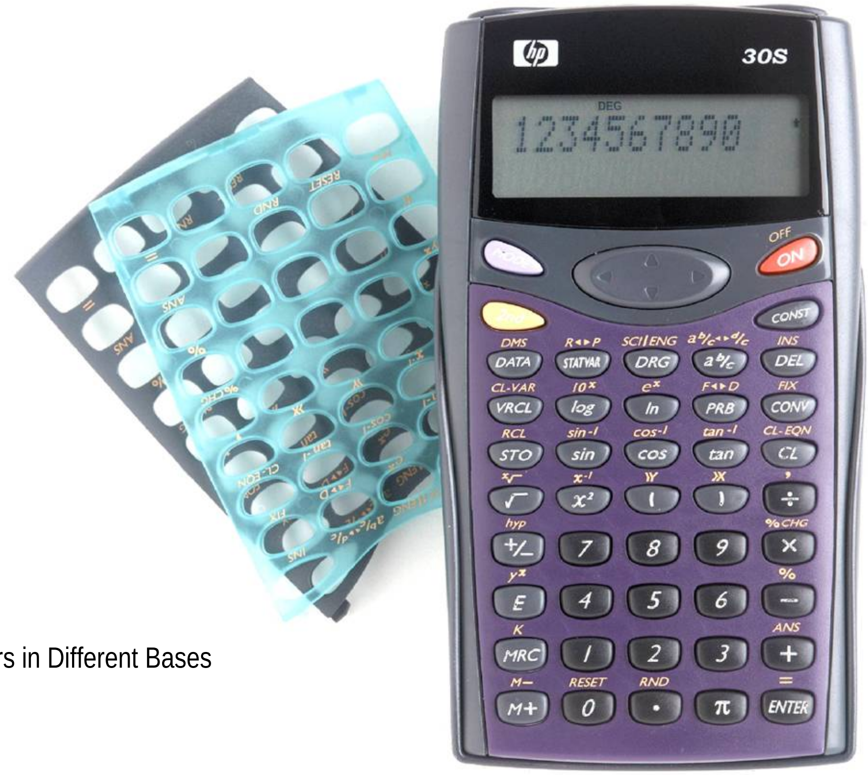
hp calculators



HP 30S Base Conversions

Numbers in Different Bases

Practice Working with Numbers in Different Bases

Numbers in different bases

Our number system (called Hindu-Arabic) is a *decimal* system (it's also sometimes referred to as denary system) because it counts in 10s and powers of 10. Its base (i.e. the number on which the number system is built), is therefore 10. While base 10 numbers are extensively used, this is not the only possible base. There have been number systems with base 20 (used by the Mayas), mixed bases of 10 and 60 (used by the Babylonians), of 5 and 10 (ancient Romans), etc. Even nowadays the sexagesimal system (base 60) is used in some measurements of time and angle. Bases 2, 8, 10 and 16 are particularly important in computing. Base 2 numbers are called binary numbers and their digits are limited to 1 and 0. A common abbreviation of binary digit is bit, which is either 1 or 0. Base 8 are called octal numbers, whose digits are 0, 1, 2, 3, 4, 5, 6 and 7. Finally, base 16 numbers are called hexadecimal numbers: 0, 1, 2, ,3 ,4 ,5 ,6 ,7 ,8 ,9, A(=10d), B(=11d), C(=12d), D(=13d), E(=14d) and F(=15d).

The main difference between all these bases is the value a digit has because of its place in a numeral. For example, in the decimal number 378 the digit 3 has value 300, 7 has value 70 and 8 has value 8. In other words:

$$378 = 3 \cdot 10^2 + 7 \cdot 10^1 + 8 \cdot 10^0$$

But if 378 were a hexadecimal number then its decimal value would be:

$$378\text{h} = 3 \cdot 16^2 + 7 \cdot 16^1 + 8 \cdot 16^0 = 888\text{d}$$

A small h, b, d or o after or before a number means that this number is expressed in hexadecimal, binary, decimal or octal base respectively.

In the HP 30S, there is no specific operating mode to operate with binary, hexadecimal or octal numbers. But, in the following examples we'll learn how to perform conversions easily on your HP 30S.

