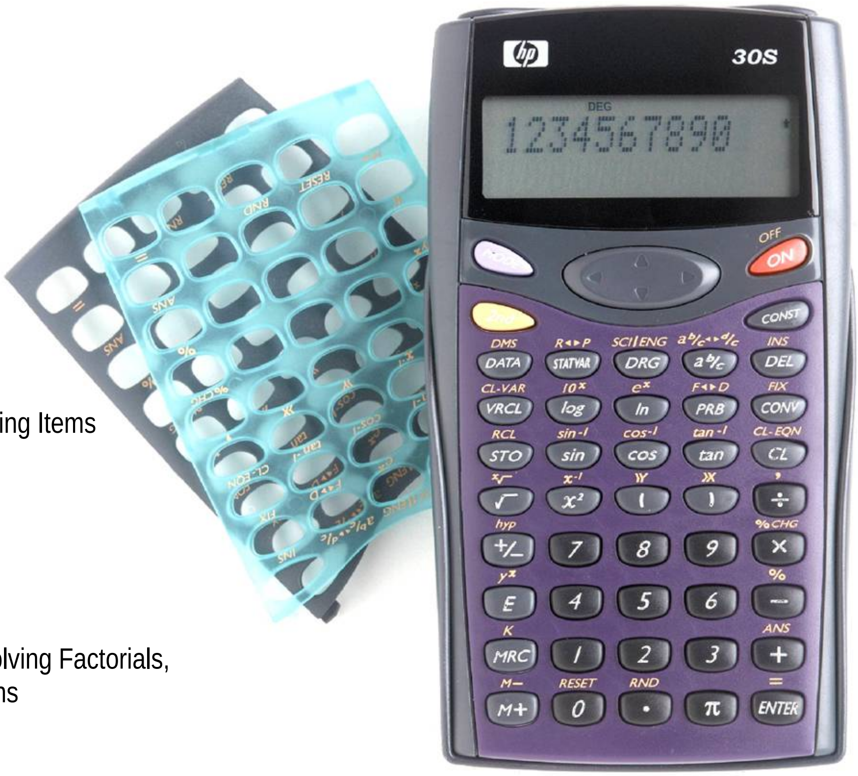
hp calculators



HP 30S Probability – Rearranging Items

Rearranging Items

Practice Solving Problems Involving Factorials, Permutations, and Combinations

Rearranging items

The calculation of probabilities often involves counting the ways in which items can be rearranged. We have three functions at our disposal for this purpose: factorial, permutations and combinations.

If *n* is a *positive integer*, its factorial (whose symbol is $n\,!$) is the product of all the positive integers up to and including *n*:

$$\boxed{n\,! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1}$$

0! is defined as 1.

A permutation of *r* from *n* is a way in which a set of *r* elements may be chosen in order from a set of *n* elements. In other words, it's an *ordered* subset of a set of distinct objects. The number of possible permutations, each containing *r* objects, that can be formed from a collection of *n* distinct objects is given by:

$$\boxed{{}^{n}P_{r} = P_{r}^{n} = {}_{n}P_{r} = P(n,r) = (n)_{r} = \frac{n!}{(n-r)!}}$$

The above expression also shows the multiple ways of symbolizing permutations. When a permutation involves all the elements of a set (i.e. r = n) then it is called a *rearrangement*, and also as *shuffle* especially when cards are used. Notice that in this case:

$$P_{r}^{n} = P_{n}^{n} = \frac{n!}{0!} = n!$$

A combination is a selection of one or more of a set of distinct objects without regard to order. These are the different ways of denoting combinations, and the number of possible combinations, each containing *r* objects, that can be formed from a collection of *n* distinct objects:

$$\boxed{{}^{n}C_{r} = C_{r}^{n} = {}_{n}C_{r} = C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}}$$

As far as rearranging items is concerned, whether order is important or not is the *only* difference between combinations and permutations.

One of the most important theorems on combinations is Vandermonde's theorem, which states that:

$$\boxed{C_{r}^{n} = C_{r-1}^{n-1} + C_{r}^{n-1}}$$

The factorial function was traditionally used for calculating permutations and combinations, which show up in many discrete probability distribution calculations, such as the binomial and hypergeometric distributions.

On the HP 30S, all these functions are in the PRB menu – just press PRB ▶ ▶ ENTER to enter the factorial function, and PRB ENTER or PRB ▶ ENTER to enter the permutations function or the combinations function respectively. The HP 30S errors for factorials of numbers greater than 69, but this covers most of the problems you will find.

Practice solving problems involving factorials, permutations, and combinations

Example 1: Calculate the number of combinations and permutations of 15 objects taken 5 at a time.

Solution: The number of combinations is calculated by first keying in the value of n (it can be the result of a calculation, in which case ANS is automatically entered into the entry line) then selecting the nCr funtion in the PRB menu and finally keying the value of r. ENTER displays the result:

1 5 PRB ▶ 5 ENTER

Likewise, the number of permutations is calculated by pressing:

1 5 PRB 5 ENTER

Note that selecting the desired function using the cursor keys suffices, you need not press the ENTER key to enter the function name into the entry line, e.g.: 1 5 PRB ENTER 5 ENTER .

