

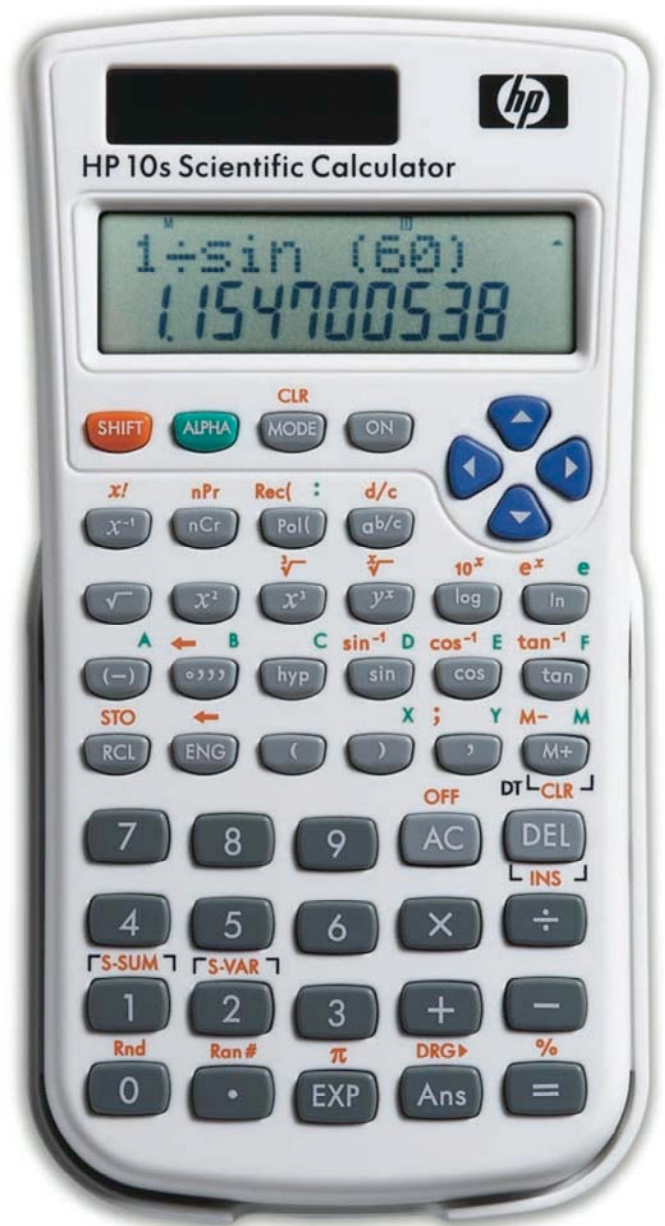


## hp calculators

### HP 10s Powers and Roots

Powers and Roots

Practice Solving Problems Involving Powers and Roots



**Powers and roots**

The number  $a$  in  $a^m$  is said to be *raised to the power*  $m$ , which is also called index or exponent. It obeys the so-called index laws, namely:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

where  $a$  is any number and  $m$  and  $n$  are rational numbers. The process of raising to a power is often referred to as *involution*. The opposite of a power is called a root. For example, if  $5^3 = 125$  then 5 is the third root of 125, and it is often written thus:

$$5 = \sqrt[3]{125}$$

The process of finding a root is known as *extraction* and also as *evolution*. Evolution is the inverse of the involution. Notice the following important relationships:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

The HP 10s has numerous functions to calculate powers and roots. These are:  $x^2$ ,  $x^3$ ,  $\text{SHIFT } 10^x$ , the exponential function  $\text{SHIFT } e^x$ ,  $x^{-1}$  (same as  $1/x$ ), the power function  $y^x$ ,  $\sqrt{\quad}$  which calculates square roots (i.e. second roots),  $\text{SHIFT } \sqrt[3]{\quad}$ , that calculates cube roots (the third root), and the  $x$ th-root function  $\text{SHIFT } \sqrt[x]{\quad}$ .

