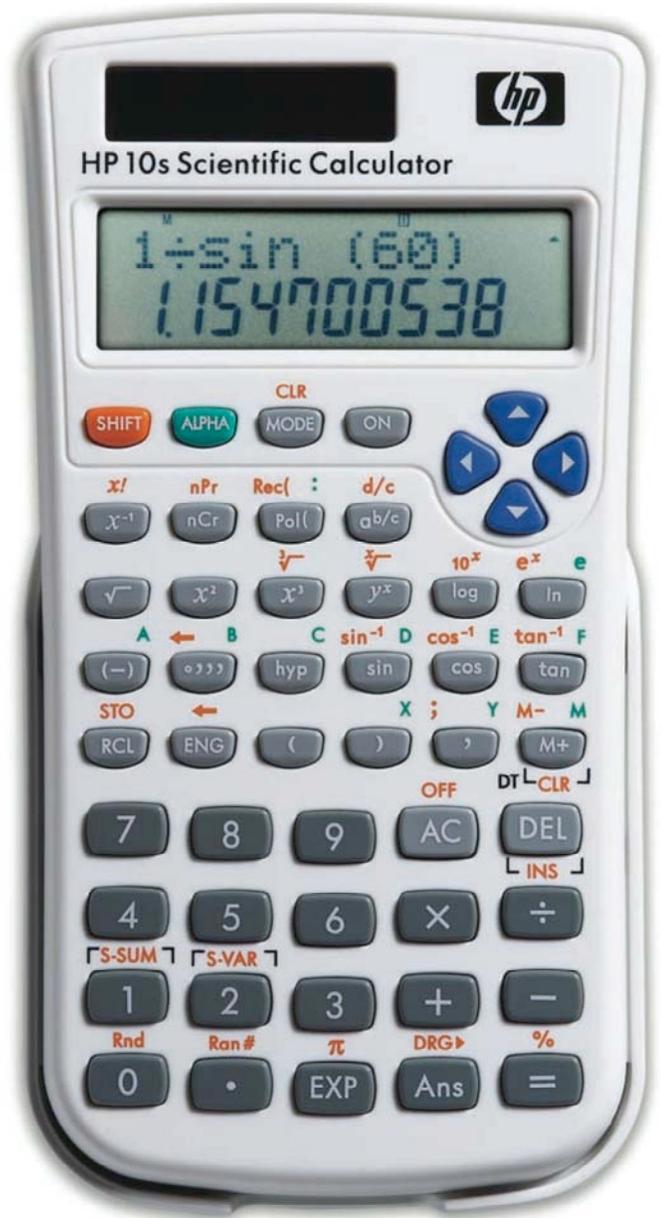**hp calculators**

HP 10s Hyperbolic Functions

Hyperbolic Functions

Practice Using Hyperbolic Functions

Hyperbolic functions

Trigonometric functions are often called "circular" functions, because the value for the cosine and sine of an angle lie on the unit circle defined by $X^2 + Y^2 = 1$ (points on the unit circle will have the X and Y coordinate of cosine(theta) and sine(theta), respectively). *Hyperbolic* trigonometric functions have a similar relationship, but with the hyperbola defined by the equation $X^2 - Y^2 = 1$.

Given a value for Z, the hyperbolic functions are defined by:

$$\sinh Z = \frac{e^Z - e^{-Z}}{2} \qquad \cosh Z = \frac{e^Z + e^{-Z}}{2} \qquad \tanh Z = \frac{\sinh Z}{\cosh Z}$$

Assume that Z is 3. The position on the unit hyperbola $X^2 - Y^2 = 1$ is defined by the point (cosh(Z),sinh(Z)), where cosh is the hyperbolic cosine and sinh is the hyperbolic sine. The value for sinh(3) is equal to 10.0179 and the value of cosh(3) is 10.0677. When $10.0677 \times 10.0677 - 10.0179 \times 10.0179$ is evaluated, the value is 1, so the point falls on the unit hyperbola. The hyperbolic tangent is defined as the hyperbolic sine divided by the hyperbolic cosine.

Hyperbolic functions are similar to circular trigonometric functions in many ways. In fact, if an identity is true for circular trigonometric functions, there is a corresponding identity in hyperbolic functions in which the sign of any product of two sines is changed (including tangent which is sine/cosine), this is known as *Osborne's rule*. For example,

cos(A+B) = cos A cos B – sin A sin B   gives   cosh(A+B) = cosh A cosh B + sinh A sinh B

and $\sin^2 A + \cos^2 A = 1$ gives $\cosh^2 A - \sinh^2 A = 1$

Another interesting property is that for all Z > 0, tanh Z < Z < sinhZ < cosh Z.

