



## hp calculators

### HP 33S Using the HP Solver – Part 1

#### The HP Solver

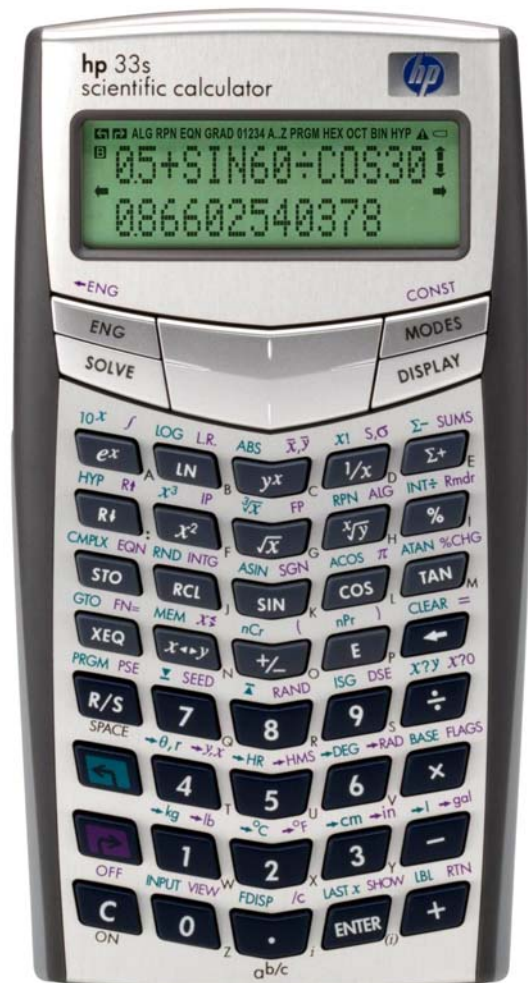
Practice Example: Finding Roots of Polynomials

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What the Solver can and can not find



## The HP Solver

Given an expression of the form:

$$f(x) = y$$

The HP Solve Application searches for a value of  $x$  that gives:

$$f(x) = y = 0$$

A value of  $x$  for which this is true is called a **root**, and it provides a **solution** of the equation  $f(x) = 0$ . Figure 1 shows this graphically – there is a root at the value of  $x$  where  $f(x)$  is zero.

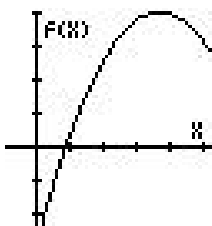


Figure 1

On the HP 33S,  $f(x)$  can be typed in Equation mode, as a formula or an equation, or it can be typed as a program.

Part 1 of this training aid provides an introduction to the Solver, using a few examples of solving in Equation mode. The second part will explain how the Solver works with programs, and in programs, providing some more examples.

In Equation mode the Solver works with  $f(x)$  as a formula containing  $x$ , for example

$$3x^2 - 3x - 15 \quad \text{or} \quad 5 \sin(x) - 7 \log(x)$$

If the Solver is given an equation with terms on both sides of the equals sign, such as:

$$3x^2 + 4x = 7x + 15$$

then it begins by moving everything to one side of the equals sign, so the above equation would become the formula:

$$f(x) = 3x^2 - 3x - 15 = 0$$

The Solver ignores the  $= 0$  part of such an equation, as it is trying to find a value for  $x$  to make the formula zero. So if an equation has  $= 0$  on the right-hand side; it is enough to type the formula at the left-hand side in Equation mode.

The variable  $x$  in the above is called the **unknown variable**. It can be represented by any of the HP 33S variables A through Z. The formula can contain more than one variable, the Solver will ask which is the unknown variable and will then ask for the known values of all the other variables.

The Solver then tries to rearrange the equation  $f(x) = 0$  to give a **direct solution** for  $x$ . (This is different from the predecessor of the HP 33S, the HP32SII, which does not try to find a direct solution.)

If a direct solution is found, it is displayed and the Solver stops. Examples of direct solutions are given in Example 3 below. If a direct solution exists but is outside the range of numbers that the HP 33S works with, NO SOLUTION FOUND is displayed and the Solver stops.

