hp calculators

HP 33S Using RPN and Algebraic Modes

Calculation modes

A simple example in RPN

Functions of a single number in RPN

A simple example in algebraic

Functions of a single number in algebraic

Arithmetic calculations with two numbers

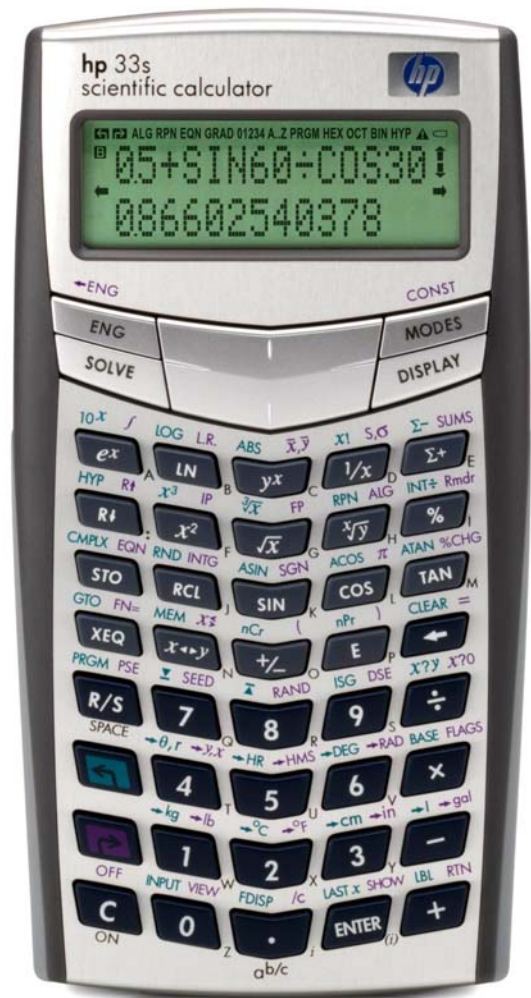Another example - the area of a piece of carpet

RPN mode in detail

Algebraic mode in detail

Making corrections

Functions of two numbers

Algebraic operator precedence

Example – which stepladder?

HP 33S Using RPN and Algebraic Mode

Calculation modes

The HP 33S allows calculations to be made in "RPN" mode, in "algebraic" mode, or in "Equation" mode.

RPN mode is the traditional way most HP calculators work. To add 1 and 2, keys are pressed in the order
[1] [ENTER] [2] [+]. This mode is very suitable for calculations where the user is working towards a solution, without having a particular formula to work on.

Algebraic mode is the way many newer HP calculators work. It is also the way most other calculators work. To add 1 and 2, keys are pressed in the order [1] [+] [2] [ENTER] as shown in Figure 1. This way of working is most suitable when the user just needs to type a formula and get an answer.

Figure 1

In Equation mode, the user types a formula or an expression the way it looks in a textbook and saves it in a list of equations. Then the equation can be use once or repeatedly, for different values of variables. Equations can also be used with the integration command and the solver command.

For complicated tasks that need to be repeated, writing a program is the best solution. Programs can be written in RPN mode or in algebraic mode, and can include equations.

Equation mode and programs are described in other training aids. To work out the value of an expression it is simpler just to type it using algebraic mode or RPN mode, and get the answer, as shown by the examples in this training aid.

Note: When doing the examples press [←] [RPN] to set RPN mode, or [→] [ALG] to clear RPN mode and set algebraic mode. The selected mode is shown at the top of the calculator screen, as in Figure 1.

Functions of a single number in RPN

In RPN mode, to square a number, the number is typed, then the [x²] key is pressed. If a second number is to be squared, the steps are repeated.

A simple example in RPN

Example 1: A gardener wants to plant a triangular piece of ground and to put protective edges around the plot. The plot will have a right angle and the two shorter edges will be 1.2m long and 0.5m long. How long will the third edge be?

Solution: The length can be calculated using Pythagoras' formula. From $a^2 + b^2 = c^2$ the third side is:

$$c = \sqrt{(a^2 + b^2)}$$

To calculate this, it is first necessary to square the numbers 1.2 and 0.5. On a calculator, the square of a number is a function of a single number, it does not involve adding or multiplying two numbers.

