hp calculators

HP 30S Solving Linear Systems

The L SOLV Mode

Practice Solving Linear Systems

The L SOLV mode

The L SOLV mode ([MODE] [2]) is a special operating environment in which systems of two linear equations in two variables:

$$\boxed{\begin{cases} a\cdot x + b\cdot y = c \\ d\cdot x + e\cdot y = f \end{cases}}$$

can be solved easily. The $_L$SOLV annunciator is displayed to indicate that this mode is active. In addition to all the functions available in HOME mode (i.e. [MODE] [0] , the default operating mode), which are also available in L SOLV mode, there are two keys that are now enabled: [2nd] [Y] and [2nd] [X] . They are used for entering the x and y variables into the entry line. The two equations are separated by a comma: [2nd] [,] .

Once the equations have been entered, press [ENTER] to find the solution (if any) of the linear system. It is displayed as a menu (much like the one displayed by [CONST]): use the ◀ and ▶ keys to see the values of the X and Y variables and press [CL] to exit the menu. Since the history stack is available, you can use ▲ to reuse or edit the system.

Practice solving linear systems

<u>Example 1:</u> Solve the system $\begin{cases} 3x - y = 6 \\ x + y = 2 \end{cases}$

<u>Solution:</u> Let's select L SOLV mode – press [MODE] [2] . We can now enter the system as follows:

[3] [2nd] [X] [−] [2nd] [Y] [2nd] [=] [6] [2nd] [,] [2nd] [X] [+] [2nd] [Y] [2nd] [=] [2]

Press [ENTER] to find the answer. The X value is displayed in the result line- To see the Y value, simply press ▶ .

<u>Answer:</u> x = 2 and y = 0. This is an example of a *consistent* system of linear equations because it has exactly one common solution.

<u>Example 2:</u> Solve the system $\begin{cases} y = 25x - 3 \\ y = -5x + 5 \end{cases}$

<u>Solution:</u> The equations are now given in this other common form $y = mx + b$, where m is the slope and b is the y-intercept of a straight line. You need not rewrite the equations first to solve this system with your calculator. Just enter them as are written:

[2nd] [Y] [2nd] [=] [2] [5] [2nd] [X] [−] [3] [2nd] [,] [2nd] [Y] [2nd] [=] [+/−] [5] [2nd] [X] [+] [5] [ENTER]

<u>Answer:</u> x = 0.2666666667 and y = 3.666666667.

<u>Example 3:</u> Express the previous result as a fraction.

<u>Solution:</u> If the resulting X-Y menu is still displayed, select X and press [2nd] [F◄►D] [ENTER] to convert the x value into a fraction. Let's now edit this calculation as follows:

► [VRCL] ◄ ◄ ◄ (or ◄ ◄ ◄ ◄ if EQN contains an expression) [ENTER] [ENTER]

<u>Answer:</u> x = $\frac{4}{15}$, and y = $3\frac{2}{3}$

<u>Example 4:</u> Solve $\begin{cases} 21x + 65y - 3x = \dfrac{y}{3} - 3 \\ x - 5 - y = 0 \end{cases}$

<u>Solution:</u> Once again, you need not rewrite the equations nor collect terms. The only thing that should be borne in mind is that the coefficients must be entered before the variables, that is, enter $\frac{y}{3}$ as [1] [÷] [3] [2nd] [y] , but not as [2nd] [y] [÷] [3] . Press:

[2] [1] [2nd] [x] [+] [6] [5] [2nd] [y] [−] [3] [2nd] [x] [2nd] [=] [1] [÷] [3] [2nd] [y] [−] [3] [2nd] [,] [2nd] [x] [−] [5] [−] [2nd] [y] [2nd] [=] [0] [ENTER]

<u>Answer:</u> x = 3.875, y = –1.125.

<u>Example 5:</u> Solve $\begin{cases} 3x - y = 6 \\ 6x - 2y = 8 \end{cases}$

<u>Solution:</u> Still in L SOLV mode, press:

[3] [2nd] [x] [−] [2nd] [y] [2nd] [=] [6] [2nd] [,] [6] [2nd] [x] [−] [2] [2nd] [y] [2nd] [=] [8] [ENTER]

The error message NO SOLUTION is displayed.

<u>Answer:</u> There are no common solutions to this system of equations. Note that the second equation is the same as $\frac{6x - 2y}{2} = \frac{8}{2} \Rightarrow 3x - y = 4$, and the number $3x - y$ cannot be equal to 6 and 4 at once. This is an example of an *inconsistent* system of linear equations.

<u>Example 6:</u> Solve $\begin{cases} 3x - y = 6 \\ 12x - 4y = 24 \end{cases}$

Solution: Since the first equation is the same as in the previous example, we can edit the entry line as follows (if you have cleared the display, press ▲ until the above system is retrieved and then 2nd ▶ to move the cursor to the last character):

DEL DEL DEL DEL DEL DEL DEL 1 2 2nd X − 4 2nd Y 2nd = 2 4 ENTER

which results in the error message MULTI SOLS being displayed for a few seconds (press CL to restore the entry line).

Answer: This system is said to be a *dependent* system of linear equations. There are an infinite number of common solutions. Note that the two equations are actually equivalent: the second equation being four times the first one.

Example 7: Solve $\begin{cases} 3 - \ln(5) = x + 1 \\ x - y = \pi \end{cases}$

Solution: In this case, the first equation contains only one variable, which actually suffices:

3 − ln 5 ▶ 2nd = 2nd X + 1 2nd , 2nd X − 2nd Y 2nd = π ENTER

Answer: Rounding to two decimal digits, x = 0.39 and y = –2.75.

Example 8: Write a linear system that returns the real and the imaginary parts of $\frac{a+bi}{c+di}$.

Solution: By storing the resulting system in EQN, we will have at our disposal a convenient way of dividing complex numbers expressed in rectangular form. Since $\frac{a+bi}{c+di} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i$, the system in question is:

$$\begin{cases} x = \dfrac{ac+bd}{c^2+d^2} \\ y = \dfrac{bc-ad}{c^2+d^2} \end{cases}$$

Use the A, B, C and D variables when typing in. Since $c^2 + d^2$ cannot evaluate to zero, store 1 in C and D, first: 1 STO ▶ ▶ ENTER STO ▶ ▶ ▶ ENTER . When solving the system, the calculator will prompt us for the values of the four variables as they are found in the equations. We can set a particular order by starting the first equation with, say, ABCD – ABCD.

Let's now store the system into EQN:

MODE 2 2nd X 2nd = VRCL ENTER VRCL ▶ ENTER VRCL ▶ ▶ ENTER VRCL ▶ ▶ ▶ − VRCL ENTER VRCL ▶
ENTER VRCL ▶ ▶ ENTER VRCL ▶ ▶ ▶ + (VRCL ENTER VRCL ▶ ▶ + VRCL ▶ ENTER VRCL ▶ ▶ ▶) ÷
(VRCL ▶ ▶ x^2 + VRCL ▶ ▶ ▶ x^2) 2nd , 2nd Y 2nd = (VRCL ▶ ENTER
VRCL ▶ ▶ − VRCL ENTER VRCL ▶ ▶ ▶) ÷ (VRCL ▶ ▶ x^2 + VRCL ▶ ▶ ▶ x^2 STO ◀ ENTER

In order to check it was correctly entered, let's calculate $\frac{5+3i}{9-2i}$. Press:

2nd CL-VAR VRCL ◀ ENTER ENTER 5 ENTER 3 ENTER 9 ENTER +/− 2 ENTER

which should return the answer 0.46 + 0.44 i, rounded to two decimal digits.