

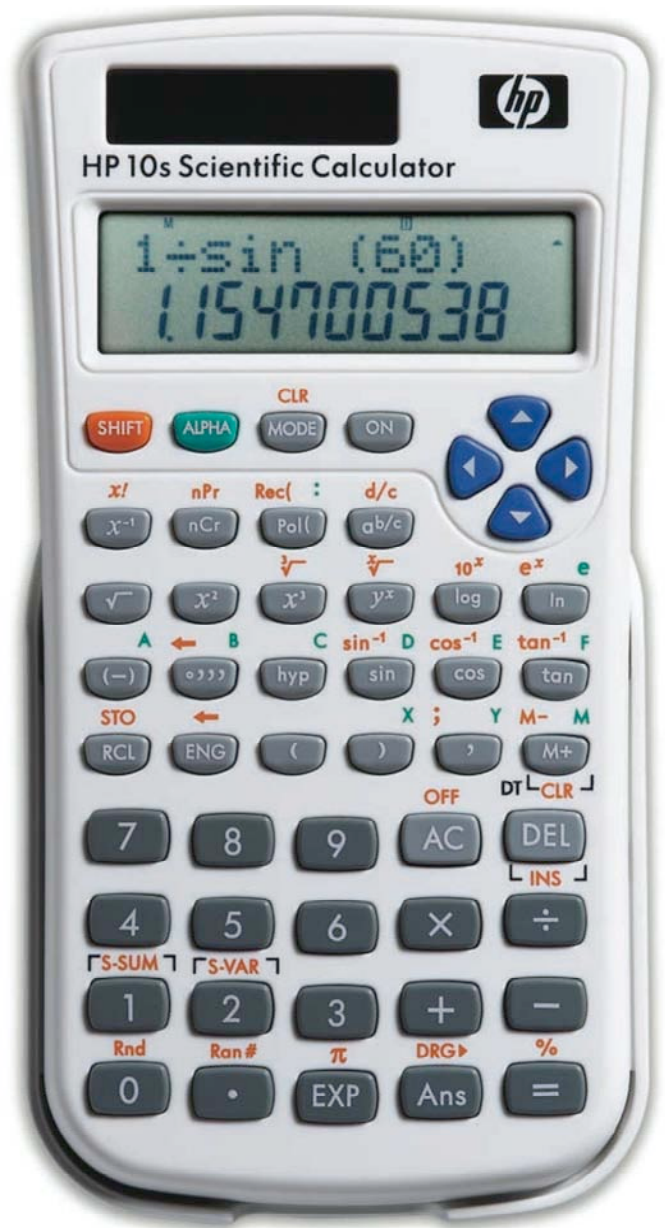


## hp calculators

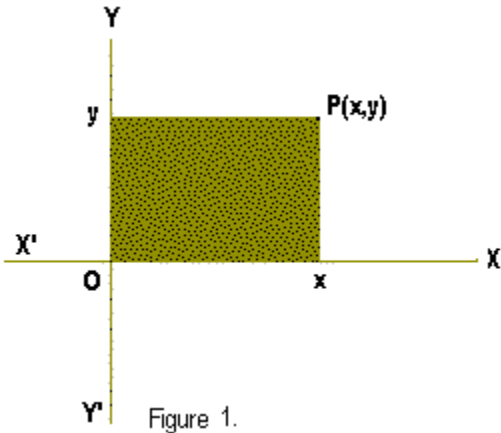
### HP 10s Polar/Rectangular Coordinate Conversions

Rectangular and Polar Coordinates

Practice Solving Problems Involving  
Coordinate Conversions

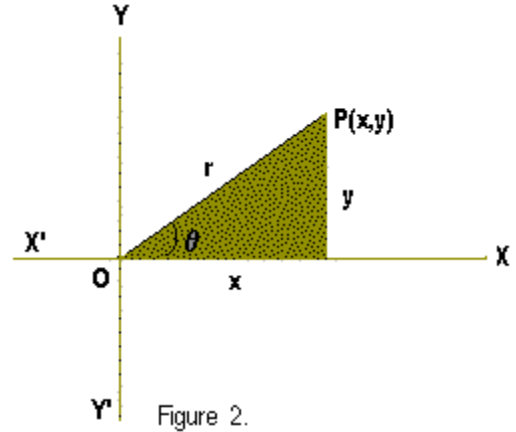


**Rectangular and polar coordinates**



Coordinates define the position of a point in space. In two dimensions, they are an ordered pair (a,b). There are two primary coordinate systems: rectangular and polar. The **rectangular** system is the commonest coordinate system and uses perpendicular lines in order to measure distances to the base lines or axes, which are also perpendicular to each other. It is often referred to as rectangular cartesian system, and even as cartesian system, even though cartesian coordinates may not be rectangular (when axes are not perpendicular to each other, coordinates are called oblique.) Figure 1 shows the plane rectangular cartesian coordinates of a two dimensional point P. They are written as (x,y). The x-coordinate is measured along or parallel to the XX'-axis and is called **abscissa**. Likewise, the y-coordinate is measure along the YY'-axis and is called **ordinate**.

