



## hp calculators

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## The normal distribution

The normal distribution is frequently used to model the behavior of random variation about a mean. This model assumes that the sample distribution is symmetric about the mean,  $M$ , with a standard deviation,  $S$ , and generates the shape of the familiar bell curve. A standardized normal distribution has a mean of 0 and a standard deviation of 1. This results in the familiar  $Z$  value used in normal distribution problems to signify the number of standard deviations above or below the mean a particular observation falls. It is computed using the formula shown below.

$$Z = \frac{X - \mu}{\sigma}$$

Figure 1

where  $X$  is the observation,  $\mu$  is the mean and  $\sigma$  is the standard deviation.  $Z$  is often called a  $Z$ -score.

## Entering the normal distribution program

Solving problems involving the normal distribution requires the entry of the program below into the HP 33S calculator. This program can be found in chapter 16 of the HP 33S RPN/ALG Scientific Calculator Owner's Manual.

Given a value  $x$ , this program calculates the probability that a random selection from the sample data will have a higher value. This is known as the upper tail area,  $Q(x)$ . This program also provides the inverse: given a value  $Q(x)$ , the program calculates the corresponding value  $x$ . This program uses the built-in integration feature of the HP 33s to integrate the equation of the normal frequency curve. The inverse is obtained using Newton's method to iteratively search for a value of  $x$  which yields the given probability  $Q(x)$ . The program as listed will work in RPN mode only and that mode is assumed throughout this training aid.

After each label's section in the program listing below, the checksum and length are displayed in parentheses to the right of the page. These should match what you see on your HP 33S if you have entered the program correctly. To see each checksum once you have completely entered the program below, while still in program mode, press  $\leftarrow$  MEM  $\rightarrow$  ENTER  $\rightarrow$  SHOW. Pressing  $\downarrow$  (found on the cursor key at the top of the HP 33S) will scroll down through each label in the program showing its length. To see each checksum, press  $\rightarrow$  SHOW when each label is displayed. For example, in the listing below, the (D72F – 48) indicates a checksum of D72F and a length of 48.

In RPN mode, press the following keys to prepare for entry of the program (**WARNING:** Doing this will erase all of program memory):

$\leftarrow$  PRGM  $\leftarrow$  CLEAR  $\leftarrow$  ENTER

Once this is done, key in the following program:

$\leftarrow$ LBL S 0 STO M $\leftarrow$ INPUT M 1 STO S $\leftarrow$ INPUT S $\rightarrow$ RTN	(D72F – 48)
$\leftarrow$ LBL D $\leftarrow$ INPUT X XEQ Q STO Q $\rightarrow$ VIEW Q $\leftarrow$ GTO D	(EA54 – 18)
$\leftarrow$ LBL T $\leftarrow$ INPUT Q RCL M STO X	(79B9 – 12)
$\leftarrow$ LBL T XEQ Q RCL – Q RCL X STO D R1 XEQ F RCL $\div$ T $\div$ STO + X	
$\leftarrow$ ABS 0 $\cdot$ 0 0 0 1 $\leftarrow$ x?y 3 $\leftarrow$ GTO T RCL X $\rightarrow$ VIEW X $\leftarrow$ GTO T	(0E12 – 63)
$\leftarrow$ LBL Q RCL M RCL X $\rightarrow$ FN= F $\rightarrow$ / D 2 $\rightarrow$ $\pi$ X $\sqrt{x}$ RCL X S STO T	
$\div$ $\div$ 0 $\cdot$ 5 + $\rightarrow$ RTN	(FA83 – 72)
$\leftarrow$ LBL F RCL D RCL – M RCL $\div$ S $\sqrt{x}$ 2 $\div$ $\div$ $e^x$ $\rightarrow$ RTN	(1981 – 42)

Press  $\leftarrow$  PRGM to exit program mode. You are now ready to work the following examples.

## Practice solving problems involving the normal distribution

Example 1: Find  $Q(x)$  for a Z value of +1. Make sure the HP 33S is in RPN mode.

Solution: With the input value given as a Z-score, we're dealing with the standardized normal distribution having a mean of 0 and a standard deviation of 1. Press  $\leftarrow$  [RPN] to enter RPN mode.

In RPN mode: [XEQ] [S]



Figure 2

Since we are dealing with a standardized normal distribution, the mean should stay equal to 0.

In RPN mode: [R/S]



Figure 3

Since we are dealing with a standardized normal distribution, the standard deviation is equal to 1.

In RPN mode: [R/S]

Now, calculate  $Q(x)$  for an x value of 1 by pressing:

In RPN mode: [XEQ] [D]



Figure 4

In RPN mode: [1] [R/S]



Figure 5

Answer: The upper tail probability for the standardized normal distribution with a value of x equal to +1 is 0.1587. This means that only 15.87% of all values would be larger than a Z-score of +1.

Example 2: Find  $Q(x)$  for a Z value of -1. Make sure the HP 33S is in RPN mode.

Solution: With the input value given as a Z-score, we're dealing with the standardized normal distribution having a mean of 0 and a standard deviation of 1. Press  $\leftarrow$  [RPN] to enter RPN mode.

