



hp calculators



HP 30S Solving Problems Involving Fractions

Basic Concepts

Fractions on the HP 30S

Practice Working Problems Involving Fractions

### Basic concepts

Those numbers that can be written as one integer over another, i.e.  $\frac{a}{b}$ , ( $b$  can't be zero) are called *rational numbers*.

When written as the quotient of two integers, rational numbers are called fractions. In arithmetic there are three basic rules for fractions:

- ◆  $\frac{a}{b} > \frac{c}{d}$  if  $ad - bc > 0$  (same for  $<$  and  $=$ )
- ◆  $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
- ◆  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b}$  is also referred to as a *vulgar* fraction when  $a$  and  $b$  are positive integers (the sign is considered apart).  $a$  is called the numerator (corresponds to the dividend in a division) and  $b$  is the denominator (which corresponds to the divisor). When the numerator is 1 (or  $-1$ ), it is a *unit* fraction. A proper fraction is a fraction in which the numerator (apart from the sign, remember) is *smaller* than the denominator. Therefore, proper fractions always lie between  $-1$  and  $1$ . If the numerator is *greater* than the denominator, the fraction is called improper.

Vulgar fractions that have the same value are called equivalent fractions: for example  $\frac{3}{4}$  and  $\frac{6}{8}$ . Reducing a vulgar fraction to its lowest terms means to find the simplest equivalent fraction, which can be done by dividing the numerator and the denominator by the same number. This process is also called *cancellation*.

Mixed numbers are those improper fractions written as an integer followed by a proper fraction. For example:  $4\frac{3}{4}$  or  $2\frac{1}{2}$ . It is important to understand that there's no implicit multiplication in  $a\frac{b}{c}$  (also written as  $a\frac{b}{c}$ ). In fact, it is an *addition* that is implicit:

$$a\frac{b}{c} = a + \frac{b}{c} = \frac{ac + b}{c}$$

When the numerator and the denominator of a fraction are not both integers then the fraction is called *complex*, for example:  $\frac{13}{3\frac{3}{4}}$ . Finally, a number where the part which is a proper fraction is expressed as a set of digits placed after a

decimal point, is called a decimal (also known as decimal fraction) e.g.  $3\frac{7}{50} = 3.14$ .

### Fractions on the HP 30S

The HP 30S has three keys to handle fractions, namely  $\frac{a}{b/c}$ ,  $\frac{a^b/c \leftrightarrow d/c}{2nd}$  and  $\frac{F \leftrightarrow D}{2nd}$ . The symbol used by the HP 30S to show a fraction in the entry line (i.e. the equivalent to the symbol " $\frac{1}{1}$ ", which is called *solidus*) is " $\frac{1}{1}$ " and is keyed in by pressing  $\frac{a}{b/c}$ . Thus,  $7\frac{1}{8}$  means  $7/8$  and is entered by pressing  $7 \frac{a}{b/c} 8$ . Mixed numbers are also keyed in using the  $\frac{a}{b/c}$  key twice, for instance:  $7\frac{8}{9}$  means  $7\frac{8}{9}$  and is entered into the entry line by pressing  $7 \frac{a}{b/c} 8 \frac{a}{b/c} 9$  but this number is displayed in the *result* line (i.e. once  $\frac{ENTER}{}$  has been pressed) as  $7\frac{8}{9}$ .

## HP 30S Solving Problems Involving Fractions

$\text{2nd} \text{ a}\% \text{ d/c}$  is used for converting a mixed number to an improper fraction and vice versa, and  $\text{2nd} \text{ F}\leftrightarrow\text{D}$  for converting a decimal to and from a fraction. Let's illustrate all this with various examples.