If x = sinh y, then y is defined to be the inverse hyperbolic sine (abbreviated as $\sinh^{-1}$ or arcsinh). The other inverse hyperbolic functions are similarly defined, but note that y = $\cosh^{-1}$ x is defined only for y $\geq$ 0.

Hyperbolic functions are useful in integration and have applications in many areas of engineering. For example, the shape formed by a wire freely hanging between two points (known as a *catenary* or *chainette* curve) is described by the hyperbolic cosine. Hyperbolic functions are also used in electrical engineering applications.

On the HP 10s hyperbolic functions are postfix functions and are accessed by pressing the [hyp] key and then the appropriate trigonometric key or inverse trigonometric key. Once [hyp] has been pressed, the HYP annunciator is displayed; to cancel this mode just press [hyp] again. Note that hyperbolic functions are *not* affected by the angle mode.

Practice using hyperbolic functions

Example 1:   Find the Guadermannian of 2. Set the angular mode to radians by pressing [MODE][MODE] 2 .

Solution: The Guadermannian function (gd) of Z is defined as $\tan^{-1}$ sinh Z. Press

   [SHIFT] [tan⁻¹] [hyp] [sin] 2 [=]

The result depends on the current angle unit because of the arctan function. The Guadermannian function is used for calculating hyperbolic functions from trigonometric functions since:

if y = gd x, then tan y = sinh x.

Answer: 1.301760336 assuming the angular mode is set to Radians.

Example 2: Find the hyperbolic cosecant and the inverse hyperbolic cosecant of 3.2

Solution: Inverse hyperbolic functions should not be mistaken for the reciprocal of the hyperbolic functions, which are sometimes referred to as:

$$\text{Hyperbolic cosecant} = \frac{1}{\sinh} \qquad \text{Hyperbolic secant} = \frac{1}{\cosh} \qquad \text{Hyperbolic cotangent} = \frac{1}{\tanh}$$

These functions also have their inverse functions, which can be written as:

$$\text{csch}^{-1} Z = \sinh^{-1} \frac{1}{Z} \qquad \sec h^{-1} Z = \cosh^{-1} \frac{1}{Z} \qquad \coth^{-1} Z = \tanh^{-1} \frac{1}{Z}$$

To find the hyperbolic cosecant of 3.2 press

$\boxed{\text{hyp}}\ \boxed{\text{sin}}\ \boxed{3}\ \boxed{\cdot}\ \boxed{2}\ \boxed{=}\ \boxed{x^{-1}}\ \boxed{=}$

and to find the inverse hyperbolic cosecant of 3.2 press

$\boxed{\text{hyp}}\ \boxed{\text{SHIFT}\ \text{sin}^{-1}}\ \boxed{3}\ \boxed{\cdot}\ \boxed{2}\ \boxed{x^{-1}}\ \boxed{=}$

Answer: The hyperbolic cosecant is 0.08166009 and the inverse hyperbolic cosecant is 0.307625067.

Example 3: A tram carries tourists between two peaks that are the same height and 437 meters apart. Before the tram latches onto the cable, the angle from the horizontal to the cable at its point of attachment is 63 degrees. How long does it take the tram to travel from one peak to the other, if the tram moves at 135 meters per minute?

Solution: The travel time in minutes is given by the following formula:

$$t = \frac{437 \times TAN(63°)}{135 \times ASINH(\tan(63°))}$$

Make sure DEG mode is active, and press:

$\boxed{4}\ \boxed{3}\ \boxed{7}\ \boxed{\times}\ \boxed{\text{tan}}\ \boxed{6}\ \boxed{3}\ \boxed{\div}\ \boxed{(}\ \boxed{1}\ \boxed{3}\ \boxed{5}\ \boxed{\times}\ \boxed{\text{hyp}}\ \boxed{\text{SHIFT}\ \text{sin}^{-1}}\ \boxed{\text{tan}}\ \boxed{6}\ \boxed{3}\ \boxed{=}$

Answer: 4.4552687994. The travel time between the peaks is just under four and one half minutes.

Example 4:   A cable is strung between two poles that are 40 feet apart, with the cable attached to each pole at a height of 30.436 feet above the level ground. At the midpoint between the poles, the cable is 18.63 feet above the level ground. What is the length of the cable required between the two poles?

Solution: The length of the cable is described by the formula below, where *a* is the lowest height of the cable and *D* is the distance between the two poles:

$$L = 2\ a\ SINH\frac{D/2}{a}$$

[2][×][1][8][·][6][3][×][hyp][sin][( ][4][0][÷][2][÷][1][8][·][6][3][=]

Answer: The length of the cable will be 48.14 feet.