If a direct solution cannot be found, the Solver begins by calculating  $f(x)$  at two guesses for the unknown variable  $x$ . The Solver uses the initial value of  $x$  as one initial guess. It can create its own second guess or the user can give a second starting guess. These starting values can be very useful, either to direct the Solver towards one root of several, or to direct the Solver away from values that would cause an error. Good guesses can also speed up the search for a root. In some cases the function varies very slowly over some values, and a good guess is needed to direct the Solver away from them and towards the range of values where a solution is expected.

Beginning from the values of  $f(x)$  obtained for the first two guesses, the Solver searches for values of the unknown variable  $x$  that make  $f(x)$  closer to zero. If two guesses have the opposite sign, the Solver tries to narrow down the region between them until it finds where the sign changes and  $f(x)$  is zero. If two guesses have the same sign, the Solver uses the difference between them to pick the direction in which to change  $x$  to look for a third value closer to zero.

The process is repeated until one of the following happens.

A value of  $x$  is found for which  $f(x)$  is exactly zero.

Two neighboring values of  $x$  are found, differing by 1 or 2 in the twelfth significant digit, such that  $f(x)$  changes sign between them. The two values can be on either side of a root, or on either side of a pole or a discontinuity in  $f(x)$ . An example of finding a pole is looking for a solution of  $\tan(b) = 0$  between 89 degrees and 91 degrees.

No value can be found, but the Solver finds a minimum of  $f(x)$ , a value closer to zero than neighboring values.

No value can be found because the Solver is looking at values of  $x$  for which  $f(x)$  is constant.

No value can be found because  $f(x)$  is decreasing asymptotically towards a non-zero value.

An error occurs, for example a negative value of  $x$  is tried when  $f(x)$  includes a logarithmic term, or the Solver tries a value of  $x$  outside the calculator's numeric range.

The examples will show some of these cases, with further explanations.

To help the user distinguish between the above, the Solver returns the last value it tried for  $x$ , the last but one value tried, and the value of  $f(x)$  at the last value. Example 2 below gives details.

### Practice Example: Finding Roots of Polynomials

**Example 1:** Solve the polynomial equation  $x^3 + 15x^2 + 47x = -33$

**Solution:** The HP 33S manual gives a program for finding the roots of polynomials up to the fifth order, but it is often simpler to use the solver. The following steps show how this can be done.

Before a solution is looked for, it is useful to store a reasonable guess in the variable to be used. In this case, the variable will be "X", and to store a zero in it, the following keys are pressed:

**0** **STO** **X**

When **STO** is pressed, the symbol A..Z is shown at the top of the screen as a signal that one of the keys marked A through Z at their bottom right must be pressed. To enter the variable X press the **2** key, which has a small “X” at its lower right to show that it is also the **X** key. (Note that the exponent key **E** and the Cancel key **C** do not enter the letters E and C.)

Next, the equation must be entered. This is done in the HP 33S “Equation mode.” Enter Equation mode by typing **EQN**. Equation mode stores a list of equations and a new equation can be typed anywhere in this list. If necessary it is possible to put the new equation in a particular place by moving up or down through the list with the up and down cursor keys below the HP 33S screen.

Enter the equation by typing:

**RCL** **X** **y<sup>x</sup>** **3** **+** **1** **5** **x** **RCL** **X** **y<sup>x</sup>** **2** **+** **4** **7** **x** **RCL** **X** **=** **+/−** **3** **3** **ENTER**

To enter a variable into an equation, press the **RCL** key and then one of the letter keys. As with **STO**, the symbol A..Z at the top of the screen is shown as a signal that one of the keys marked A through Z must be pressed. To enter the variable X in this formula press the **2** key again.

To solve the equation, press the **SOLVE** key. This is the large silver key at the lower left below the display. The HP 33S asks which variable to solve for:



Figure 2

The symbol A..Z is at the top of the screen yet again. The variable in this formula is X so press the **2** key again.



Figure 3

The word “SOLVING” is shown while a solution is looked for. If the search lasts more than a short time, the busy indicator **B** is shown as well. When a solution is found, it is displayed. It is also stored in the variable.



Figure 4

Answer: X = -1 is a solution of the equation.

The equation is a cubic, so there are two other solutions. If all three solutions are real numbers then the solution found depends on the initial guess given. To try finding a different solution store a starting value of -50 in X.

**5** **0** **+/−** **STO** **X**

Enter equation mode again by typing **▢** **EQN**. The equation is shown again. As Equation mode stores a list of equations, it is possible to select a different equation to solve by moving up or down through the list with the up and down cursor keys below the screen. In this case the same equation is to be solved.