In RPN mode: **XEQ** **S**



Figure 6

Since we are dealing with a standardized normal distribution, the mean should stay equal to 0.

In RPN mode: **R/S**



Figure 7

Since we are dealing with a standardized normal distribution, the standard deviation is equal to 1.

In RPN mode: **R/S**

Now, calculate Q(x) for an x value of -1 by pressing:

In RPN mode: **XEQ** **D**



Figure 8

In RPN mode: **1** **+/-** **R/S**



Figure 9

**Answer:** The upper tail probability for the standardized normal distribution with a value of x equal to -1 is 0.8413. This means that 84.13% of all values would be larger than a Z-score of -1. Conversely, 15.87% of all values would be smaller than a Z-score of -1.

**Example 3:** The average number of claims processed per hour by an insurance adjuster is 15 with a standard deviation of 4 and follows the normal distribution. If an adjuster processes 20 claims per hour, what percentage of adjusters is this person performing faster than?

**Solution:** This is a normal distribution problem where the input is not standardized. Press **◀** **RPN** to enter RPN mode. Then execute label S and enter the mean and standard deviation.

In RPN mode: **XEQ** **S** **1** **5** **R/S** **4** **R/S**

Now execute label D and enter the value of  $x$  for which we wish to compute the value of  $Q(x)$ .

In RPN mode: **XEQ** **D** **2** **0** **R/S**



Figure 10

**Answer:** The upper tail probability with a value of  $x$  equal to 20 is 0.1056. This means that 10.56% of all insurance adjusters would be performing faster than the individual under consideration. The person being considered is nearly in the top 10%.

**Example 4:** Find  $x$  given a  $Q(x)$  of 0.65. Assume a standardized normal distribution. Make sure the HP 33S is in RPN mode.

**Solution:** With the input value given as a  $Q(x)$  probability, we'll need to execute label I which will determine the appropriate value for  $x$ . Since this is a standardized normal distribution, execute label S first and enter values of 0 for the mean and 1 for the standard deviation. Press **◀** **RPN** to enter RPN mode.

In RPN mode: **XEQ** **S** **0** **R/S** **1** **R/S**

Now execute label I and enter the value for  $Q(x)$ . Note that the previous value computed for  $Q(x)$  is displayed at the prompt.

In RPN mode: **XEQ** **I**



Figure 11

In RPN mode: **0** **.** **6** **5** **R/S**



Figure 12

**Answer:** The value of  $x$  for which the upper tail probability is equal to 0.65 is  $-0.3853$ . Since the normal distribution is symmetrical around the mean, 50% of the area / probability will be above the mean and 50% will be below the mean. In this example, since we were looking for a value of  $x$  for which upper tail probability would be 65%, the value of  $x$  would have to be less than 0.

**Example 5:** The average number of claims processed per hour by an insurance adjuster is 15 with a standard deviation of 4 and follows the normal distribution. Within what range, evenly distributed on either side of the average, would you expect to find 50% of the adjusters performing?

**Solution:** With the input value given as a  $Q(x)$  probability, we'll need to execute label I which will determine the appropriate value for  $x$ . Since this is not a standardized normal distribution, execute label S first and enter values of 15 for the mean and 4 for the standard deviation. Press **◀** **RPN** to enter RPN mode.

In RPN mode: **XEQ** **S** **1** **5** **R/S** **4** **R/S**

Now execute label I and enter the value for Q(x). Note that the previous value computed for Q(x) is displayed at the prompt.

In RPN mode: **XEQ** **I**



Figure 13

We're looking for a range within which 50% of the probability falls that is evenly spread around the average. Since the normal distribution is symmetrical, this means that 25% would be below the mean and 25% would be above the mean. The input values for Q(x), however, are upper tail probabilities. This means the values of Q(x) that need to be input will be 0.75 and 0.25.

In RPN mode: **0** **.** **2** **5** **R/S**



Figure 14

In RPN mode: **XEQ** **I**



Figure 15

In RPN mode: **0** **.** **7** **5** **R/S**



Figure 16

The values of x are  $-0.6745$  and  $+0.6745$ . These are the number of standard deviations above and below the average, between which 50% of the adjusters would be expected to be performing. To translate these values of x into actual claims processed per hour. We will need to add 0.6745 standard deviations to the average and also subtract 0.6745 standard deviations from the average.

In RPN mode: **0** **.** **6** **7** **4** **5** **ENTER** **4** **x** **1** **5** **+**

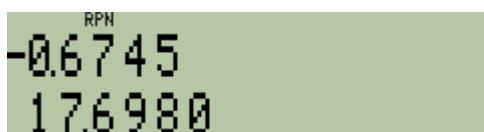


Figure 17

In RPN mode: **0** **.** **6** **7** **4** **5** **+/−** **ENTER** **4** **x** **1** **5** **+**

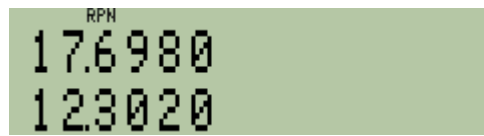


Figure 18

Answer: The middle 50% of the adjusters would average processing between 12.3020 and 17.6980 claims per hour.