Practice working with numbers in different bases

Example 1: Convert the hexadecimal number F9014 to decimal base.

Solution: Here's one way of converting an integer $d_n d_{n-1} \ldots d_2 d_1$ in base b to a decimal integer:

$$((d_n b + d_{n-1})b + \ldots + d_2)b + d_1$$

In this example, the calculation is: $(((15 \cdot 16 + 9)16 + 0)16 + 1)16 + 4$, which can be done by pressing:

((1 5 × 1 6 + 9) 1 6 x^2 + 1) 1 6 + 4 ENTER

It is possible to automate this method by storing the above expression in the EQN variable:

$$B - B + A + B(C + B(D + B(X_1 + B(X_2 + B(X + B(Y + B(Y_1 + BY_2))))))) \rightarrow EQN$$

Then when recalling and executing EQN, the calculator will prompt us for the value of B, the base, first (because it is the first variable used in the expression) and then for the value of nine variables more, namely A, C, D, X_1, X_2, X, Y, Y_1 and Y_2 which represent the digits d_1 through d_9, respectively. Enter the digits backwards, i.e. from right to left. If the number has less than nine digits, simply key in 0 to the remaining variables. To store this expression in EQN press:

[2nd] [CL-EQN] [VRCL] ▶ [−] [VRCL] ▶ [+] [VRCL] [+] [VRCL] ▶ [(] [VRCL] ▶ ▶ [+] [VRCL] ▶ [(] [VRCL] ▶ ▶ ▶ [+] [VRCL] ▶ [(] [VRCL] ▶ ▶ ▶ ▶ [+] [VRCL] ▶ [(] [VRCL] ▶ ▶ ▶ ▶ ▶ [+] [VRCL] ▶ [(] [VRCL] ◀ ◀ ◀ ◀ [+] [VRCL] ▶ [(] [VRCL] ◀ ◀ ◀ [+] [VRCL] ▶ [(] [VRCL] ◀ ◀ [+] [VRCL] ▶ [×] [VRCL] ◀ [ENTER] [STO] ◀ [ENTER]

Note that you need not enter the trailing parentheses because the STO command operates on the whole entry line. (For additional information on the EQN register, please refer to the HP 30S learning module *Working with Expressions*).

Let's do the above conversion again. Press [2nd] [CL-VAR] to clear all variables (except EQN) so we won't have to clear the entry line nor enter the zero values:

[VRCL] ◀ [ENTER] [ENTER] [1] [6] [ENTER] [4] [ENTER] [1] [ENTER] [ENTER] [9] [ENTER] [1] [5] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER]

Answer: 1019924

Example 2: Express the numbers 111011101b and 7052014o in base 10.

Solution: These conversions can be performed by the expression we stored into EQN in the previous example, because they are conversions to decimal base of positive[1] integers with up to nine digits. Let's convert the former by pressing:

[2nd] [CL-VAR] [VRCL] ◀ [ENTER] [ENTER] [2] [ENTER] [1] [ENTER] [ENTER] [1] [ENTER] [1] [ENTER] [1] [ENTER] [ENTER] [1] [ENTER] [1] [ENTER] [1] [ENTER]

As to the octal number, press:

[2nd] [CL-VAR] [VRCL] ◀ [ENTER] [ENTER] [8] [ENTER] [4] [ENTER] [1] [ENTER] [ENTER] [2] [ENTER] [5] [ENTER] [ENTER] [7] [ENTER] [ENTER] [ENTER]

Answers: 477 and 1856524, respectively.

Example 3: Convert $3.0002_8 \cdot 8^{11}$ and $\mathrm{ACDC}_{16} \cdot 16^{34}$ to decimal.

Solution: Integers with exponent, $d_n.d_{n-1}...d_2d_1 \cdot b^p$, can also be converted using the above expression. Simply multiply the result of converting $d_nd_{n-1}...d_2d_1$ to decimal by b^{p-n+1}. Therefore, in order to convert the octal number, let's convert the mantissa first by pressing:

[2nd] [CL-VAR] [VRCL] ◀ [ENTER] [ENTER] [8] [ENTER] [2] [ENTER] [ENTER] [ENTER] [ENTER] [3] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER]

which returns 12290, which must now be multiplied by 8^{11-5+1}:

[×] [8] [2nd] [y^x] [(] [1] [1] [−] [5] [+] [1] [ENTER]

[1] To convert a negative number, change its sign, convert, and change the sign of the result.