Figure 5

To solve the equation, press **SOLVE**, then press **2** again to show that the formula is to be solved for X.



Figure 6

Answer: X = -11 is a second solution of the equation. There is a third solution, or root, which can be found if a suitable guess is tried.

### Practice Example: Finding the Root of a Log Equation

Example 2: Polynomial equations of order up to 5 can also be solved using the polynomial solver program in the HP 33S manual, but many equations can only be solved using the Solver. Try solving the equation:  
 $4x^2 - 14 \text{ LOG}(x) = 33$

Solution: Enter the equation as before. Go to equation mode by typing **▢** **EQN**. Type the new equation immediately below the previous one, or move up or down through the list with the up and down cursor keys below the screen to place the new equation somewhere else in the list.

Enter the equation by typing:

**4** **x** **x^2** **RCL** **X** **▢** **−** **1** **4** **x** **▢** **LOG** **RCL** **X** **▢** **▢** **▢** **=** **3** **3** **ENTER**

This time,  $x^2$  has been typed by use of the **x^2** key. Note that the function name is shown as SQ in Equation mode:

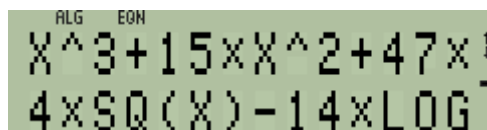


Figure 7

To solve the equation, press the **SOLVE** key. Then press **X** to show that this is the variable for which a solution is desired. A different variable name could have been used, but the same variable can be used in different equations.

The word “SOLVING” is shown as before. This time an error message follows. Although the solver knows which equation is being solved, it has started the search from the previous value stored in X, and that was -11, or one of the other negative roots of the polynomial in Example 1.



Figure 8

This shows how important it is to begin with a good guess. Obviously, only positive numbers should be used in the search for a solution this time. To make sure that only positive numbers are used in the search, two guesses can be given. One is the number in the variable being solved for. The other is the number displayed in the lower line of the screen. If these are the same, the HP 33S generates its own second guess. In this example, try using 1 and 10 as the guesses: Press the cancel key **[C]** key at the bottom left of the keyboard twice, to cancel the error message, and to leave Equation mode. Then enter the two guesses, and solve again.

**[C]** **[C]** **[1]** **[STO]** **[X]** **[←]** **[10<sup>x</sup>]** **[→]** **[EQN]** **[SOLVE]** **[X]**

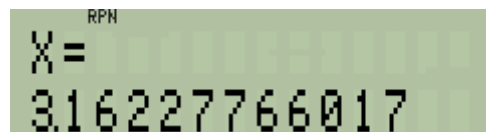


Figure 9

**Answer:** Unlike the results of examples 1, the answer is not a whole number. To 12 significant figures it is 3.16227766017. The true answer is  $\sqrt{10}$  which is an irrational number and can not be displayed exactly on a calculator.

The Solver provides additional information about the solution it has found. If the example was tried in RPN mode, press the back-arrow key **[←]** to remove the text “X=” from the upper line.

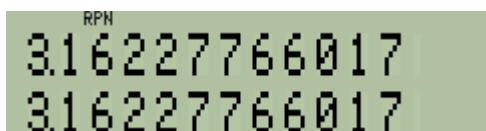


Figure 10

In RPN mode the lower line is stack register X and shows the best value that the solver could find. The upper line, stack register Y, shows the previous value tried. If these two are the same then this is an exact solution. Register Z shows the value of the formula using the best answer. Press **[R↓]** to see the value in register Z.



Figure 11

The number in Z is now on the upper line. It is zero, which means that 3.16227766017 is an exact solution to the 12 digit precision of the HP 33S.

The Solver works in the same way in algebraic mode, but to see the previous value tried and the best answer, press the  $\boxed{\text{R}\downarrow}$  key to see a menu.

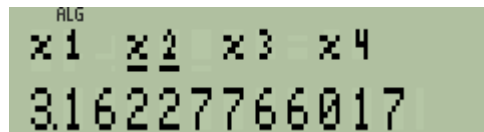


Figure 12

The number in x1 is the best answer, the number in x2, shown in Figure 12, is the previous value, and the number in x3 is the value of the formula. Use the left and right arrow keys to see each of these numbers, and press  $\boxed{\text{ENTER}}$  to copy the number shown into a calculation.

