



hp calculators

HP 33S Using the formula solver – part 1

What is a solution?

Practice Example: Finding roots of polynomials

Practice Example: Finding the root of a log equation

Practice Example: Where there is no solution

What the solver can and can not find



What is a solution?

Given a formula or equation such as:

$$7x + 12 = 33$$

it is easy to find that:

$$x = 3$$

This is the **solution** or root of the formula. It answers the question “What value of x makes the formula correct?”

It is rarely this easy to solve a formula or equation. For example look at this one:

$$4x^2 - 14\text{LOG}(x) = 33$$

It seems likely that there is a value of x for which the formula is true, but there is no quick and easy way to find what this value is. A special method to find the solution is needed.

The HP 33S has a Formula Solver built into it. This can be used to solve problems like the example above, and many others. This practice aid will show some simple applications. Part 2 will show further details.

The Formula Solver is one part of the HP Solve application. Other practice aids show other uses of the HP Solver, for example its use to solve a formula typed as a program.




Practice Example: Finding roots of polynomials



Example 1: Solve the polynomial equation $x^3 + 15x^2 + 47x = -33$

Solution: The HP 33S manual gives a program for finding the roots of polynomials up to the fifth order, but it is often simpler to use the formula solver. The following steps show how this can be done.

Before a solution is looked for, it is useful to store a guess in the variable to be used. In this case, the variable will be “X”, and to store a zero in it, the following leys are pressed:

When  is pressed, the symbol A..Z is shown at the top of the screen as a signal that one of the keys marked A through Z at their bottom right must be pressed. To enter the variable X press the  key, which has a small “X” at its lower right to show that it is also the  key.

Next, the equation must be entered. This is done in the HP 33S “Equation mode.” Go to equation mode by typing  . Equation mode stores a list of equations and a new equation can be typed anywhere in this list. If necessary it is possible to put the new equation in a particular place by moving up or down through the list with the up and down cursor keys below the HP 33S screen.

Enter the equation by typing:

RCL **X** **y^x** **3** **+** **1** **5** **x** **RCL** **X** **y^x** **2** **+** **4** **7** **x** **RCL** **X** **↵** **=** **↵** **3** **3** **ENTER**

To enter a variable into an equation, press the **RCL** key and then one of the letter keys. As with **STO**, the symbol A..Z at the top of the screen is shown as a signal that one of the keys marked A through Z must be pressed. To enter the variable X in this formula press the **2** key again.

To solve the equation, press the **SOLVE** key. This is the lower left key below the display. The HP 33S asks which variable to solve for:



Figure 1

The symbol A..Z is at the top of the screen yet again. The variable in this formula is X so press the **2** key again to fill in X.



Figure 2

The word “SOLVING” is shown while a solution is looked for. If the search lasts more than a short time, the busy indicator **B** is shown as well. When a solution is found, it is displayed. It is also stored in the variable.



Figure 3

Answer: X = -1 is a solution of the equation.

The equation is a cubic, so there are two other solutions. If all three solutions are real numbers then the solution found depends on the initial guess given. Try finding a different solution.

Example 2: Solve the same polynomial equation with a guess of -50.

Solution: Store the new guess in the variable:

5 **0** **↵** **STO** **X**

Enter equation mode again by typing **↵** **EQN**. The equation is shown again. As equation mode stores a list of equations, it is possible to select a different equation to solve by moving up or down through the list with the up and down cursor keys below the screen. In this case the same equation is to be solved.



Figure 4

To solve the equation, press **[SOLVE]**, then press **[2]** again to show that the formula is to be solved for X.



Figure 5

Answer: X = -11 is a second solution of the equation. There is a third solution, or root, which can be found if a suitable guess is tried.

Practice Example: Finding the Root of a Log Equation

Example 3: Polynomial equations of order up to 5 can also be solved using the polynomial solver program in the HP 33S manual, but many equations can only be solved using the function solver. Try solving the equation given at the beginning: $4x^2 - 14 \text{ LOG}(x) = 33$

Solution: Enter the equation as before. Go to equation mode by typing **[EQN]**. Type the new equation immediately below the previous one, or move up or down through the list with the up and down cursor keys below the screen to place the new equation somewhere else in the list.

