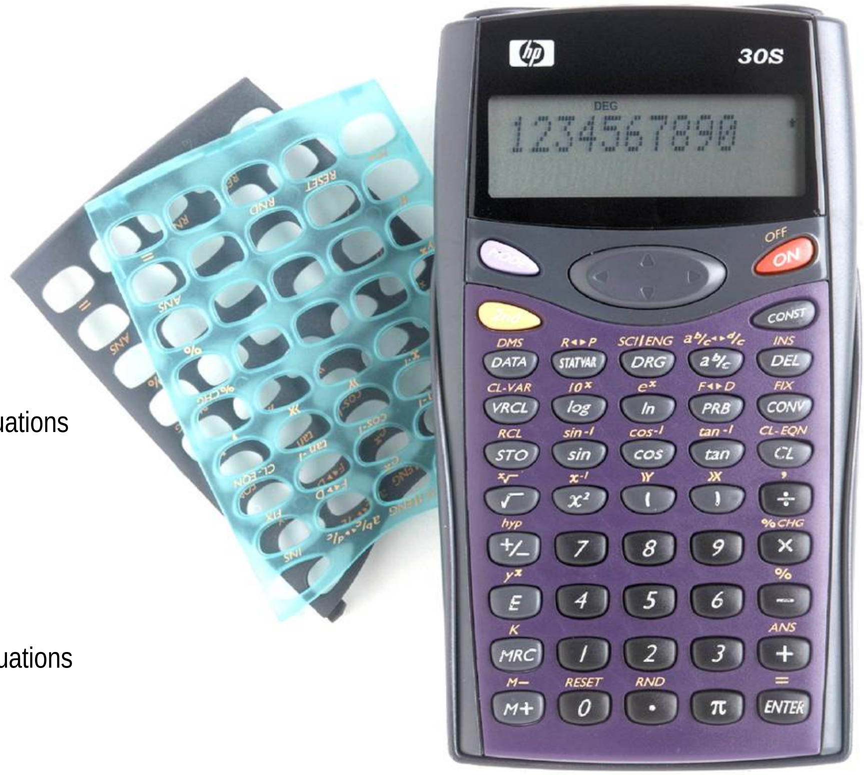
hp calculators

HP 30S Solving Quadratic Equations

The Q SOLV Mode

Practice Solving Quadratic Equations



The Q SOLV mode

Pressing [MODE] [3] selects the Q SOLV mode: an operating environment in which the quadratic equation:

$$\boxed{ax^2 + bx + c = 0}$$

can be solved easily. The $_Q$SOLV annunciator is displayed to indicate that this mode is active. In addition to all the functions available in HOME mode (the default operating mode), which are also available in Q SOLV mode, there are two keys that are now enabled: [2nd] [Y] and [2nd] [X]. They are used to enter the quadratic equation into the entry line. The equation can be written using either [2nd] [X] or [2nd] [Y] as the variable, but not both. The equal sign ([2nd] [=]) is mandatory (for instance, $x^2 + x - 1$ generates an error), so is the [x^2] function unless the equation is linear (e.g. the equation $x \cdot x + x - 1 = 0$ would also be an error condition—it must be entered as $x^2 + x - 1 = 0$).

Once the equation has been entered, press [ENTER] to find the two real solutions (if any) of the quadratic equation. They are displayed as a menu (much like the one displayed by [CONST]): use the ◄ and ► keys to see the values of the x_1 and x_2 variables (or y_1 and y_2, if the equation was written in Y), and press [CL] to exit this menu. Since the history stack is available, you can use ▲ to reuse or edit the equation.

Practice solving quadratic equations

Example 1: Find the roots of the polynomial $x^2 - 4x - 5$

Solution: First of all, let's select Q SOLV mode by pressing [MODE] [3] if it is not already the current mode (the calculator stays in this mode even if power is turned off). The roots of the polynomial $x^2 - 4x - 5$ are the x values for which the equation $x^2 - 4x - 5 = 0$ is true. Let's now enter the equation by pressing:

[2nd] [X] [x^2] [−] [4] [2nd] [X] [−] [5] [2nd] [=] [0]

To find the roots, simply press [ENTER] and then ► to see the second root.