To square 1.2 and 0.5, it is enough to type:

$\boxed{1}\;\boxed{\cdot}\;\boxed{2}\;\boxed{x^2}\;\boxed{\cdot}\;\boxed{5}\;\boxed{x^2}$

RPN
1.44
0.25

Figure 2

Answer: The second result, 0.25, is on the lower line of the calculator screen, called the X register. The previous result, 1.2 squared, is on the upper line, called the Y register. Registers are places where numbers are held in the calculator; X and Y are the first two registers in the RPN stack. The stack is the most important tool in RPN calculations. Its use is shown later in this example.

Other functions of one number, for example sine, log or square root, work the same way. The number is typed and shows up in the lower line, in the X register. Then the function key, for example $\boxed{\text{SIN}}$, $\boxed{\text{LOG}}$ or $\boxed{\sqrt{x}}$ is pressed to give the answer. The square and square root keys are labeled with an "x" as a reminder that they work on the number in the X register.

Functions of a single number in algebraic

In algebraic mode, to square a number, the number is typed, then the $\boxed{x^2}$ key is pressed. If a second number is to be squared, the steps are repeated.

A simple example in algebraic

Example 2:   A gardener wants to plant a triangular piece of ground and to put protective edges around the plot. The plot will have a right angle and the two shorter edges will be 1.2m long and 0.5m long. How long will the third edge be?

Solution: The length can be calculated using Pythagoras' formula. From $a^2 + b^2 = c^2$ the third side is:

$$c = \sqrt{(a^2 + b^2)}$$

To try this, first set algebraic mode, pressing $\boxed{\text{ }}$ $\boxed{\text{ALG}}$. Now to square 1.2 and 0.5, type:

$\boxed{1}\;\boxed{\cdot}\;\boxed{2}\;\boxed{x^2}\;\boxed{\cdot}\;\boxed{5}\;\boxed{x^2}$

ALG
0.5²
0.25

Figure 3

Answer: The second result, 0.25, is on the lower line of the calculator screen. In algebraic mode, the formula or expression used to calculate this result is shown on the upper line. The previous result, 1.2 squared, has vanished, though it can still be used, as will be shown.

Functions of one number are calculated the same way in algebraic mode as in RPN mode. First the number is typed and it shows up on the lower line, then the function key is pressed and the result is seen on the lower line. In algebraic mode, the formula used to calculate the result is seen on the upper line.

Note: When functions are printed some come before the number, for example $\sqrt{2}$ or sin(30). Others are printed with the function after the number, examples are 6! or 5^2. Calculators rarely work like this; on many algebraic calculators, all functions of one number are calculated by typing the function first, followed by the number, usually in parentheses (brackets). On the HP 33S in algebraic mode the function key follows the number. This is often quicker because it saves the need for typing parentheses or pressing an extra key to finish the calculation.

Arithmetic calculations with two numbers

Now that the lengths of the two sides have been squared, it is necessary to add the two squares. To see how this is done in RPN mode, press [◄] [RPN] to set RPN mode. The last result calculated in algebraic mode is left in the X register, but the rest of the RPN stack is cleared. So it is necessary to calculate the square of 1.2 again.

[1] [·] [2] [x²]

RPN
0.25
1.44

Figure 4

The numbers in the X register and the Y register are now the other way round compared to Figure 2. This does not matter if they are added, but the order matters if they need to be subtracted or divided. Pressing [x↔y] exchanges the numbers in the X and Y registers.

To add the two numbers, it is enough to press the add key.

[+]

RPN
0
1.69

Figure 5

In RPN mode addition takes the number from the X register, uses it, and puts the result in the X register, so the result is seen on the lower line. This is like a function of one number, but because addition uses two numbers, it takes the second number from the Y register. The number in the Y register is removed. There is a third stack register above Y, called the Z register, and the number in the Z register moves down to the Y register.

Subtraction, multiplication and division work the same way. The number in X is subtracted from, multiplied by, or divided into the number in Y. If the two numbers are in the wrong order, [x↔y] can be used to exchange them.

In algebraic mode, addition works differently. First press [◄] [ALG] to return to algebraic mode again. The number in the X register of the RPN stack is left in the lower line of the screen, ready for use in algebraic mode. This can be helpful to users who change between RPN and algebraic modes often. For this example, it is necessary to return to the numbers in Figure 3, by pressing the same keys again:

1 . 2 x² . 5 x²

To add two numbers in algebraic mode, it is necessary to type or calculate the first number, press +, get the second number and press ENTER to finish the calculation.