Enter the equation by typing:

[4] [X] [x²] [RCL] [X] [EQN] [] [-] [1] [4] [X] [EQN] [LOG] [RCL] [X] [EQN] [] [EQN] [] [3] [3] [ENTER]

This time, x^2 has been typed by use of the **[x²]** key. Note that the function name is shown as SQ in equation mode:

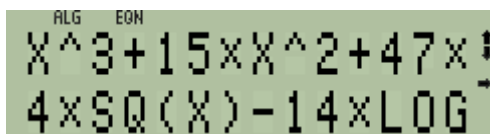


Figure 6

To solve the equation, press the **[SOLVE]** key. Then press **[X]** to show that this is the variable for which a solution is desired. A different variable name could have been used, but the same variable can be used in different equations.

The word “SOLVING” is shown as before. This time an error message follows. Although the solver knows which equation is being solved, it has started the search from the previous value stored in X, and that was -11, or one of the other negative roots of the polynomial in examples 1 and 2.



Figure 7

This shows that it can be very important to begin with a good guess. Obviously, only positive numbers should be used in the search for a solution this time. To make sure that only positive numbers are used in the search, two guesses can be given. One is the number in the variable being solved for. The other is the number displayed in the lower line of the screen. These can be the same, in which case the HP 33S generates its own second guess. In this example, try using 1 and 10 as the guesses: Press the cancel key C key at the bottom left of the keyboard twice, to cancel the error message, and to leave equation mode. Then enter the two guesses, and solve again.

C C 1 STO X \leftarrow 10 X \rightarrow EQN SOLVE X

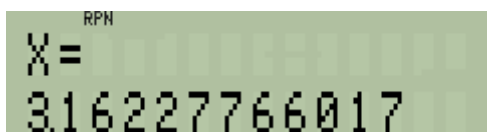


Figure 8

Answer: Unlike the results of examples 1 and 2, the answer is not a whole number. To 12 significant figures it is 3.16227766017. The true answer is $\sqrt{10}$, which is an irrational number and can not be displayed exactly on a calculator.

The Solver provides additional information about the solution it has found. If the example was tried in RPN mode, press the back-arrow key \leftarrow to remove the text "X=" from the upper line.

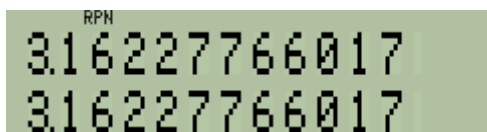


Figure 9

In RPN mode the lower line is stack register X and shows the best value that the solver could find. The upper line, stack register Y, shows the previous value tried. If two are the same then this is an exact solution. Register Z shows the value of the formula using the best answer. Press $\text{R}\downarrow$ to see the value in register Z.

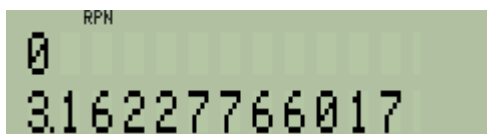


Figure 10

The number in Z is now on the upper line. It is zero, which means that 3.16227766017 is an exact solution to the 12 digit precision of the HP 33S.

The Solver works in the same way in Algebraic mode, but to see the previous value tried and the best answer, press the $\text{R}\downarrow$ key to see a menu.

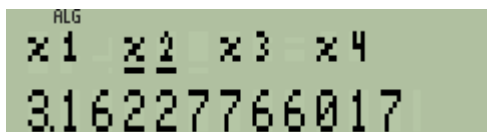


Figure 11

The number in x1 is the best answer, the number in x2, shown in Figure 11, is the previous value, and the number in x3 is the value of the formula. Use the left and right arrow keys to see each of these numbers, and press **ENTER** to copy the number shown into a calculation.