Answer: 5 and –1.

Example 2: Find the roots of the polynomial $x^2 - 2x + 1$

Solution: In Q SOLV mode, press:

[2nd] [X] [x^2] [−] [2] [2nd] [X] [+] [1] [2nd] [=] [0] [ENTER]

The x_1 and x_2 variables contain the same value, which means that the roots of the polynomial are repeated.

Answer: 1, which is a multiple root, i.e. $x^2 - 2x + 1 = (x - 1)^2$

Example 3: Solve the quadratic equation $\frac{3}{4}y + 2 + 8y - y^2 = 7.25y^2 + 2$

Solution: In Q SOLV mode, press:

[3] [a b/c] [4] [2nd] [Y] [+] [2] [+] [8] [2nd] [Y] [−] [2nd] [Y] [x²] [2nd] [=] [7] [•] [2] [5] [2nd] [Y] [x²] [+] [2] [ENTER]

Note that you need not rewrite the equation first, nor collect terms. It is done by the calculator.

Answer: The equation has two real solutions: 0, and 1.060606061 (= $1\frac{2}{33}$, use [2nd] [F◄►D] to express the decimal as a mixed number).

Example 4: Solve $-x^2 + 25 = 0$

Solution: Press:

[+/−] [2nd] [X] [x²] [+] [2] [5] [2nd] [=] [0] [ENTER]

Answer: –5 and 5 satisfy the given equation.

As you can see, any coefficient can be zero, including the leading one (i.e. the coefficient of the second power term), in which case what we are solving is a linear equation.

Example 5: Solve $3x - 4x + 36 = 5x$

Solution: [3] [2nd] [X] [−] [4] [2nd] [X] [+] [3] [6] [2nd] [=] [5] [2nd] [X] [ENTER]

Being a linear equation, there can only be one solution (or none) . Even though the calculator keeps displaying both x_1 and x_2 , one of them will always be zero when solving linear equations: the another one (i.e. the non-zero value) will be the solution. If both are zero, the solution is of course zero.

Answer: x = 6 .

The following example illustrates the various error conditions you may encounter .

Example 6: Solve: $x/6 + x^2 = 1$, $x = x$, $5 - 3 = 2$, $x - x = 5$ and $x^2 = -1$.

Solution: Let's enter the equations as written:

- $x/6 + x^2 = 1$. Pressing [2nd] [X] [÷] [6] [+] [2nd] [X] [x²] [2nd] [=] [1] [ENTER] generates a *syntax* error because the coefficients must be placed before the variable. Simply enter it as $\frac{1}{6}x + x^2 = 1$. The correct sequence is then: [1] [÷] [6] [2nd] [X] [+] [2nd] [X] [x²] [2nd] [=] [1] [ENTER] . Note that parentheses are not necessary: 1/6X actually means (1/6)·X.

- $x = x$. Pressing [2nd] [x] [2nd] [=] [2nd] [x] [ENTER] gives MULTI SOLS, i.e. there is an infinite number of solutions (just pick one!)
- $5 - 3 = 2$. Press [5] [−] [3] [2nd] [=] [2] [ENTER]. Even though the equation *is* true, it is an error condition on the HP 30S (syntax error) because there is no variable in the equation. The x or y variable must be used at least once (even if it disappears when simplifying, as in the previous case).
- $x - x = 5$. Press [2nd] [x] [−] [2nd] [x] [2nd] [=] [5] [ENTER]. The message NO SOLUTION is displayed. Quite eloquent. $0 \neq 5$.
- $x^2 = -1$. Pressing [2nd] [x] [x²] [2nd] [=] [+/−] [1] [ENTER] gives NO REAL SOL. The solutions are a pair of conjugate complex numbers, which the built-in solver cannot find. But read on…

To remove an error message and display the entry line, simply press [CL]. (The message disappears after a few seconds anyway).