## Practice working problems involving fractions

Example 1: Enter the proper fractions  $\frac{3}{9}$  and  $\frac{21}{124}$

Solution: Note that pressing  $\text{3} \text{ } \div \text{ } \text{9} \text{ } \text{ENTER}$  does not return a fraction but a decimal number which is the result—within the accuracy of the calculator—of dividing 3 by 9. As stated above, fractions are entered with the  $\text{a}\% \text{ c}$  key, which separates the numerator from the denominator. Press:

$\text{3} \text{ } \text{a}\% \text{ c} \text{ } \text{9}$

The entry line now reads  $3 \text{ } \div \text{ } 9$ . Press  $\text{ENTER}$  to display the result line. The fraction now displayed is  $1/3$ , which is equivalent to the entered fraction but reduced to its simple form. The HP 30S always tries to find the simplest equivalent fraction. Let's enter the second fraction by pressing:

$\text{2} \text{ } \text{1} \text{ } \text{a}\% \text{ c} \text{ } \text{1} \text{ } \text{2} \text{ } \text{4} \text{ } \text{ENTER}$

No reduction is possible this time, therefore the fraction displayed in the result line is  $21/124$ .

Example 2: Enter the improper fraction  $\frac{1000}{101}$  and the complex fraction  $\frac{1.8}{9}$

Solution: These fractions are entered exactly as in the previous example, but the results displayed after pressing the  $\text{ENTER}$  key are different. Let's enter the first fraction by pressing:

$\text{1} \text{ } \text{E} \text{ } \text{3} \text{ } \text{a}\% \text{ c} \text{ } \text{1} \text{ } \text{0} \text{ } \text{1} \text{ } \text{ENTER}$

The fraction displayed in the entry line ( $1\text{E}3 \text{ } \div \text{ } 101$ ) changes to  $9 \text{ } \div \text{ } 91/101$  which means  $9 \frac{91}{101}$ . Improper fractions are always converted to mixed numbers (i.e. an integer plus a proper fraction) after pressing  $\text{ENTER}$ . As with proper fractions, the HP 30S tries to give the simplest form. As to the complex fraction, press:

$\text{1} \text{ } \text{.} \text{ } \text{8} \text{ } \text{a}\% \text{ c} \text{ } \text{9} \text{ } \text{ENTER}$

It returns 0.2 and not the fraction  $\frac{1}{5}$ . This is because calculations containing both fractions and decimals are calculated in decimal format. As long as both the numerator and the denominator evaluate to an integer, the result will be in fraction format, though. For example  $\sqrt{(4)} \text{ } \div \text{ } 3$  will return  $2/3$ .

Example 3: Enter the mixed numbers  $7 \frac{2}{18}$ ,  $-1 \frac{57}{125}$  and  $3 \frac{19}{5}$ .

Solution: When entering mixed numbers, remember that the  $\text{a}\% \text{ c}$  key is used for separating both the integer from the proper fraction and the numerator from the denominator. To enter the first fraction press:

## HP 30S Solving Problems Involving Fractions

$\boxed{7}$   $\boxed{a/b/c}$   $\boxed{2}$   $\boxed{a/b/c}$   $\boxed{1}$   $\boxed{8}$   $\boxed{ENTER}$

The fraction returned is  $7\frac{1}{9}$ , i.e.  $7\frac{1}{9}$  which is the same as  $7\frac{2}{18}$  after doing a cancellation.

Let's now enter the second fraction  $-1\frac{57}{125}$ . On the HP 30S negative fractions and mixed numbers are keyed in by pressing the  $\boxed{+/-}$  key before entering either part (integer, numerator or denominator). It is usually pressed just before the integer of a mixed number or the numerator of a fraction. In this example press:

$\boxed{+/-}$   $\boxed{1}$   $\boxed{a/b/c}$   $\boxed{5}$   $\boxed{7}$   $\boxed{a/b/c}$   $\boxed{1}$   $\boxed{2}$   $\boxed{5}$   $\boxed{ENTER}$

No cancellation is possible this time, so the result is  $-1\frac{57}{125}$ .

