

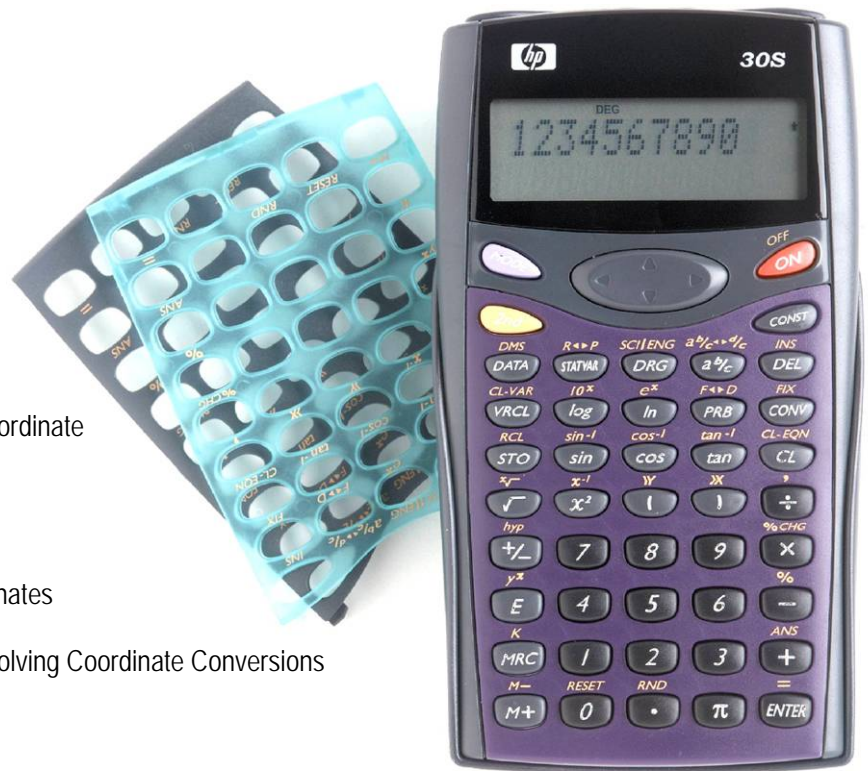


hp calculators

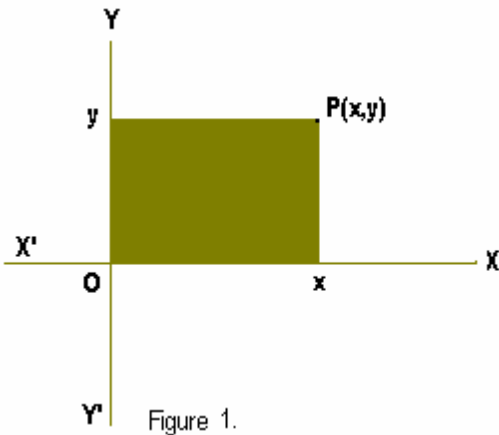
HP 30S Polar/Rectangular Coordinate Conversions

Rectangular and Polar Coordinates

Practice Solving Problems Involving Coordinate Conversions

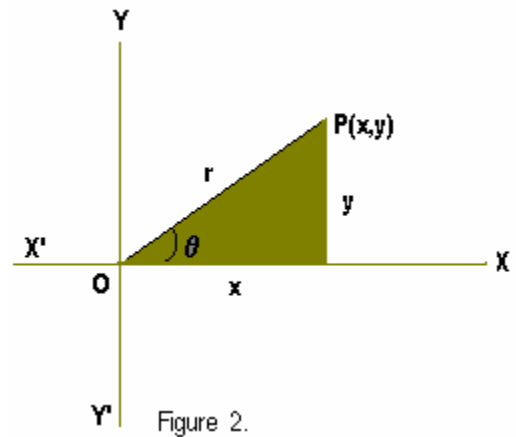


Rectangular and polar coordinates



Coordinates define the position of a point in space. In two dimensions, they are an ordered pair (a,b). There are two primary coordinate systems: rectangular and polar. The **rectangular** system is the commonest coordinate system and uses perpendicular lines in order to measure distances to the base lines or axes, which are also perpendicular to each other. It is often referred to as rectangular cartesian system, and even as cartesian system, even though cartesian coordinates may not be rectangular (i.e. when axes are not perpendicular to each other, coordinates are called oblique.) Figure 1 shows the plane rectangular cartesian coordinates of a two dimensional point P. They are written as (x,y). The x-coordinate is measured along or parallel to the XX'-axis and is called **abscissa**. Likewise, the y-coordinate is measure along the YY'-axis and is called **ordinate**.