If the numbers in stack registers X and Y, or in menu values x1 and x2, differ by 1 in the last digit then there is no solution exactly correct to 12 digits, and the two values are on either side of the exact answer. This is confirmed if the value in stack register Z, or in menu value x3, is very small. Press  $\boxed{\text{C}}$  to cancel the menu.

If an error condition has occurred, such as the log of a negative number, and the Solver has not yet calculated  $f(x)$  at two values of  $x$ , then the three values will not have been put on the stack or in the menu.

### Practice Example: Direct Solutions

**Example 3:** To show that the HP 33S looks for a direct solution before starting to search for a root, try to solve  $\text{LN}(z) = 0$  beginning from a negative number for the guess.

**Solution:** Store -5 in Z. Then store  $\text{LN}(Z)$  as the formula to solve. This means that a solution is wanted for the equation  $\text{LN}(Z) = 0$ .

$\boxed{5} \boxed{+/} \boxed{\text{STO}} \boxed{Z} \boxed{\text{R}\downarrow} \boxed{\text{EQN}} \boxed{\text{LN}} \boxed{\text{RCL}} \boxed{Z} \boxed{\text{R}\downarrow} \boxed{)} \boxed{\text{ENTER}}$

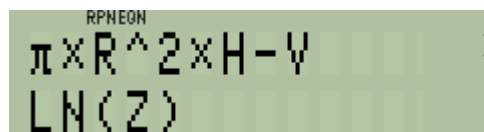


Figure 13

To solve the equation, press  $\boxed{\text{SOLVE}} \boxed{Z}$ . The Solver immediately displays the answer:



Figure 14

**Answer:**  $Z = 1$  is the solution to  $\text{LN}(Z) = 0$ . This is obvious, the point of this example is that the answer was found immediately, and the negative guess was not tried. If the negative guess had been tried, it would have caused a LOG(NEG) error, as happened in Example 2. Instead, the Solver recognized that  $Z$  appears only once in the formula, and that  $\text{LN}(Z) = 0$  can therefore be rewritten as  $Z = \exp(0)$  to solve for  $Z$  directly. Such direct solutions can speed up the use of the Solver, especially when a complicated formula with several variables is being solved several times for different variables.

Note 1: A direct solution might exist but be outside the range of numbers that the HP 33S works with. A simple example would be solving  $\sqrt{x} - 10^{-300}$ . The root is at  $x = 10^{-600}$ . This is a correct root but it lies outside the range of the HP 33S. The range is negative numbers between  $-9.999999999 \times 10^{499}$  and  $-10^{-499}$ , exact zero, and positive numbers between  $+10^{-499}$  and  $+9.999999999 \times 10^{499}$ . This is sufficient to cover all physical measurements, and even all numbers in government finances, but problems in number theory and in combinatorial operations may require numbers outside this range.



Note 2: If a calculation causes an error, the Solver stops at once. The error can be due to an out-of-range value as described above, or to an invalid value, for example a negative number used in a logarithmic term. The error can occur during the calculation of a direct solution or during a search for a solution. If the Solver stops before any results have been calculated, then the best value, previous value, and function value, will not be placed on the stack or in a menu, as described in Example 2. On the other hand, the error may occur only when the Solver has got as close to a good solution as is possible, so the stack (or menu) may contain reasonable values. Nevertheless, stack register Z (or menu item x3) may contain a zero only because the Solver has previously been used and has found a solution, even though an error has occurred and the Solver has found no good values this time. It is therefore very important to check the result carefully whenever the Solver stops with an error.

Note 3: Where more than one solution is possible, for example  $\text{ASIN}(Y)=0$ , the direct solution is the “principal” value. For example, for  $\text{ASIN}(Y)=0$ , this is 0 degrees, not 180 degrees, or  $-180$  degrees, or any other possible value. In the same way, an equation such as  $X^2=4$  is solved directly and returns the positive root 2. To find other roots, it is necessary to write the expression in such a way that the Solver does not find a direct solution. An easy way to achieve this is to add  $0 \times$  the unknown variable into an expression, for example  $\text{ASIN}(Y) + 0 \times Y = 0$  or  $X^2 + 0 \times X = 0$ . This is because the Solver stops looking for a direct solution as soon as it sees the unknown variable more than once in an expression.

