



hp calculators

HP 30S Logarithmic Functions

Logarithms and Antilogarithms

Practice Solving Problems Involving Logarithms



Logarithms and antilogarithms

The logarithm of x to the base a (written as $\log_a x$) is defined as the inverse function of $x = a^y$. In other words, the logarithm of a given number is the exponent that a base number must have to equal the given number. The most usual values for a are 10 and e , which is the exponential constant and is defined by the infinite sum: $1 + 1/1! + 1/2! + 1/3! + \dots + 1/n! + \dots$. Its value is approximately 2.718 and is a transcendental number, that is to say: it cannot be the solution of a polynomial equation with rational coefficients.

Logarithms to base 10 are called common logarithms and also Briggsian logarithms. They are usually symbolized as $\log_{10} x$ or simply \log 10, and on the HP 30S, they correspond to the log key. These logarithms are used in calculations.

Logarithms to base e are called natural logarithms, Napierian logarithms and also hyperbolic logarithms. Their symbol is $\ln x$ or $\log_e x$. They are calculated with the ln key on the HP 30S. This kind of logarithms is the most used in mathematical analysis. There is still another kind of logarithms, though less frequently used; they are the binary logarithms, which are logarithms with base 2 ($\log_2 x$).

The following formula is very useful to change logarithms from one base to another:

$$\log_n x = \frac{\log_m x}{\log_m n}$$

The denominator, $\log_m n$, is known as the *modulus*.

The inverse function of the logarithm is called the antilogarithm. If $y = \log_a x$, then $x = a^y$ is the antilogarithm of y . If the base is e then the inverse function is called the exponential function, e^x , which is also known as the compound interest function and the growth (if $x > 0$) or decay (if $x < 0$) function. Perhaps the most important property of the exponential function is that its derivative is also e^x , that is, it's the solution of the differential equation $dy/dx = y$ for which $y = 1$ when $x = 0$.

On the HP 30S, the keys that perform these functions are ln , log , 2nd 10^x and 2nd e^x . The function y^x (y 2nd y^x x ENTER) can be considered the generic antilogarithm function: if 10^x is the inverse of $\log_{10} x$ and e^x is the inverse of $\log_e x$, then y^x is the inverse of $\log_y x$. Refer to the HP 30S learning module *Solving Problems Involving Powers and Roots* for more information on the y^x function.

Practice solving problems involving logarithms

Example 1: Find the common logarithm of 2

Solution: log 2 ENTER

Answer: 0.301029996

Example 2: What is the numerical value of the base of the natural logarithms?

Solution: 2nd e^x $/$ ENTER

Answer: 2.718281828. Note that the pattern 18-28-18-28 is really easy to remember!

Example 3: Verify that $\ln(8) + \ln(5) = \ln(8 \times 5)$

Solution: Before electronic calculators replaced logarithmic tables, logarithms were used for multiplying and dividing large numbers quickly because the logarithm of the product is equal to the sum of logarithms of the multiplicand and the multiplier. Press:

\ln 8 \blacktriangleright + \ln 5 ENTER and
 \ln 8 \times 5 ENTER

Answer: Both expressions evaluate to 3.688879454

Example 4: Find the log to base 3 of 5. Confirm the result using the y^x function.

Solution: Using the formula given above, the log to base 3 of 5 can be calculated as $\frac{\log_{10} 5}{\log_{10} 3}$:

\log 5 \blacktriangleright \div \log 3 ENTER

Let's confirm this result by calculating 3^{ANS} :

3 2nd y^x 2nd ANS ENTER

We obtain 5, which means that the logarithm was correctly calculated.

Answer: 1.464973521

Example 5: What is the value of x in the equation $18^x = 324$??

Solution: To solve this equation, we will use an important property of logarithms which states that the logarithm of a number raised to a power is equal to the power multiplied by the logarithm of the number. This involves taking the logarithm of both sides of the equation. The original equation would then look like this:

$$\log 18^x = \log 324 \Rightarrow x \log 18 = \log 324$$

and x is therefore equal to:

$$x = \frac{\log 324}{\log 18}$$

\log 324 \blacktriangleright \div \log 18 ENTER

Answer: 2. Note that the same answer will be found using natural logarithms instead.

Example 6: A rare species of tree has a trunk whose cross-section changes as $1/x$ with the height x . (Obviously this breaks down at ground level and at the tree top.) The cross section for any such tree is given by A/x , where A is the cross-section calculated at 1 meter above the ground. What is the volume of the trunk between 1 meter and 2 meters above ground?

Solution: The volume is obtained by integrating the cross-section along the length, so it is given by the integral:

$$\int_1^2 \frac{A}{x} dx$$

Although the HP 30S has no built-in integration function, this integral can be easily evaluated if we remember that the indefinite integral of $1/x$ is $\ln(x)$. The result is therefore:

$$V = A \times \ln(2) - \ln(1)$$

And since $\ln(1) = 0$:

$$V = A \times \ln(2)$$

As no one is likely to measure tree heights to an accuracy of more than three significant digits, let's set our HP 30S to display the answer with just 3 digits after the decimal point, by pressing $\text{2nd} \text{ FIX } \text{3}$.

Now let's find $\ln(2)$:

$\text{ln} \text{ 2 } \text{ENTER}$

Answer: The log to base e of 2 is close to 0.693, so the volume is $0.693A$ cubic meters. Press $\text{2nd} \text{ FIX } \text{0}$ to return to the default display format, once you have finished this example.

Example 7: An activity of 200 is measured for a standard of Cr^{51} (with a half-life of 667.20 hours). How much time will have passed when the activity measured in the sample is 170?

Solution: This is the formula for half-life computations:

$$A = A_0 \times \left(\frac{1}{2}\right)^{\frac{t}{\tau}}$$

Let's rearrange the equation to solve for t :

$$t = \tau \frac{\ln \frac{A}{A_0}}{\ln \frac{1}{2}}$$

The sequence of keystrokes is then:

$\text{6} \text{ 6} \text{ 7} \text{ .} \text{ 2} \text{ ln} \text{ 1} \text{ 7} \text{ 0} \text{ } \div \text{ 2} \text{ 0} \text{ 0} \text{ } \blacktriangleright \text{ } \div \text{ ln} \text{ 1} \text{ } \div \text{ 2} \text{ ENTER}$

Notice the implicit multiplication between 667.2 and $\ln \frac{170}{200}$.

Answer: 156.4352172 hours. Approximately, 156 hours and 26 minutes (which can be obtained by pressing 2nd DMS ENTER ENTER provided that DEG is the current angular mode).