**Polar** coordinates describe the position of a point P by its distance to a fixed point O (the pole) and the angle that OP makes with the base line (XX' in figure 2). The angle is measured in the positive or counterclockwise direction from the base line. Coordinates are written as (r, θ). OP is known as the **radius vector** and θ as **vectorial angle**.



By learning what these functions actually do, we will be able to use them in different contexts. Figure 2 shows the relationship between rectangular and polar coordinates. Note that:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan \frac{y}{x}, \quad x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

The HP 10s provides two functions for converting between polar and rectangular coordinates. They are R→P ( $\text{Pol}$ ) and P→R ( $\text{Rec}$ ), which take two arguments that are input using the  $\text{◊}$  key to separate them. R→P and P→R return r and x, respectively; the θ and y values are shown by pressing  $\text{RCL}$  key, r and x can be reviewed by pressing  $\text{RCL}$  key.

	R→P		P→R	
	INPUT	OUTPUT	INPUT	OUTPUT
<b>e</b>	x	r	r	x
<b>f</b>	y	θ	θ	y

## Practice solving problems involving coordinate conversions

**Example 1:** Convert the rectangular coordinates  $(-7.5, 13)$  into polar coordinates.

**Solution:** The function that we need to use in this example is  $\text{Pol}()$ , which converts the pair of rectangular coordinates  $(x, y)$  into the polar coordinates  $(r, \theta)$ .

$\text{Pol}()$   $(-)$   $7$   $.$   $5$   $,$   $1$   $3$   $)$   $=$

$r$  is displayed. To view the value of  $\theta$  simply press  $\text{RCL}$   $\text{F}$ . Note that  $\theta$  is expressed in the current angle unit. If you need to view  $r$  again, press  $\text{RCL}$   $\text{E}$ .

**Answer:** Rounding to four decimal digits,  $r = 15.0083$  and  $\theta = 119.9816^\circ$

**Example 2:** Express the point whose polar coordinates are  $(\sqrt{3}, \frac{2\pi}{3})$  in rectangular coordinates.

**Solution:** The function we will now use is  $\text{SHIFT RecI}$ , which will return the abscissa and the ordinate respectively. Its arguments are  $r$  and  $\theta$ . Since  $\theta$  is an angle, you must make sure that the appropriate angle unit is set before starting the calculation. Since the angle is radians, press  $\text{MODE MODE}$   $2$ .

$\text{SHIFT RecI}$   $\sqrt{\phantom{x}}$   $3$   $,$   $2$   $\times$   $\text{SHIFT}$   $\pi$   $\div$   $3$   $)$   $=$

**Answer:**  $(-0.8660, 1.500)$  rounded to four decimal digits.

**Example 3:** Find the hypotenuse of a right triangle whose catheti are 9 and 40.

**Solution:** Pythagoras' theorem states that the hypotenuse is given by  $\sqrt{x^2 + y^2}$  ( $x$  and  $y$  being the catheti of a right triangle). But this is exactly what the  $\text{P} \rightarrow \text{R}$  function returns. Let's compare both methods:

$9$   $x^2$   $+$   $4$   $0$   $x^2$   $=$   $\sqrt{\phantom{x}}$  (8 keystrokes)  
 $\text{Pol}()$   $9$   $,$   $4$   $0$   $)$   $=$  (7 keystrokes)

The latter method is one keystroke shorter. In fact, since the closing parenthesis is not required, it could be done in 6 keystrokes.

**Answer:** 41. (The set 9, 40 and 41 is another example of Pythagorean triples: refer to the HP 10s learning module *Powers and Roots*).

**Example 4:** A vector has components  $-8$  in the X direction and  $-5$  in the Y direction. In what direction does it point?

**Solution:** The angle is given (in the current angle unit) by  $\text{R} \rightarrow \text{P}(-8, -5)$ : Press  $\text{MODE MODE}$   $1$  for degrees:

$\text{Pol}()$   $8$   $,$   $5$   $)$   $=$   $\text{RCL}$   $\text{F}$

**Answer:**  $-147.9946^\circ$  which is the same as the positive angle  $212.0054^\circ$  (because  $147.9946^\circ + 212.0054^\circ = 360^\circ$ ).