If the numbers in stack registers X and Y, or in menu values x1 and x2, differ by 1 in the last digit then there is no solution exactly correct to 12 digits, and the two values are on either side of the exact answer. This is confirmed if the value in stack register Z, or in menu value x3, is very small. Press **C** to cancel the menu.

If an error condition has occurred, such as the log of a negative number, and the Solver has not yet calculated $f(x)$ at two values of x , then the three values will not have been put on the stack or in the menu.

Practice Example: Where there is no solution

Example 4: Sometimes a formula or equation has no exact or approximate solution. For example $a^2 = -4$ clearly has two complex roots $a = (0,2)$ and $(0,-2)$, but no real root that the solver can find. Try solving this to see how the formula solver handles such cases.

The formula solver always begins by moving everything from the right of the equals sign to the left side, so the above equation would become $a^2 + 4 = 0$. Then it looks for a value of the variable to make the left hand side equal to zero. If the formula to be solved already has zero to the right of the equals sign, then there is no need to include “= 0”, only the formula to be solved needs to be typed.

Solution: Go to equation mode and enter the formula $A^2 + 4$

→ **EQN** **RCL** **A** **y^x** **2** **+** **4** **ENTER**

The calculator display shows 'RPN EQN' at the top. The main display area contains the formula '4xSQ(X)-14xLOG' on the top line and 'A^2+4' on the bottom line.

Figure 12

To solve the formula, press **SOLVE** and **A**. **A** is on the **e^x** key.

The word “SOLVING” is shown as before. There is no root, so the search can take some time. To interrupt a long-lasting search, press the cancel key **C**. If the search is not interrupted, it will finally display:

The calculator display shows 'RPN' at the top. The main display area contains the message 'NO ROOT FND'.

Figure 13

What did the solver find? In RPN mode press **←** to cancel the error message and see:

The calculator display shows 'RPN' at the top. The main display area contains two lines of text, both showing the value '-1.0259003487E-8'.

Figure 14

The HP 33S displays two small numbers very close to each other. The value of the numbers might vary, depending on the guess or guesses for A. Press $\boxed{\text{RPN}} \boxed{\text{SHOW}}$ to see all the digits of the last value tried, then $\boxed{\text{X} \leftrightarrow \text{Y}} \boxed{\text{RPN}} \boxed{\text{SHOW}}$ to see the previous value tried by the solver. They are very close to each other, but not the same. Press $\boxed{\text{RT}}$ to see the value of the formula $A^2 + 4$ at this value of A. In algebraic mode, use $\boxed{\text{RT}}$ to see the menu of these values, as in example 3.



Figure 15

4 is as close as the solver could get to 0 using this formula. Squaring and adding any number much smaller than 10^{-7} makes no difference to a precision of 15 digits used internally by the solver, so it can spend a long time looking for the best value before it decides there is no solution.

Answer: Although the formula solver has found no solution, it has located a minimum of the formula, a point closer to zero than any other point near it. This can be useful, as will be mentioned in the second part of this training aid.

What the Solver can and can not find

The examples above have shown the basics of what the formula solver can find and what it can not find.

In a formula or equation with one unknown variable the solver can find one or more roots if there are any.

If the solution can not be represented exactly, the solver finds the two nearest numbers on either side of it.

The solver can not find a solution if two or more variables are unknown.

The solver can not find complex roots, as these have two unknown variables, the real and imaginary parts. Note that the HP 33S manual has a polynomial root finder program that will find complex roots.

The solver can not find roots of matrix equations.

Note that the HP 33S manual has a matrix program for solving three simultaneous equations.

The solver can not find a root if there is no root, but in this case it can find a minimum.

If there is a solution that is not zero but is less than 10^{-499} the solver returns zero.

If there is a solution that is greater than 10^{499} the solver gives an OVERFLOW error.

If an error condition occurs in a calculation, for example the log of a negative number, the solver stops.

In addition there are some special cases that are explained in the second part of this aid.

The solver has many more features. The second part of this training aid will describe some of them.