### Practice Example: a Formula with Several Variables



**Example 4:** A factory is to produce tin cans with a volume of 100 cubic centimeters. The designer estimates that the height should be 10 cm and the radius about 2 cm. Calculate the exact volume of this can, and if it is not close to 100 cubic centimeters then recalculate the radius to give the required volume.

**Solution:** The equation for a cylinder’s volume  $V$ , given its radius  $r$ , and height  $h$ , is  $V = \pi r^2 h$ . Enter this as the formula  $\pi r^2 h - V$  in equation mode and then use the Solver.

Go to Equation mode by typing  . If necessary, put the new equation in a particular place in the list of equations by moving up or down through the list with the up and down cursor keys below the HP 33S screen.

Enter the formula by typing:

To enter a variable into an equation, press the  key and then one of the letter keys. Again, the symbol A..Z at the top of the screen is shown as a reminder that one of the keys marked A through Z must be pressed. For example press the  key to enter the variable V.



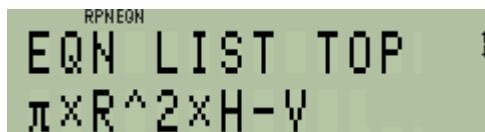


Figure 15

To solve the equation, press the **[SOLVE]** key. The Solver asks which variable to solve for:



Figure 16

The symbol A..Z is at the top of the screen once more. The variable in this formula is V so press **[6]** again. The Solver now knows that V is the unknown variable and it asks for the values of the known variables.



Figure 17

The value that is already stored in R is shown too. If this is the required value then it is enough to press **[R/S]**. If the variable R has not been used before, then its value is zero. In this example, type the radius 2 and press **[R/S]**.



Figure 18

The Solver asks for the other known variable. Type the height, 10, and press **[R/S]** again. The HP 33S displays SOLVING for a moment, then the result.

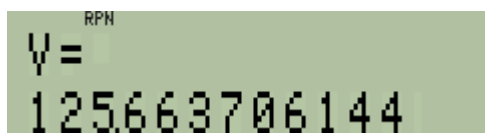


Figure 19

The volume is over 125 cubic centimeters, considerably more than the intended 100. Repeat the calculation, but this time use the known volume of 100, and solve for the radius. Solve the equation again by pressing **[>] [EQN] [SOLVE]**. The Solver asks for the unknown variable, press **[R]**. The Solver then asks for the known variables, first H.



Figure 20

The present value of H is the value previously given. As this is to remain the same, just press **[R/S]** again. The Solver now asks for the other variable, V.



Figure 21

The present value of V is shown; this is the volume just calculated. As the volume should be 100, type 100 and press **[R/S]**. The Solver calculates and displays the radius needed to give the required volume.

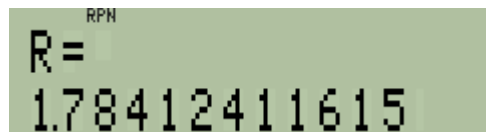


Figure 22

Answer: The cans should have a radius of 1.78 cm.

### What the Solver can and can not find

The examples above have shown the basics of what the solver can find and what it can not find. In summary:

In a formula or equation with one unknown variable the solver can find one or more roots if there are any.

If the solution can not be represented exactly, the solver finds the two nearest numbers on either side of it.

If no root is found but the Solver locates a pole or a discontinuity across zero it returns that as a solution.

The solver can not find a solution if two or more variables are unknown.

The solver can not find complex roots, as these have two unknown variables, the real and imaginary parts. Note that the HP 33S manual has a polynomial root finder program that will find complex roots.

The solver can not find roots of matrix equations.

Note that the HP 33S manual has a matrix program for solving two or three simultaneous equations.

The solver can not find a root if there is no root, but in this case it can find a minimum or an asymptote.

If an error occurs, such as calculating the log of a negative number, the Solver stops with an error message.

If a search finds a solution that is not zero but is less than  $10^{-499}$  the solver underflows and returns zero.

If a search finds a solution that is greater than  $9.999999999 \times 10^{499}$  the solver gives an OVERFLOW error.

When the Solver stops, it returns the best value found for the unknown variable, the previous best value found, and the value of the function at the best value. If an error occurs during calculation of a direct solution or during a search then the Solver stops at once. In such a case, these best values might not be available.

The second part of this training aid shows how the Solver works with programs and in them.