As to the hex number, note that: $ACDC_{16} \cdot 16^{34} = A.CDC_{16} \cdot 16^{37}$. The method is now exactly the same as before. The sequence:

[2nd] [CL-VAR] [VRCL] ◀ [ENTER] [ENTER] [1] [6] [ENTER] [1] [2] [ENTER] [1] [3] [ENTER] [1] [2] [ENTER] [1] [0] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER] [ENTER]

converts ACDCh to 44252d, which must be multiplied by 16^{37-4+1} so as to complete the conversion:

[×] [1] [6] [2nd] [y^x] [(] [3] [7] [−] [4] [+] [1] [ENTER]

<u>Answers:</u> $2.5770399808 \cdot 10^{10}$ and $3.854892877 \cdot 10^{45}$, respectively.

<u>Example 4:</u> Convert 0.11011011_2 to decimal.

<u>Solution:</u> Note that $0.11011011_2 = 1.1011011_2 \cdot 2^{-1}$. So first of all, let's convert 11011011_2 to decimal:

[2nd] [CL-VAR] [VRCL] ◀ [ENTER] [ENTER] [2] [ENTER] [1] [ENTER] [1] [ENTER] [ENTER] [1] [ENTER] [1] [ENTER] [ENTER] [1] [ENTER] [1] [ENTER] [ENTER]

and now let's multiply the result (210) by 2^{-1-8+1}, that is to say, divide by 2^8:

[÷] [2] [2nd] [y^x] [8] [ENTER]

<u>Answer:</u> 0.85546875

<u>Example 5:</u> Convert 1046 to hexadecimal.

<u>Solution:</u> This is the general procedure to convert a decimal integer x into base b:

- Find the integer $f \le \frac{\ln x}{\ln b}$
- Find $E_1 = \frac{x}{b^f}$. Let d_1 be the integer part of E_1
- Continue for $E_{i+1} = (E_i - d_i)b$. The result is $d_1 d_2 \ldots d_{n-1} d_n$, where d_i is the integer part of E_i. The last digit, d_n, is found when i = f + 1.

In this case, as [ln] [1] [0] [4] [6] ▶ [ln] [1] [6] [ENTER] returns 2.507666784, f = 2, which means that the hex number has *three* digits. The first digit of the result will be the integer part of $\frac{1046}{16^2}$:

[1] [0] [4] [6] [÷] [1] [6] [x²]

which results in 4.0859375. Therefore, the first digit is 4. The second one is calculated as follows:

[−] [4] [ENTER] [×] [1] [6] [ENTER]

which returns 1.375. Then, the second digit is 1. Finally, the last digit is the integer part of:

[−] [1] [ENTER] [×] [1] [6] [ENTER]

which gives 6.

Answer: 416h.

Example 6: Convert $6.022 \cdot 10^{23}$ to octal.

Solution: Let's calculate $\frac{\ln x}{\ln b}$ by pressing

[ln] [6] [.] [0] [2] [2] [E] [2] [3] [►] [÷] [ln] [8] [ENTER]

which returns 26.33152963. Therefore f = 26, which is also the exponent in base 8. There is no need to find all the 27 digits of the answer because the decimal number is given to four significant digits. The first digit will be the integer part of:

[6] [.] [0] [2] [2] [E] [2] [3] [÷] [8] [2nd] [y^x] [2] [6]

which results in 1.99251266. Therefore the first digit is 1. Let's compute four digits more:

[−] [1] [ENTER] [×] [8] [ENTER] which returns 7.940101276, therefore $d_2 = 7$,
[−] [7] [ENTER] [×] [8] [ENTER] which returns 7.520810209, therefore $d_3 = 7$,
[−] [7] [ENTER] [×] [8] [ENTER] which returns 4.166481673, therefore $d_4 = 4$,
[−] [4] [ENTER] [×] [8] [ENTER] which returns 1.331853385, therefore $d_5 = 1$.

Answer: $1.7741_8 \cdot 8^{26}$