Example 7: Solve $x^2 - 2x + 2 = 0$

Solution: Let's enter the equation in Q SOLV mode:

[2nd] [x] [x²] [−] [2] [2nd] [x] [+] [2] [2nd] [=] [0] [ENTER]

The error message NO REAL SOL is displayed. In general, if solving $ax^2 + bx + c = 0$ in Q SOLV mode gives NO REAL SOL, then the two complex solutions can be calculated as follows:

$$x_1 = \frac{-b}{2a} + \frac{\sqrt{-b^2 + 4ac}}{2a} i, \quad \text{and} \quad x_2 = \frac{-b}{2a} - \frac{\sqrt{-b^2 + 4ac}}{2a} i$$

Note that their real parts are the same and their imaginary parts only differ in the sign.

This process can be automated by means of the EQN variable. Let's store the following *linear system* in EQN (refer to the HP 30S learning modules *Solving Linear Systems* and *Working with Expressions* for more information about the L SOLV mode and the EQN variable, respectively):

$$\begin{cases} X = 0.5A^{-1}B \\ Y = \dfrac{\sqrt{-B^2 + 4AC}}{2A} \end{cases}$$

First of all, let's store 1 in A, B and C so that these variables can't cause a math error when storing the second expression (remember that expressions are evaluated before being stored in EQN):

[1] [STO] [ENTER] [STO] [▶] [ENTER] [STO] [▶] [▶] [ENTER]

Next, we'll store the linear system in L SOLV mode by pressing:

[MODE] [2] [2nd] [x] [2nd] [=] [.] [5] [VRCL] [2nd] [x⁻¹] [VRCL] [▶] [2nd] [,] [2nd] [y] [2nd] [=] [√] [+/−] [VRCL] [▶] [x²] [+] [4] [VRCL] [ENTER] [VRCL] [▶] [▶] [ENTER] [▶] [÷] [2] [VRCL] [ENTER] [STO] [◀] [ENTER]

Finally, press [MODE] [3] to return to Q SOLV mode. That's all. Next time we get a NO REAL SOL message when attempting to solve a quadratic expression, we can find the complex numbers as follows:

[MODE] [2] [VRCL] ◀ [ENTER] [ENTER] , the calculator now prompts for the A, B and C coefficients:
[CL] [1] [ENTER] [CL] [+/−] [2] [ENTER] [CL] [2] [ENTER]

The resulting menu contains the values of the X and Y variables, which are the real and the imaginary part of one of the complex solutions. The other solution is the conjugate number.

Answer: –1 + i and –1 – i .

Example 8: Write a generic expression to find the real solutions of the equation $ax^2 + bx + c = 0$.

Solution: You may find it more convenient to input only the coefficients of a quadratic equation than to key in the whole equation. If so, store the generic equation into EQN in Q SOLV mode:

[MODE] [3] [VRCL] [2nd] [X] [x^2] [+] [VRCL] ▶ [2nd] [X] [+] [VRCL] ▶ ▶ [2nd] [=] [0] [STO] ◀ [ENTER]

Then, to solve an equation, say $x^2 - 15x + 50 = 0$, simply press:

[VRCL] ◀ [ENTER] [ENTER] [CL] [1] [ENTER] [CL] [+/−] [1] [5] [ENTER] [CL] [5] [0] [ENTER]

The solutions $x_1 = 10$ and $x_2 = 5$ are displayed.

Example 9: Solve $y^4 - 5y^2 + 4 = 0$

Solution: This quartic or biquadratic equation can be reduced to a quadratic form by making the following substitution: $x = y^2$. Then what we have to solve is $x^2 - 5x + 4 = 0$. If the EQN stored in the previous example is still in memory, press:

[VRCL] ◀ [ENTER] [ENTER] [CL] [1] [ENTER] [CL] [+/−] [5] [ENTER] [CL] [4] [ENTER]

which results in $x_1 = 4$ and $x_2 = 1$. Since $x = y^2$, then $y = \pm\sqrt{x}$.

Answer: $y_1 = 2$, $y_2 = -2$, $y_3 = 1$ and $y_4 = -1$.