Note: On some Algebraic calculators, a key marked = finishes a calculation, but on the HP 33S, ENTER is used. The = key is used only when an equation is being typed in Equation mode.

In this example, the two numbers have been calculated already. One is shown on the lower line, so it is enough to press + to add it. The other number is the result of the previous calculation and pressing ← LASTx will bring it back. Then ENTER must be pressed to finish the addition.
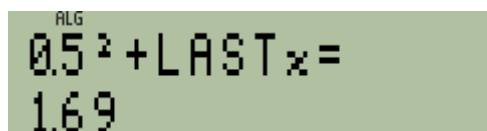
+ ← LASTx ENTER

```
ALG
0.5²+LASTx=
1.69
```
Figure 6

Both in Algebraic and in RPN mode the result of the addition, 1.69, is shown on the lower line. It is enough to press the square root key to get the final answer. As before, this is a function of a single number, so it is enough to press the √x function key. In algebraic mode, the ENTER key can be pressed to finish the calculation, though it is not needed with a function of a single number.

√x ENTER

```
ALG
√1.69=
1.3
```
Figure 7

Answer: The length of the third edge of the garden plot is 1.3m.

Another example - the area of a piece of carpet

Here is another example to show how arithmetical calculations involving addition and multiplication are carried out using parentheses in algebraic mode and the stack in RPN mode.

Example 3:   A new carpet is needed for two rooms, one 6 yards long, one 8 yards long, both 5 yards wide. What is the total area of carpet to be bought?

Solution: The area is calculated using the expression $(6 + 8) \times 5$.

In algebraic mode the total length can be calculated first, 6 yards + 8 yards. Then the length is multiplied by the width, 5 yards. So these keys would be typed:

↱	(		This opens the parentheses to put around (6+8)
6	+	8	This gives the 6+8
↱	)		This closes the parentheses

⊠   5   This multiplies by 5
ENTER   This tells the calculator that the formula is finished so it can now get
          the answer

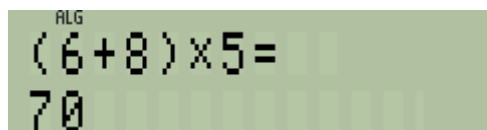and the formula and answer would be shown

```
 ALG
(6+8)×5=
70
```
Figure 8

In RPN the total length can be calculated first too. 6 is typed and put into the Y register, then 8 is typed and added. Then 5 is typed and multiplied:

6   ENTER   This types 6, ENTER separates it from the next number and copies it to Y
8   ⊞   This puts 8 in the X register and adds it to the 6 in the Y register
5   ⊠   This gives the 5 and multiplies by it

The answer is shown like this:

```
      RPN
0
70
```
Figure 9

Algebraic mode is simple and follows the formula, and it takes 10 keys.

RPN mode is simple too, and often uses fewer keys, only 6 in this example.

Answer: The total area of carpet is 70 square yards.

In general, algebraic mode is better when working out known formulas, and if it is enough to type them and get the results.

RPN mode is best for users going through calculations a step at a time, and sometimes even changing their minds during the calculation.

To see more detail of how RPN and algebraic modes work, the example above can be repeated, a step at a time. Press DISPLAY 1 then 2 to set FIX 2 mode, in which two digits will appear after the decimal point, as below. This is especially useful when calculations are made with prices in dollars and cents.

RPN mode in detail

First try the example in RPN mode. Press ◢ RPN to switch to RPN mode, if it is not already set. Now press the keys.

6   ENTER   This puts the 6 in the calculator, ENTER separates it from the next number
          The number 6 is in X and in Y after ENTER is pressed

RPN
6.00
6.00

Figure 10

8   This puts 8 in X, a new number replaces the previous one after ENTER is pressed

RPN
6.00
8_

Figure 11

+   This adds the 8 to the 6

RPN
0.00
14.00

Figure 12

5   This puts 5 in X. After a calculation, a new number pushes the old one into Y.

RPN
14.00
5_

Figure 13

×   This multiplies the 14 by the 5 and leaves the result in X

RPN
0.00
70.00

Figure 14

There is a clear pattern here. A number is typed, then something is done with that number. Then another number is typed, and something is done again. In this example, a new number is added to a number that was typed before, or multiplied by it, but RPN works the same way for other actions too.