Answer: 3003 and 360360.

Example 2: Calculate the number of ways that six people can line up for a photograph.

Solution: Since the order does matter in this example, the problem is solved as a permutation: P_6^6. Press:

6 PRB 6 ENTER

Note that $P_6^6 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$, so this example can be solved using the factorial function, which is a postfix function on the HP 30S:

6 PRB ▶ ▶ ENTER ENTER

Answer: 720

Example 3: In how many different ways can five seats be filled by a group of ten persons?

Solution: Once again, the order is important ,so the answer is P_5^{10}. Press:

1 0 PRB 5 ENTER

Answer: 30,240

Example 4: Verify Vandermonde's theorem for n = 4 and r = 3.

Solution: Pressing 4 PRB ▶ 3 ENTER returns the same result as pressing 3 PRB ▶ 2 + 3 PRB ▶ 3 ENTER .

Answer: C_3^4=4 and $C_2^3 + C_3^3$=4

Example 5: How many different hands of 5 cards could be dealt from a standard deck of 52 cards? Assume the order of the cards in the hand does not matter.

Solution: Since the order of the cards in the hand does not matter, the problem is solved as a combination: C_5^{52}

`5` `2` `PRB` ▶ `5` `ENTER`

Answer: 2,598,960 different hands.

Example 6: If five cards are dealt from a standard deck of 52 cards, calculate the probability of these five cards containing four-of-a-kind.

Solution: The probability is the number of favorable events divided by the number of possible events. There are 13 ranks of four-of-a-kind, the fifth card being any of the remaining 48 cards. The number of possible five-card hands has been calculated in the previous example: it is given by the combination the C_5^{52}. Therefore, the probability of being dealt four-of-a-kind is:

$$P = \frac{\text{possible four-of-a-kind hands}}{\text{possible five-card hands}} = \frac{13 \times 48}{\binom{52}{5}}$$

The keystroke sequence is:

`1` `3` `×` `4` `8` `÷` `5` `2` `PRB` ▶ `5` `ENTER`

If the result of the previous example is still displayed (and therefore stored in ANS), then you can press this one instead:

`1` `3` `×` `4` `8` `÷` `2nd` `ANS` `ENTER`

Answer: 0.000240096. Approximately, 1 in every 4165 cards (which is calculated by inverting the result).

Example 7: Find the probability that at least two of 23 people have the same birthday.

Solution: The probability that at least two of *n* persons have the same birthday is given by: $1-\frac{P_n^{365}}{365^n}$. In this example n = 23:

`1` `−` `3` `6` `5` `PRB` `2` `3` `÷` `3` `6` `5` `2nd` `yˣ` `2` `3` `ENTER`

Note that parentheses are not required because nCr and nPr have priority over the multiplication and division.

Answer: 0.507297234. The probability is greater than 50%. You can easily verify that for $n \leq 22$, the probability is smaller than 50%, so 23 is the smallest number of persons such that the probability of at least two of them having the same birthday is greater than 50%!

Example 8: Evaluate the binomial density function f(x) at x=4 for a constant probability of success (p) of 0.49 and 6 Bernoulli trials (n).

Solution: $f(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$. In our case: $f(4) = \binom{6}{4} \cdot 0.49^4 \cdot (1-0.49)^{6-4}$

6 PRB ▶ 4 × • 4 9 2nd y^x 4 × (1 − • 4 9) 2nd
y^x (6 − 4 ENTER

Answer: f(4) = 0.224913711

Example 9: If you flip a coin 10 times, what is the probability that it comes up tails exactly 4 times?

Solution: It is an application of the previous example. In this case, n = 10, x = 4 and p = 0.5:

1 0 PRB ▶ 4 × • 5 2nd y^x 4 × • 5 2nd y^x (1 0 − 4 ENTER

Answer: 0.205078125. If you flip a coin 10 times, there is a 20.51% chance of seeing heads 4 times.

Example 10: An urn contains eight white, five red and six black balls. If three balls are drawn at random, find the probability that all three are white.

Solution: This problem can be solved using either using combinations or permutations with the same result, because: $\frac{C_3^8}{C_3^{19}} = \frac{P_3^8}{P_3^{19}}$. Press either

8 PRB 3 ÷ 1 9 PRB 3 ENTER or
8 PRB ▶ 3 ÷ 1 9 PRB ▶ 3 ENTER

Answer: 0.057791538

Example 11: In the previous example, find the probability that one ball is black and two are white.

Solution: $\frac{C_1^6 \cdot C_2^8}{C_3^{19}}$

6 PRB ▶ 1 × 8 PRB ▶ 2 ÷ 1 9 PRB ▶ 3 ENTER

Answer: 0.173374613