**Polar** coordinates describe the position of a point P by its distance to a fixed point O (the pole) and the angle that OP makes with the base line (XX' in figure 2). The angle is measure in the positive or counterclockwise direction from the base line. Coordinates are written as (r,θ). OP is known as the **radius vector** and θ as **vectorial angle**.



The HP 30S provides four functions for converting between polar and cartesian coordinates. They are in the R↔P menu (2nd R↔P): R ▶ Pr, R ▶ Pθ, P ▶ Rx, P ▶ Ry. All these functions take two arguments e.g. R ▶ Pθ (x , y), which are separated by commas (2nd ,). By learning what these functions actually do, we will be able to use them in different contexts. Figure 2 shows the relationship between rectangular and polar coordinates. Note that:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan \frac{y}{x}, \quad x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

Practice solving problems involving coordinate conversions

Example 1: Convert the rectangular coordinates (-7.5,13) into polar coordinates.

Solution: To find the corresponding radius vector and vectorial angle, we can use the R ▶ Pr and R ▶ Pθ functions respectively. Press:

2nd R↔P +/- 7 . 5 2nd , 1 3 ENTER which returns r, and  
 2nd R↔P ▶ +/- 7 . 5 2nd , 1 3 ENTER which returns θ.

θ is expressed in the current angle unit. Note that the latter keystroke sequence can be shortened if we edit the previous calculation instead of entering it anew. If r has just been calculated, then to find θ press:

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$\blacktriangleright$   $\text{2nd}$   $\text{INS}$   $\text{2nd}$   $\text{R}\leftrightarrow\text{P}$   $\blacktriangleright$   $\text{ENTER}$   $\text{DEL}$   $\text{DEL}$   $\text{ENTER}$

Answer: Rounding to four decimal digits,  $r = 15.0083$  and  $\theta = 119.9816^\circ$

Example 2: Express the point whose polar coordinates are  $(\sqrt{3}, \frac{2\pi}{3})$  in rectangular coordinates.

Solution: The functions we will now use are  $\text{P} \blacktriangleright \text{Rx}$  and  $\text{P} \blacktriangleright \text{Ry}$ , which will return the abscissa and the ordinate respectively. Its arguments are  $r$  and  $\theta$ . Since  $\theta$  is an angle, you must make sure that the appropriate angle unit is set. Alternatively, you can specify the angle unit just after keying in the value. Let's now find the abscissa by pressing:

$\text{2nd}$   $\text{R}\leftrightarrow\text{P}$   $\blacktriangleright$   $\sqrt{\phantom{x}}$   $3$   $\text{2nd}$   $\text{,}$   $($   $2$   $\pi$   $\div$   $3$   $)$   $\text{2nd}$   $\text{DMS}$   $\blacktriangleright$   $\blacktriangleright$   $\text{ENTER}$   $\text{ENTER}$

To find the ordinate just replace  $\text{P} \blacktriangleright \text{Rx}$  with  $\text{P} \blacktriangleright \text{Ry}$  in the above calculation:

$\blacktriangleright$   $\text{2nd}$   $\text{INS}$   $\text{2nd}$   $\text{R}\leftrightarrow\text{P}$   $\blacktriangleleft$   $\text{ENTER}$   $\text{DEL}$   $\text{DEL}$   $\text{ENTER}$

Answer:  $(-0.8660, 1.500)$  to four decimal places.

Example 3: Find the hypotenuse of a right triangle whose catheti are 9 and 40.