The RPN stack: At each step, RPN takes one or more numbers it needs from the stack of numbers, for example 6 and 8 above to add them. Then it puts the answer on the stack, ready for use at the next step. The stack holds 4 numbers, in registers called X, Y, Z and T. X is the number most recently typed or calculated and is shown on the lower line of the screen. Y is the number typed or calculated before X, and is shown on the upper line. Numbers in Z and T are not shown but are ready to be used if necessary. When a calculation uses the numbers in both the X and the Y registers, and puts the result in X, then the number in Z is copied into Y. The number in T is copied to Z, but also stays in T. The LASTx key fetches a copy of the last number that was in X before the most recent calculation, so it can be used again or to correct mistakes. Two other keys are used with the RPN stack. R↓ is called "roll down" and moves the number in Y down into X, Z to Y, T to Z and X to T. R↑ is called "roll up" and moves the stack in the opposite direction, X up to Y and so on.

The ENTER key has a special task in RPN. It copies into register Y the number typed or calculated in register X. This is useful if two numbers are being put on the stack, for example to add 1 and 2 1 ENTER 2 + is typed, and ENTER separates the 1 from the 2 so they are treated as two separate numbers, not as the one number 12. The number in Y is pushed into Z, the number in Z is pushed into T, and the number previously in T is lost. After ENTER is typed, another number immediately typed into the X register or recalled into it with the RCL function (described in a separate training aid) replaces the number in X. This allows ENTER to separate two numbers, and the second number goes in the X register.

As ENTER copies the number from X to Y, it can also be used to make a copy for other purposes, such as doubling a number by means of ENTER +. Pressing ENTER repeatedly pushes the number in X into the Z and then the T registers.

Why "RPN?" You might have noticed that calculating the carpet area in RPN did not use brackets, and RPN is actually designed to work without brackets. This often makes it a little quicker to use than algebraic mode. Mathematical notation without brackets was introduced by the Polish mathematician Jan Lukasiewicz in the 1920s. Since the operations such as +, − or × are done *after* the number is entered, this method of calculation is called Reverse Polish Notation, or RPN. The numbers it uses are on the stack, not inside brackets, so it is also called "Stack notation".

Algebraic mode in detail

To compare the above to algebraic mode, press ▶ ALG to switch to algebraic mode, if it is not already set. Now pressing the following keys gives the results shown.

▶   (This opens the brackets to put around (6+8)
6   +   8   This gives the 6+8 and the calculator screen looks as shown in Figure 15
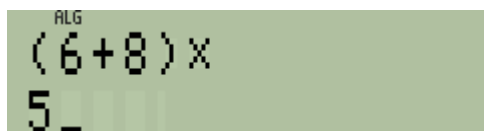
Figure 15

▶   )   This closes the brackets and the intermediate value is calculated as in Figure 16

Figure 16

×   5   This prepares the calculator to multiply by 5, as in Figure 17

Figure 17

ENTER   This tells the calculator to finish the calculation and show the result as in Figure 18

Figure 18

The whole expression (6+8)×5 was typed and then ENTER was pressed to get the answer. The HP 33S helped by calculating intermediate values on the lower line, and putting the expression typed so far on the upper line.

To continue a calculation after pressing the ENTER it is enough to press another calculation key. For example if the carpet costs $17.32 per square yard, it is easy to continue the calculation and get the total price of the carpet. First press ×. Then finish the calculation by pressing 1 7 · 3 2 ENTER, as below.

Figure 19

The total price is $1,212.40.

To begin a new calculation, the first number in that calculation is typed. The LASTx key brings the answer from the last calculation into the new calculation. For example another way to calculate the price of the carpet would be to type 1 7 · 3 2 × ⇦ LASTx.

Figure 20

Why "Algebraic"? You can see that the formula to calculate was typed and is displayed in the way it looks when it is written down on paper in algebraic notation. That is why this calculation mode is called Algebraic Mode. Algebraic notation was developed over centuries as a shorthand way of writing things such as "add the number eight to the number six and multiply the result by the number five".

Making corrections

In both modes, digits in a number that is being typed can be deleted with the ⇦ key. If a number has been completed or calculated, this key deletes it completely.

In RPN mode, LASTx brings back the number that was in X before the most recent calculation. This can be used to correct a calculation. For example if a wrong number was added, then LASTx − corrects the error. If the number was to be used for multiplication instead, pressing LASTx again, and then × will give the correct answer.

In algebraic mode, addition, subtraction, multiplication and division are not carried out immediately. If a wrong key was pressed, + and − should be followed by a 0 and × or ÷ should be followed by a 1.