Note that the third number is not actually a mixed number strictly speaking because its fraction part is not proper. Nevertheless, your HP 30S can handle it as well and will return the reduced, proper form. Press:

$\boxed{3}$   $\boxed{a/b/c}$   $\boxed{1}$   $\boxed{9}$   $\boxed{a/b/c}$   $\boxed{5}$   $\boxed{ENTER}$

which returns the mixed number  $6\frac{4}{5}$ .

Fractions can be entered and displayed wherever ordinary decimal numbers can be used. For example fractions can be used in ordinary arithmetic, in calculations with logarithmic and trigonometric functions, etc.

Example 4: Add  $1\frac{3}{4}$  to  $2\frac{5}{8}$

Solution: We will enter the two mixed numbers as explained above. No parenthesis is necessary because fractions (i.e. the  $\boxed{a/b/c}$  key) take priority over the addition. Press:

$\boxed{1}$   $\boxed{a/b/c}$   $\boxed{3}$   $\boxed{a/b/c}$   $\boxed{4}$   $\boxed{+}$   $\boxed{2}$   $\boxed{a/b/c}$   $\boxed{5}$   $\boxed{a/b/c}$   $\boxed{8}$

The entry line now reads  $1\frac{3}{4} + 2\frac{5}{8}$ . Press  $\boxed{ENTER}$  to carry out the calculation.

Answer: The result line reads  $4\frac{3}{8}$  which means  $4\frac{3}{8}$ . Since this calculation only contains fractions then the result is expressed as a fraction too.

Example 5: Express the previous result as a decimal.

Solution:  $F \leftrightarrow D$  ( $\boxed{2nd}$   $\boxed{F \leftrightarrow D}$ ) is a one-argument function,  $a F \leftrightarrow D$ , that converts a decimal to a fraction or vice versa. Since the result of the previous example is stored in ANS, you need not enter it again, simply press:

$\boxed{2nd}$   $\boxed{F \leftrightarrow D}$   $\boxed{ENTER}$

Answer: 4.375. Since pressing the  $\boxed{ENTER}$  key repeats the most recent calculation, if you press  $\boxed{ENTER}$  now, the result line will contain the mixed number again. Press  $\boxed{ENTER}$  one more time to display the decimal fraction.

HP 30S Solving Problems Involving Fractions

---

**Example 6:** Are the fractions  $\frac{147}{489}$  and  $\frac{539}{1793}$  equivalent to each other?

**Solution:** There are several ways of testing whether two fractions are equivalent. We can subtract one from the other and see if the result is zero, or we can calculate the cross multiplication since if  $a/b = c/d$  then  $ad = bc$ . Doing the division is another way. But let your HP 30S do the hard part, and just enter both fractions. Since the HP 30S carries out an automatic simplification, if the fractions displayed in the result line are the same then they are equivalent. Press:

$\left(\frac{1}{5}\right) \left(\frac{4}{3}\right) \left(\frac{7}{9}\right) \left(\frac{a}{b} \div \frac{c}{d}\right) \left(\frac{4}{1}\right) \left(\frac{8}{7}\right) \left(\frac{9}{9}\right) \left(\frac{ENTER}{3}\right)$  which returns 49/163

**Answer:** Since the latter results in 49/163 too, they are equivalent fractions.

**Example 7:** Convert the mixed number  $2\frac{89}{133}$  to an improper fraction.

**Solution:** We have seen how improper fractions are automatically converted to mixed numbers when evaluated. But the opposite is also possible. The  $a\frac{b}{c} \leftrightarrow \frac{d}{e}$  function ( $\left(\frac{2nd}{a} \frac{a}{b} \leftrightarrow \frac{c}{d}\right)$ ) carries out conversions between mixed numbers and improper fractions. Press:

$\left(\frac{2}{2}\right) \left(\frac{a}{b} \div \frac{c}{d}\right) \left(\frac{8}{8}\right) \left(\frac{9}{9}\right) \left(\frac{a}{b} \leftrightarrow \frac{c}{d}\right) \left(\frac{1}{1}\right) \left(\frac{3}{3}\right) \left(\frac{3}{3}\right) \left(\frac{2nd}{a} \frac{a}{b} \leftrightarrow \frac{c}{d}\right) \left(\frac{ENTER}{ENTER}\right)$

**Answer:**  $355/133$ , i.e.  $\frac{355}{133}$ . Incidentally, this is a good approximation to  $\pi$  (within 8.47 millionths of one percent) and very easy to remember because it's made by duplicating the first three odd numbers and inserting a division sign in the middle  $133\overline{)355}$ . And it was also the very first example in the HP-35 operating manual!

**Note.**  $a\frac{b}{c} \leftrightarrow \frac{d}{e}$  and  $F \leftrightarrow D$  must be the *last* functions entered into the entry line, otherwise a syntax error occurs. However, chain calculations are always possible using the ANS function. For example:

**Example 8:** Calculate  $\sin(30^\circ) + \frac{3}{4}$  and express the result as a fraction.

**Solution:** The calculation in question is:  $(\sin(30^\circ) + 3 \div 4) \rightarrow F \leftrightarrow D \rightarrow A \frac{b}{c} \leftrightarrow \frac{d}{e}$ . Trouble is that  $F \leftrightarrow D$  cannot be placed in the middle of a calculation. We have to split it this way:

$\left(\frac{sin}{sin}\right) \left(\frac{3}{3}\right) \left(\frac{0}{0}\right) \left(\frac{2nd}{2nd}\right) \left(\frac{DMS}{DMS}\right) \left(\frac{ENTER}{ENTER}\right) \left(\frac{+}{+}\right) \left(\frac{3}{3}\right) \left(\frac{a}{b} \div \frac{c}{d}\right) \left(\frac{4}{4}\right) \left(\frac{2nd}{2nd}\right) \left(\frac{F \leftrightarrow D}{F \leftrightarrow D}\right) \left(\frac{ENTER}{ENTER}\right)$

We now have the mixed number  $1\frac{1}{4}$  in ANS. (The sequence  $\left(\frac{2nd}{2nd}\right) \left(\frac{DMS}{DMS}\right) \left(\frac{ENTER}{ENTER}\right)$  inserts the symbol  $^\circ$  after 30, so as not to be affected by the current angle mode). We can now convert it to an improper fraction by pressing:

$\left(\frac{2nd}{2nd}\right) \left(\frac{a}{b} \leftrightarrow \frac{c}{d}\right) \left(\frac{ENTER}{ENTER}\right)$

## HP 30S Solving Problems Involving Fractions

Answer:  $\frac{5}{4}$

Example 9: Find a fraction which approximates  $\pi$  to four decimal places.

Solution: First of all, set the number of displayed decimal places to 4 by pressing  $\text{2nd}$   $\text{FIX}$   $4$ . Notice that  $\text{2nd}$   $\pi$   $\text{2nd}$   $\text{F}\leftrightarrow\text{D}$   $\text{ENTER}$  will *not* return a fraction, because  $\pi$  is calculated to 22 significant digits. Too many digits for F $\leftrightarrow$ D to handle.  $\pi$  has to be rounded to four decimal places before attempting the conversion. To do so, use the  $\text{2nd}$   $\text{RND}$  key :

$\text{2nd}$   $\text{RND}$   $\pi$   $\text{2nd}$   $\text{F}\leftrightarrow\text{D}$   $\text{ENTER}$

The mixed number  $3\frac{177}{1250}$  is returned. If a fraction is preferred, convert this number to an improper fraction by pressing:

$\text{2nd}$   $\text{a}/\text{b}/\text{c}$   $\leftrightarrow$   $\text{d}/\text{c}$   $\text{ENTER}$

Answer:  $3\frac{177}{1250} = \frac{3927}{1250} = 3.1416$