**Practice solving problems involving powers and roots**

Example 1: Find the hypotenuse of a triangle the catheti of which are 8 and 15 (see figure 1).

Solution: The hypotenuse of a right triangle is given by Pythagoras' theorem (even though the Babylonians already knew this relationship!):

$$\text{Hypotenuse} = \sqrt{a^2 + b^2}$$

where  $a$  and  $b$  are the two catheti. In this example:

$$\sqrt{\quad} (\quad) 8 \ x^2 \ + \ 1 \ 5 \ x^2 \ =$$

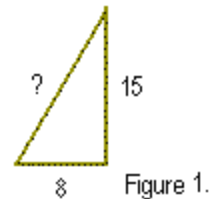


Figure 1.

Answer: 17. (The set of numbers 8, 15 and 17 is an example of a Pythagorean triple, i.e. integers that can be the sides of the same right triangle. Some of the simpler sets were already known by the ancient Egyptians.) Refer to the HP 10s learning module *Polar/Rectangular Coordinate Conversions* to learn a faster way of calculating the hypotenuse.

Example 2: Calculate  $0^0$

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Solution: Although some calculators return 1, on the HP 10s  $0^0$  is a Math ERROR condition because  $0^0$  is mathematically an indeterminate (or undetermined) form, much like  $0 / 0$  or  $\log 0$ . Press  $\text{ON}$  to remove the error indicator.

Example 3: Find the probability that at least one of 253 people has the same birthday as you.

Solution: The probability that the birthday of at least one of  $n$  persons is a particular day is given by:

$$1 - \left(\frac{354}{365}\right)^n$$

In this example  $n = 253$ :

$1 - \left(\frac{354}{365}\right)^{253}$

Answer: 0.500477154. The probability is greater than 50%. You can easily check that for  $n \leq 252$ , the probability is smaller than 50%, so 253 is the smallest number of persons such that the probability of at least one of them having the same birthday as you is greater than 50%!

Example 4: Find the geometric mean of the set of numbers { 2, 3.4, 3.41, 7, 11, 23 }

Solution: For a set of  $n$  positive numbers  $\{ a_1, a_2, \dots, a_n \}$  the geometric mean is defined as

$$G = (a_1 \cdot a_2 \cdot \dots \cdot a_n)^{\frac{1}{n}}$$

To find  $G$  press:

$2 \times 3 \cdot 4 \times 3 \cdot 4 \cdot 1 \times 7 \times 1 \cdot 1 \times 2 \cdot 3 = \sqrt[n]{\dots}$

Answer:  $G = 5.873725441$

Example 5: In a rather overoptimistic effort to break the speed of sound, high-flying pilot Ike Daedalus cranks open the throttle on his surplus Hawker Siddeley Harrier aircraft. From his instruments he reads a pressure altitude (PALT) of 25,500 feet with a calibration airspeed (CAS) of 350 knots. What is the flight mach number

$$M = \frac{\text{speed of aircraft}}{\text{speed of sound}}$$

if the following formula is applicable?

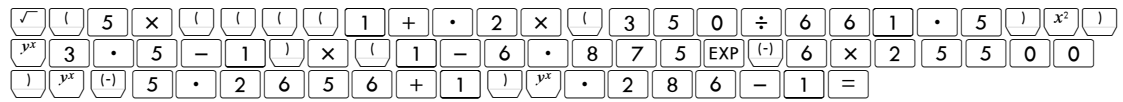
$$M = \sqrt{5 \left[ \left( \left( \left( 1 + 0.2 \left[ \frac{350}{661.5} \right]^2 \right)^{3.5} - 1 \right) \left[ 1 - (6.875 \times 10^{-6}) 25500 \right]^{-5.2656} \right) + 1 \right]^{0.286} - 1}$$

Solution: If the problem seems very complex and you are not sure that your HP 10s can handle it, evaluate the problem in parts, displaying intermediate results by pressing  $\text{=}$ . In this particular case, though, the HP 10s can evaluate this formula as it is stated from left to right, without having to press the  $\text{=}$  key at all:

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Answer: M = 0.835724535