Solution: Pythagoras' theorem states that the hypotenuse is given by  $\sqrt{x^2 + y^2}$  ( $x$  and  $y$  being the catheti of a right triangle). But this is exactly what the  $\text{R} \blacktriangleright \text{Pr}(x, y)$  function returns. Let's compare both methods:

$\sqrt{\phantom{x}}$   $9$   $x^2$   $+$   $4$   $0$   $x^2$   $\text{ENTER}$   
 $\text{2nd}$   $\text{R}\leftrightarrow\text{P}$   $9$   $\text{2nd}$   $\text{,}$   $4$   $0$   $\text{ENTER}$

Both take the same number of keystrokes...

Answer: 41. (The set 9, 40 and 41 is another example of Pythagorean triples. Refer to the HP 30S learning module *Powers and Roots*).

Example 4: A vector has components  $-8$  in the X direction and  $-5$  in the Y direction. In what direction does it point?

Solution: The angle is given (in the current angle unit) by  $\text{R} \blacktriangleright \text{P}\theta(-8, -5)$ :

$\text{2nd}$   $\text{R}\leftrightarrow\text{P}$   $+/-$   $8$   $\text{2nd}$   $\text{,}$   $+/-$   $5$   $\text{ENTER}$

Answer:  $-147.9946^\circ$  which is the same as the positive angle  $212.0054^\circ$  (because  $147.9946^\circ + 212.0054^\circ = 360^\circ$ ).

Notice that the calculation  $\arctan(\frac{-5}{-8})$  won't give the right result – you have to add  $180^\circ$  because the arctangent function does not take into account the quadrant where the vector lies, unlike the coordinate conversion function  $\text{R} \blacktriangleright \text{P}\theta$ .

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**Example 5:** Add two vectors having polar coordinates  $(8,30^\circ)$  and  $(12,60^\circ)$ . Represent the sum in terms of magnitude and angle  $(r, \theta)$ .

**Solution:** Vectors can be easily added (or subtracted) when expressed in rectangular form. The components of the resultant will be the sum of the corresponding components of the vectors. In other words, the sum vector has the following components.

$$P \blacktriangleright Rx(8,30^\circ) + P \blacktriangleright Rx(12,60^\circ) \text{ and } P \blacktriangleright Ry(8,30^\circ) + P \blacktriangleright Ry(12,60^\circ)$$

First of all, let's set the angle unit to degrees (press  $\text{DRG}$ , select DEG and press  $\text{ENTER}$ ). And now we can find the components and store them in the variables A and B by pressing:

$\text{CL}$   $\text{2nd}$   $R\leftrightarrow P$   $\blacktriangleright\blacktriangleright$   $8$   $\text{2nd}$   $\text{,}$   $30$   $\text{0}$   $\text{+}$   $\text{2nd}$   $R\leftrightarrow P$   $\blacktriangleright\blacktriangleright$   $12$   $\text{2nd}$   $\text{,}$   $60$   $\text{0}$   $\blacktriangleright$   
 $\text{STO}$   $\text{ENTER}$  and

$\text{2nd}$   $R\leftrightarrow P$   $\blacktriangleleft$   $8$   $\text{2nd}$   $\text{,}$   $30$   $\text{0}$   $\text{+}$   $\text{2nd}$   $R\leftrightarrow P$   $\blacktriangleleft$   $12$   $\text{2nd}$   $\text{,}$   $60$   $\text{0}$   $\blacktriangleright$   
 $\text{STO}$   $\text{ENTER}$

(Remember that you can also obtain the latter expression by editing the former). In order to find the magnitude and the angle of the resultant we have to express the resultant in polar coordinates, i.e. we have to calculate  $R \blacktriangleright Pr(X,Y)$  and  $R \blacktriangleright P\theta(X,Y)$ :

$\text{2nd}$   $R\leftrightarrow P$   $\text{ENTER}$   $\text{VRCL}$   $\text{ENTER}$   $\text{2nd}$   $\text{,}$   $\text{VRCL}$   $\blacktriangleright$   $\text{ENTER}$   $\text{ENTER}$  and  $\text{2nd}$   $\text{INS}$   $\text{2nd}$   $R\leftrightarrow P$   $\blacktriangleright$   $\text{ENTER}$   $\text{DEL}$   $\text{DEL}$   $\text{ENTER}$

**Answer:** Rounding to four decimal digits,  $(19.3462, 48.0675^\circ)$ .

**Example 6:** The coordinates of the point P (see fig. 3) after the axes have been rotated  $30^\circ$  are  $(4.5, 1.5)$ . Calculate the old coordinates.

