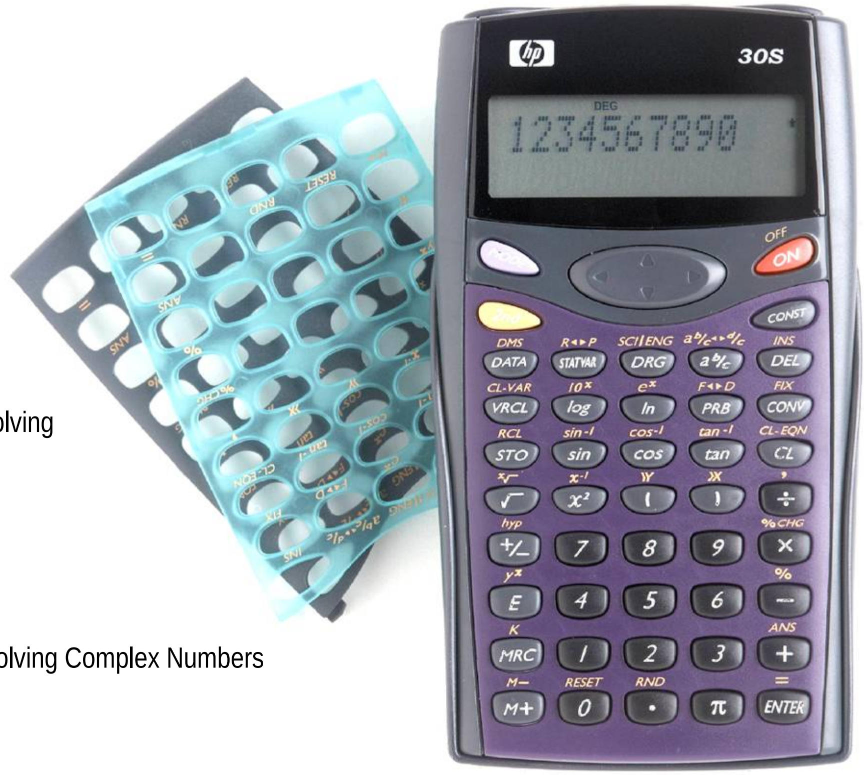
hp calculators



HP 30S Solving Problems Involving
Complex Numbers

Basic Concepts

Practice Solving Problems Involving Complex Numbers

Basic concepts

There is *no* real number *x* such that $x^2 + 1 = 0$. To solve this kind of equations a new set of numbers must be introduced. A complex number is a number of the form $a + bi$ where *a* and *b* are real numbers and *i* is the square root of –1, i.e. $i^2 = -1$, and is called the imaginary unit. Since *i* is used for representing the intensity of current in electromagnetism, engineers often write the imaginary unit as *j*. The real *a* is called the real part of the complex number, and *b*, also real, is the imaginary part. When both *a* and *b* are integers, the complex number $a + bi$ is called a Gaussian integer (e.g. $-4 + 3i$). Notice that real numbers can be thought as the subset of complex numbers whose imaginary part is zero. Here are the most basic rules:

- $a + bi = c + di$ if and only if a = c and b = d
- $(a + bi) + (c + di) = (a + c) + (b + d)i$
- $(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$
- $\dfrac{a + bi}{c + di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{bc - ad}{c^2 + d^2} i$

The conjugate complex number of $a + bi$ is $a - bi$. Note that the product of a pair of conjugate numbers is a real number ($a^2 + b^2$). The modulus or absolute value of the complex number $a + bi$ is defined as $\sqrt{a^2 + b^2}$.