If two numbers are entered in the wrong order, x↔y can be used to change the order both in RPN and in algebraic modes. For example, if 2 ÷ 7 is required in algebraic mode, but 7 ÷ 2 has been typed by mistake, it is enough to press x↔y before ENTER and the correct answer will be obtained.

If a calculation has gone completely wrong in RPN, it is enough to start over again. Any numbers left on the stack from previous calculations can be ignored. In Algebraic, the ☐C☐ key clears a calculation that has gone wrong.

Functions of two numbers

Unlike functions of a single number, the arithmetical operations ☐+☐, ☐−☐, ☐×☐ and ☐÷☐ use two numbers, but there are also some mathematical functions that use two numbers.

An often-used two-number function is the exponentiation or "power" function, and this works in the same way as ☐+☐, ☐−☐, ☐×☐ and ☐÷☐ both in RPN and in algebraic mode.

Example 4:   What is 2 to the power 10?

Solution: The calculation is 2^{10} using the ☐y^x☐ key.

In RPN mode, put 2 in the Y register, put 10 in the X register, then press ☐y^x☐

☐2☐ ☐ENTER☐ ☐1☐ ☐0☐ ☐y^x☐

In algebraic mode, type 2, press ☐y^x☐, type 10, and press ☐ENTER☐ to complete the calculation.
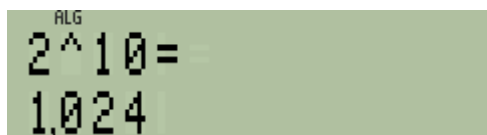
☐2☐ ☐y^x☐ ☐1☐ ☐0☐ ☐ENTER☐

```
ALG
2^10=
1,024
```

Figure 21

Answer: Two to the power 10 is 1,024, often called 1k in computing.

☐y^x☐ works exactly like the arithmetical operators and is often considered to be one of them. Most other two-number functions work this way too. These are the root (or involution) function ☐$\sqrt[x]{y}$☐, quotient ☐INT÷☐ and remainder ☐Rmdr☐, combinations ☐nCr☐ and permutations ☐nPr☐.

Percentage functions are used in algebraic mode in combination with ☐+☐, ☐−☐ and ☐×☐ as is explained in the business training guide on Percentages.

Working with complex numbers, with pairs of numbers representing rectangular or polar coordinates and with pairs of numbers in statistical operations are described in training aids on these subjects.

Algebraic operator precedence

In algebraic mode the HP 33S calculates using "operator precedence". This means that a combination of several ☐+☐ and ☐−☐ operations (or several ☐×☐ and ☐÷☐ operations) is calculated from left to right, but ☐×☐ and ☐÷☐ have a higher "precedence" and are carried out before ☐+☐ and ☐−☐.

Example 5:   What is the result of calculating $1 \div 2 \div 3$ in algebraic mode?
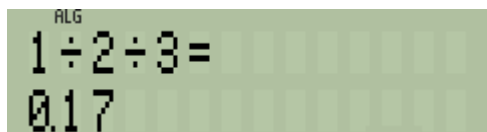
Solution: Type the keys ☐1☐ ☐÷☐ ☐2☐ ☐÷☐ ☐3☐ ☐ENTER☐ .

Figure 22

Answer: The result is equal to 1 ÷ 6, in other words the calculation goes from left to right, 1 ÷ 2, and the result divided by 3. If the calculation went the other way, it would be 1 ÷ (2 ÷ 3), or 3 ÷ 2, giving 1.5 as the result.

The rules of precedence are that algebraic calculations are carried out in the following order.

1.   Expressions in parentheses.
2.   All single number functions, also complex number functions, percentages, and co-ordinate transformations.
3.   The two-number functions $\boxed{\sqrt[x]{y}}$ and $\boxed{y^x}$.
4.   The other two-number functions, $\boxed{nPr}$, $\boxed{nCr}$ and $\boxed{\%CHG}$.
5.   Multiplication, division and integer quotient and remainder, $\boxed{\times}$, $\boxed{\div}$, $\boxed{INT\div}$ and $\boxed{Rmdr}$.
6.   Addition and subtraction.

Operations with the same level of precedence are carried out from left to right, as Example 5 showed.

Example 6:   In what order is the calculation $1 + (0.5 + 1.5) \times 3^{4!}$ carried out?

