



hp calculators

HP 33S Solving numeric integration problems

Numeric integration

Using the integration function

Practice solving numeric integration problems



Numeric integration

Numeric integration has many uses in different areas of science. One of the more common ways to visualize integration is that of the area under a curve to the X-axis between two points.

Using the integration function

The HP 33S has a very powerful numeric integrator built into the calculator. This function is found above the e^x key and is accessed by pressing 2ND \int . The method used in this training aid will be to enter the function to integrate as an equation and then to integrate it between an upper and lower limit of integration.

The general approach to integrate an equation will be:

- Step 1: If the equation that defines the integrand's function isn't stored in the equation list, key it in and leave Equation mode. The equation usually contains just an expression.
- Step 2: Enter the limits of integration:
 in RPN mode, key in the lower limit and press ENTER , then key in the upper limit;
 in algebraic mode, key in the lower limit, press $\text{X}\leftrightarrow\text{Y}$, then key in the upper limit.
- Step 3: Display the equation: Press 2ND EQN and, if necessary, scroll through the equation list (press the DOWN or UP keys found on the cursor key at the top of the HP 33S) to display the desired equation.
- Step 4: Select the variable of integration: Press 2ND \int and then press the appropriate key on the HP 33S to indicate the proper variable (the 2 key would choose X , for example). This starts the calculation.

Note that using the integration function uses much more of the calculator's memory than any other operation and, although highly unlikely, if a MEMORY FULL message is shown, refer to appendix B in the HP 33S manual for more information on what steps to take.

You can halt a running integration calculation by pressing C or R/S . However, no information about the integration is available until the calculation finishes normally.

The display format setting chosen through the DISPLAY menu affects the level of accuracy assumed for your function and used for the result. The integration is more precise but takes much longer in the **ALL** setting (DISPLAY 4) and in the **FIX** (DISPLAY 1), **SCI** (DISPLAY 2), and **ENG** (DISPLAY 3) modes with more digits displayed. The uncertainty of the result ends up in the Y-register, pushing the limits of integration up into the T- and Z-registers.

This training aid cannot begin to illustrate the wide range of applications available using the built-in numeric integration function, but it can illustrate some of the more common uses. For additional information, see chapters 8 and 14 of the HP 33s RPN/ALG Scientific Calculator Owner's Manual.

Practice solving numeric integration problems

Example 1: Integrate the function $1/X$ from 1 to 10. Use FIX 4 as the display setting.

Solution: In either RPN or algebraic mode: DISPLAY 1 4 2ND EQN $1/x$ RCL X 2ND \int ENTER

The display should look similar to the one shown in Figure 1. Note, if you have other equations already in the HP 33S calculator, the top line of the display may not indicate "EQN LIST TOP" but may show another equation.

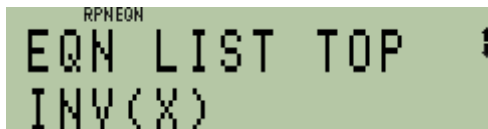


Figure 1

To show the checksum and length of this equation, press the following in RPN or algebraic mode. Note that the symbol \rightarrow means to press the right arrow direction of the cursor key at the top of the HP 33S.

In RPN or algebraic mode: $\left[\text{MEM} \right] \left[\rightarrow \right] \left[\text{ENTER} \right] \left[\rightarrow \right] \left[\text{SHOW} \right]$

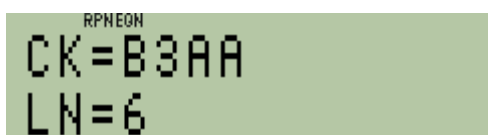


Figure 2

If the checksum of the equation just entered does not equal B3AA, then you have not entered it correctly. To exit equation mode, press:

In RPN or algebraic mode: $\left[\rightarrow \right] \left[\text{EQN} \right]$

Now enter the lower and upper limits of the integration.

In RPN mode: $\left[1 \right] \left[\text{ENTER} \right] \left[1 \right] \left[0 \right] \left[\rightarrow \right] \left[\text{EQN} \right]$

In algebraic mode: $\left[1 \right] \left[x \leftrightarrow y \right] \left[1 \right] \left[0 \right] \left[\rightarrow \right] \left[\text{EQN} \right]$

(Note: At this point, use the \downarrow or \uparrow keys to scroll through the equation list and make sure the equation is displayed as shown in Figure 3 below). Note, if you have other equations already in the HP 33S calculator, the top line of the display may not indicate "EQN LIST TOP" but may show another equation.



Figure 3

Integrate the function using X as the variable of integration.

$\left[\rightarrow \right] \left[/ \right] \left[X \right]$

After a few moments, the HP 33S will display the answer shown below.



Figure 4

Now view the uncertainty of the result.

In RPN mode: **C**

In algebraic mode: **R↓**

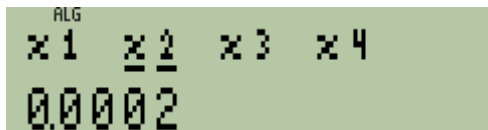


Figure 5

Answer: The area under the $1/X$ curve from 1 to 10 is approximately 2.3026. Figure 5 shows the uncertainty of the result assuming algebraic mode. In RPN mode, the uncertainty is shown in the second level of the stack.

Example 2: Integrate the function $\text{Sin}^2(X)$ from 0 to Π . Use FIX 4 as the display setting. Make sure the HP 33S is in radians mode.