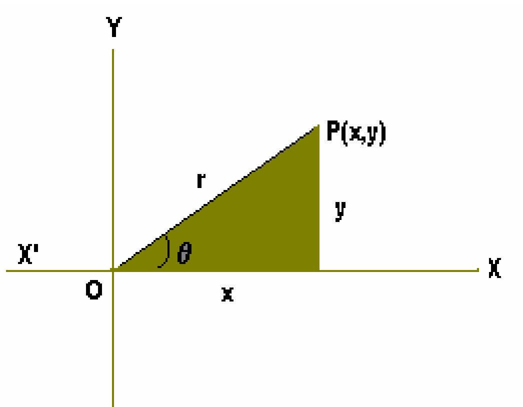


Figure 1.

Figure 1 shows the complex plane, also known as the *Argand diagram*. It is a representation of the complex number z = $x + yi$. XX' is the real axis, and YY' is the imaginary axis. The point P whose cartesian coordinates are (x, y) is called the affix of the complex number z. Note that r (the distance of the affix from the origin O) is equal to the modulus of z. The angle θ is called the argument of z. From the figure: $\tan\theta = \dfrac{y}{x}$

A complex number $x + yi$ can be represented in various ways:

- (x, y) i.e. as cartesian coordinates – the rectangular form.
- $r\cos\theta + ir\sin\theta$. This is the modulus-argument form or polar form, often written as $(r, \angle\theta)$, i.e. the polar coordinates.
- $re^{i\theta}$. It is the exponential form, and is based on Euler's formula $\cos\theta + i\sin\theta = e^{i\theta}$ (where θ is expressed in radians). The function $\cos\theta + i\sin\theta$ is sometimes referred to as cis θ. Note that if $\theta = \pi$ then we obtain the well-known identity $e^{i\pi} = -1$.

Practice solving problems involving complex numbers

The HP 30S has no specific functions for operating with complex numbers. Having said that, this calculator is powerful enough to carry out calculations with complex numbers easily. It is the purpose of this learning module to introduce some methods of doing such calculations on your HP 30S.

Example 1: Find the modulus and the argument of the complex number 5 + 6·i.

Solution: The R↔P menu (2nd R↔P) contains four functions that are very useful in operating with complex numbers. They are the rectangular to and from polar conversion functions, which are described in greater detail in the HP 30S learning module *Polar/Rectangular Coordinate Conversions*. For our purpose, R▸Pr and R▸Pθ return the modulo and the argument, respectively, of a complex number expressed in rectangular form. The real and imaginary parts are separated by commas (2nd ,), e.g. R▸Pr(a,b). To find the modulus in this example, press:

2nd R↔P ENTER 5 2nd , 6 ENTER

and to find the argument, press:

2nd R↔P ▶ ENTER 5 2nd , 6 ENTER

In fact, pressing the ENTER key to enter the function name into the entry line is not necessary if the next keystroke is a number entry key, e.g.: 2nd R↔P ▶ 5 2nd , 6 ENTER .

Remember that the argument of a complex number is an angle, and therefore its value depends on the angle unit. The R▸Pθ function always returns the angle in the current angular mode.

Answer: Rounding to four decimal digits, r = 7.8102 and θ = 50.1944º

Example 2: The voltage in a circuit is 45 + 5j volts and the impedance is 3 + 4j ohms. Find the total current.

Solution: The current is given by the following formula:

$$I = \frac{E}{Z} = \frac{45+5j}{3+4j}$$

We therefore have to divide two complex numbers here. One way of doing this calculation is to use the basic formula given above, which divides two complex numbers expressed in rectangular form. While additions and subtractions of complex numbers are easily done in rectangular form, the product and division are much easier if the numbers are in exponential form because:

$$re^{j\theta} \cdot qe^{j\varphi} = rq \cdot e^{j(\theta+\varphi)}$$

To multiply (divide) two complex numbers we just have to multiply (divide) their moduli and add (subtract) their arguments. But, be warned that *<u>you must work in radians</u>* when using this formula. So, press DRG , select RAD and then press ENTER . In our case, the modulus of the current will be the modulus of E divided by the modulus of Z, i.e. R▸Pr(45,5) ÷ R▸Pr (3,4). To store this modulus in the variable A, press:

2nd R↔P 4 5 2nd , 5 ▶ ÷ 2nd R↔P 3 2nd , 4 ▶ STO ENTER .