Solution: Type the keys $\boxed{1}\boxed{+}\boxed{\rightarrow}\boxed{(}\boxed{0}\boxed{.}\boxed{5}\boxed{+}\boxed{1}\boxed{.}\boxed{5}\boxed{\rightarrow}\boxed{)}\boxed{\times}\boxed{3}\boxed{y^x}\boxed{4}\boxed{\leftarrow}\boxed{x!}\boxed{ENTER}$.
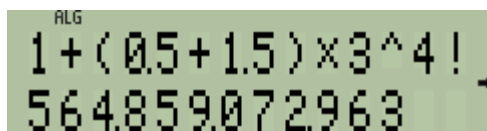

Figure 23

Answer: The calculation in brackets is carried out first, giving the result 2. The rest of the calculation proceeds from right to left, as the factorial function has highest precedence, followed by the power function, followed by multiplication, with addition coming last. The answer 564,859,072,963 can be confirmed if 1 is subtracted first, then the result is divided by 2. That result is equal to 3^{24}, which is 3 to the power 4 factorial.

In RPN mode there are no rules of precedence, each operation is carried out when its key is pressed.

In algebraic mode there is no stack and the keys $\boxed{ENTER}$, $\boxed{x \leftrightarrow y}$ and $\boxed{LAST x}$ work as described earlier. The commands $\boxed{R\downarrow}$ and $\boxed{R\uparrow}$ provide a menu of four numbers produced by the most recent equation, program or integration. The arrow keys can be used to see each number, and pressing 1, 2, 3 or 4 copies the corresponding number to a calculation.

Example – which stepladder?

Some people will wonder why RPN is worth using if algebraic mode and Equation mode work the way expressions are printed. Here is an example to show how RPN mode is useful and fast in solving step-by-step problems where there is no formula, and where the next calculation depends on the result of the previous one.

Example 7:   You need to fix a tile that has fallen off your roof. The roof is 28 feet up, and you have a stepladder that is 29 feet long. You could also borrow your neighbor's 38 foot ladder. Is either ladder good for the job? You

could try leaning each one in turn against the roof and seeing which is better, but it's raining, so why not work it out first on your HP 33S by seeing what angle each ladder will make with the vertical when you lean it against the roof?

Solution: First try it for your own ladder. Switch to RPN mode if it is not set. Enter the ladder length:

[2] [9] [ENTER]

Then enter the height and divide so you can get the angle, [2][8][÷]. You get 1.04. That can not be right; sines and cosines should be smaller than 1. Use [◄] [LAST𝑥] [×] [◄] [LAST𝑥] to get the numbers back, then [𝑥◄►𝑦] to swap them. Now press [÷] to divide them again, in the right order, and see 0.97.

Actually, pressing [1/𝑥] would get the same result, making the correction more quickly. Anyway, this is the sine of the angle between the ladder and the vertical. Or is it the cosine? Press [◄] [ASIN] and see the answer, 74.91 degrees. No, that is not right, it must be the arc cosine that you need. You could press [◄] [◄] [LAST𝑥] again to undo the arc sine, but it is as quick to press [SIN], then [◄] [ACOS].

<div align="center">

RPN
0.00
15.09

</div>

Figure 24

Your ladder would be only 15 degrees away from the vertical. That is uncomfortably steep. So try the same calculation for your neighbor's 38 foot ladder.

Type [2][8][ENTER][3][8][÷] then [◄] [ACOS]. This time the answer is 42.54 degrees. The ladder would be at a rather shallow angle and might slip away as you stand on it.

Answer: Neither ladder is really suitable. Maybe you should ask some other neighbors if they would lend you a ladder with a better length. What would be a good length? 30 degrees would probably still be too much, about 20 degrees would be about right. So type the height again and divide by the cosine of 20 degrees. Type [2][8][ENTER][2][0][COS][÷]. That gives 29.80. A 30 foot long ladder would be almost ideal if a neighbor has one.

The same calculations could be done in algebraic mode with no difficulty. Nevertheless, many users find that algebraic mode is less well suited to such step-by-step calculations, especially because [◄] [LAST𝑥] must be used repeatedly to bring back the results of previous calculations.

Once all the examples have been completed, it is worth returning the calculator to the modes you normally use.

The practice examples have shown how the user can switch between the two modes as necessary, with the result in the lower line of the screen transferred from one mode to the other. It is possible to work entirely in one mode or the other, but the algebraic mode and the RPN mode need not be treated as exclusive, one or the other. It is possible to treat them as complimentary and use whichever one suits any given task best.