Solution: In either RPN or algebraic mode: **DISPLAY 1 4 MODES 2**
EQN SIN RCL X EQN 0 yx 2 ENTER

Note: It is possible to write the equation using the x^y function, but the equation displayed using the y^x function is may be clearer to read. The display should look similar to the one shown in Figure 6.



Figure 6

To show the checksum and length of this equation, press the following in RPN or algebraic mode. Note that the symbol **→** means to press the right arrow direction of the cursor key at the top of the HP 33S.

In RPN or algebraic mode: **MEM → ENTER EQN SHOW**

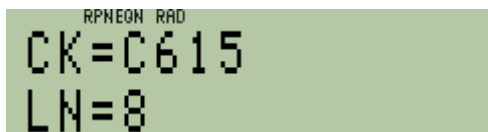


Figure 7

If the checksum of the equation just entered does not equal C615, then you have not entered it correctly. To exit equation mode, press:

In RPN or algebraic mode: **EQN**

Now enter the lower and upper limits of the integration.

In RPN mode: **0 ENTER EQN π EQN**

In algebraic mode: **0 x↔y EQN π EQN**

(Note: At this point, use the \downarrow or \uparrow keys to scroll through the equation list and make sure the equation is displayed as shown in Figure 8 below). Note, if you have other equations already in the HP 33S calculator, the top line of the display may not indicate "EQN LIST TOP" but may show another equation.



Figure 8

Integrate the function using X as the variable of integration.



After a few moments, the HP 33S will display the answer shown below.

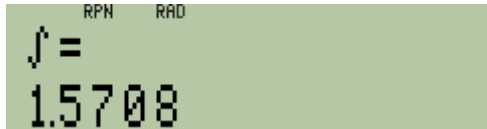


Figure 9

Now view the uncertainty of the result.

In RPN mode: \boxed{C}

In algebraic mode: $\boxed{R\downarrow}$



Figure 10

Answer: The area under $\text{Sin}^2(X)$ from 0 to Π is approximately 1.5708. The uncertainty of the result is 0.0002, as shown in the Y level of the stack, in Figure 10 (assuming RPN mode).

Example 3: Integrate the function shown below from 0 to 2Π . Use FIX 4 as the display setting. Make sure the HP 33S is in radians mode.

$$\frac{1}{1 - \text{COS}(X) + 0.25}$$

Figure 11

Solution: In either RPN or algebraic mode: $\boxed{\text{DISPLAY}} \boxed{1} \boxed{4} \boxed{\text{MODES}} \boxed{2}$
 $\boxed{\rightarrow}$ $\boxed{\text{EQN}} \boxed{1} \boxed{\div}$
 $\boxed{\rightarrow}$ $\boxed{1} \boxed{-} \boxed{\text{COS}} \boxed{\text{RCL}} \boxed{X} \boxed{\rightarrow}$ $\boxed{0} \boxed{.}$ $\boxed{2} \boxed{5} \boxed{\rightarrow}$ $\boxed{\text{ENTER}}$

The display should look similar to the one shown in Figure 12.

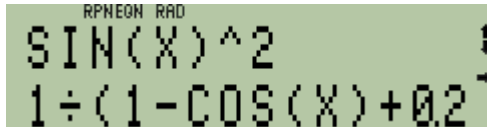


Figure 12

To show the checksum and length of this equation, press the following in RPN or algebraic mode. Note that the symbol \rightarrow means to press the right arrow direction of the cursor key at the top of the HP 33S.

In RPN or algebraic mode: \leftarrow MEM \rightarrow ENTER \rightarrow SHOW



Figure 13

If the checksum of the equation just entered does not equal BB03, then you have not entered it correctly. To exit equation mode, press:

In RPN or algebraic mode: \rightarrow EQN

Now enter the lower and upper limits of the integration. Note that the algebraic keystrokes are to allow for the computation of the upper limit of integration.

In RPN mode: 0 ENTER \rightarrow π 2 \times \rightarrow EQN

In algebraic mode: 2 \times \rightarrow π ENTER \leftrightarrow 0 \leftrightarrow \rightarrow EQN

(Note: At this point, use the \downarrow or \uparrow keys to scroll through the equation list and make sure the equation is displayed as shown in Figure 14 below). Note, if you have other equations already in the HP 33S calculator, the top line of the display may not indicate "EQN LIST TOP" but may show another equation.

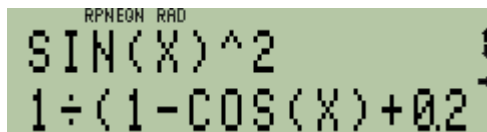


Figure 14

Integrate the function using X as the variable of integration.

\rightarrow \leftarrow \times

After a few moments, the HP 33S will display the answer shown below.

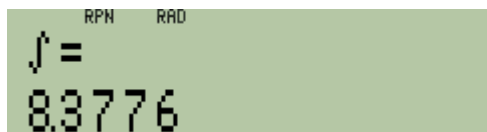


Figure 15

Now view the uncertainty of the result.

In RPN mode: \leftarrow

In algebraic mode: \leftarrow



Figure 16

Answer: The area under the function from 0 to 2π is approximately 8.3776. The uncertainty in the result is 0.0008. Figure 16 assumes algebraic mode. In RPN mode, the uncertainty is shown in the second level of the stack.