The argument of the current is simply the difference between the arguments: R▸Pθ(45,5) – R▸Pθr (3,4). Store the resulting argument in B by pressing:

2nd R↔P ▶ 4 5 2nd , 5 ▶ − 2nd R↔P ▶ 3 2nd , 4 ▶ STO ▶ ENTER

Now we have to convert the current vector to rectangular form using these formulae: P ▸ Rx(A, B) and P ▸ Ry(A, B), that is to say:

[2nd] [R↔P] ▶ ▶ [ENTER] [VRCL] [ENTER] [2nd] [,] [VRCL] ▶ [ENTER] [ENTER] and
[2nd] [R↔P] ◀ [ENTER] [VRCL] [ENTER] [2nd] [,] [VRCL] ▶ [ENTER] [ENTER] respectively.

In this case, pressing the [ENTER] key to confirm the selection in the R↔P menu is mandatory.

(Remember that the latter expression can be obtained by *editing* the former: ▶ [2nd] [INS] [2nd] [R↔P] ◀ [ENTER] [DEL] [DEL] [ENTER]).

Answer: $9.0554e^{-j0.8166} = 6.2 - 6.6j$ amperes.

Example 3: Calculate $\sqrt{5+3i}$

Solution: When expressed in exponential form, powers of complex numbers are very easy to calculate because *in radian mode*:

$$(a+b\cdot i)^n = \left(r\cdot e^{i\theta}\right)^n = r^n\cdot e^{i\cdot n\cdot\theta}$$

Therefore, the result is a complex number whose modulus and argument are r^n and $n\cdot\theta$ respectively, where r and θ are the modulus and the argument of the original complex number. In this case, n= 0.5. The result *in rectangular form* is given by:

$P \blacktriangleright Rx(\sqrt{R \blacktriangleright Pr(5,3)}, \frac{R \blacktriangleright P\theta(5,3)}{2})$ (abscissa) and $P \blacktriangleright Ry(\sqrt{R \blacktriangleright Pr(5,3)}, \frac{R \blacktriangleright P\theta(5,3)}{2})$ (ordinate).

Incidentally, the HP 30S coordinate conversion functions are not nestable, i.e. they cannot appear in their own arguments.[1] This means that the above calculations have to be split. Make sure the RAD annunciator is still displayed and press:

[2nd] [R↔P] [5] [2nd] [,] [3] ▶ [2nd] [y^x] [.] [5] [STO] [ENTER]
[2nd] [R↔P] ▶ [5] [2nd] [,] [3] ▶ [÷] [2] [STO] ▶ [ENTER]

A and B now contain $\sqrt{R \blacktriangleright Pr(5,3)}$ and $\frac{R \blacktriangleright P\theta(5,3)}{2}$ respectively. The result is once again P ▸ Rx(A, B) and P ▸ Ry(A, B):

[2nd] [R↔P] ▶ ▶ [ENTER] [VRCL] [ENTER] [2nd] [,] [VRCL] ▶ [ENTER] [ENTER] and
▶ [2nd] [INS] [2nd] [R↔P] ◀ [ENTER] [DEL] [DEL] [ENTER]

Answer: $2.3271 + 0.6446\cdot i$, rounded to four decimal places.

Note. The HP 30S learning modules *Solving Quadratic Equations* (example 7) and *Solving Linear Systems* (example 8) illustrate the use of expressions in Q SOLV and L SOLV modes to automate the process of finding the complex roots of a quadratic polynomial, and the division of two complex numbers expressed in rectangular form, respectively.

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[1] And these functions cannot be enclosed in parentheses either, that is why we must use the power function, i.e. [2nd] [y^x] [.] [5] , instead of the square root function [√] , when calculating A.