answer: If both the radius of the sphere and that of the base of the cylindrical can are 7.5 then the sphere and the can will have the same volume. The number in the twelfth digit of the answer will depend on the guesses used.

The Solver can be used for many kinds of problems, further information is given in the HP 33S manual, and a detailed description of the Solver is provided in Appendix D of the manual.