**Solution:** The old coordinates are given by:

$$x = 4.5 \cos 30 - 1.5 \sin 30 \text{ and}$$

$$y = 4.5 \sin 30 + 1.5 \cos 30$$

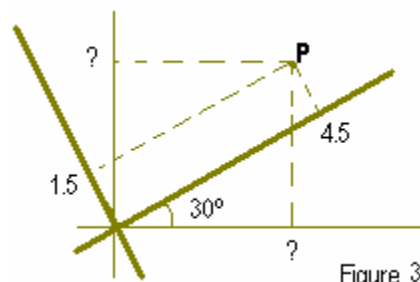


Figure 3.

Make sure DEG is the current angle mode. Then press:

$4$   $\cdot$   $5$   $\text{cos}$   $30$   $\blacktriangleright$   $-$   $1$   $\cdot$   $5$   $\text{sin}$   $30$   $\text{ENTER}$  and  
 $4$   $\cdot$   $5$   $\text{sin}$   $30$   $\blacktriangleright$   $+$   $1$   $\cdot$   $5$   $\text{cos}$   $30$   $\text{ENTER}$

Notice that the  $P \blacktriangleright Rx(r,\theta)$  function can be used to calculate  $r \cdot \cos\theta$ . Likewise,  $P \blacktriangleright Ry(r,\theta)$  returns  $y \cdot \sin\theta$ . Therefore, the equations that describe the transformation of coordinates can be written as:

$$x = P \blacktriangleright Rx(4.5,30) - P \blacktriangleright Ry(1.5,30) \text{ and}$$

$$y = P \blacktriangleright Ry(4.5,30) + P \blacktriangleright Rx(1.5,30)$$

which can be evaluated by pressing:

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$\overset{2nd}{\curvearrowright}$   $\overset{R\leftrightarrow P}{\blacktriangleright}$   $\blacktriangleright$   $\overset{4}{\square}$   $\overset{\cdot}{\square}$   $\overset{5}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{,}{\square}$   $\overset{3}{\square}$   $\overset{0}{\square}$   $\blacktriangleright$   $\overset{-}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{R\leftrightarrow P}{\blacktriangleleft}$   $\blacktriangleleft$   $\overset{1}{\square}$   $\overset{\cdot}{\square}$   $\overset{5}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{,}{\square}$   $\overset{3}{\square}$   $\overset{0}{\square}$   
 $\overset{ENTER}{\square}$  and  $\overset{CL}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{R\leftrightarrow P}{\blacktriangleleft}$   $\blacktriangleleft$   $\overset{4}{\square}$   $\overset{\cdot}{\square}$   $\overset{5}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{,}{\square}$   $\overset{3}{\square}$   $\overset{0}{\square}$   $\blacktriangleright$   $\overset{+}{\square}$   $\overset{2nd}{\curvearrowright}$   $\overset{R\leftrightarrow P}{\blacktriangleright}$   $\blacktriangleright$   $\overset{1}{\square}$   $\overset{\cdot}{\square}$   $\overset{5}{\square}$   $\overset{2nd}{\curvearrowright}$   
 $\overset{,}{\square}$   $\overset{3}{\square}$   $\overset{0}{\square}$   $\overset{ENTER}{\square}$  respectively.

Answer: (3.1471, 3.5490) to four decimal places.

**Note.** The HP 30S learning module *Solving Problems Involving Complex Numbers* demonstrates the use of coordinate conversion functions to perform calculations with complex numbers.